

Mathematical Modelling of Steady-State Heat Transfer within a Glazed Flat-Plate PV-Thermal Air Collector

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Abstract: This paper presents a mathematical modelling method of steady-state heat transfer within a glazed flat-plate PV-Thermal collector (GFPVT-C) exposed to solar radiation and using air as a cooling fluid. The main objective of the mathematical modelling is to determine the best configuration of the PV-Thermal collector (such as the optimal thickness of the ventilation duct) and the proper properties of the cooling air (i.e. inlet temperature and flow speed) that allows for an effective reduction of the operating temperature of PV cells. Moreover, with the mathematical model developed, estimations are made of the temperature reduction of PV cells within the glazed PV-Thermal collector (compared to the operating temperature of PV cells inside standard solar PV collectors) and the resulting productivity gain. The results show that the GFPVT-C could make it possible to maintain the operating temperature of the PV cells at $25\text{ }^{\circ}\text{C} \pm 5\text{ }^{\circ}\text{C}$, even in hot ambient conditions, which should result in an increase in the PV cells productivity of more than 25 %.

Key words: PV-Thermal, heat transfer, steady-state, mathematical modelling.

Nomenclature

A	Collector area (m^2)
C	Specific heat of cooling air ($\text{J} \cdot \text{kg}^{-1} \cdot ^{\circ}\text{K}^{-1}$)
D	Hydraulic diameter or characteristic length (m)
e	Thickness (m)
F'	Thermal efficiency factor of the PV-Thermal collector
F''	Flow factor of the PV-Thermal collector
F_R	Heat removal factor of the PV-Thermal collector
G	Solar radiation ($\text{W} \cdot \text{m}^{-2}$)
g	Gravitational acceleration ($g = 9.81\text{ m/s}^2$)
h_c	Convection heat transfer coefficient ($\text{W} \cdot \text{m}^{-2} \cdot ^{\circ}\text{K}^{-1}$)
h_r	Radiation heat transfer coefficient ($\text{W} \cdot \text{m}^{-2} \cdot ^{\circ}\text{K}^{-1}$)
k	Conduction heat transfer coefficient ($\text{W} \cdot \text{m}^{-1} \cdot ^{\circ}\text{K}^{-1}$)
L	Length of the PV-Thermal collector
l	Width of the PV-Thermal collector
n	Number of fins
Nu	Nusselt number
$\dot{m}$	Mass flow rate of the cooling air (kg/s)
Pr	Prandtl number
Q_u	Heat flux transferred to the cooling air (W)
q_u	Heat flux density transferred to the cooling air ($\text{W} \cdot \text{m}^{-2}$)
Ra	Rayleigh number
Re	Reynolds number
S	Absorbed solar radiation ($\text{W} \cdot \text{m}^{-2}$)

T	Temperature ($^{\circ}\text{K}$)
U	Heat transfer coefficient ($\text{W} \cdot \text{m}^{-2} \cdot ^{\circ}\text{K}^{-1}$)
U_L	Overall heat loss coefficient to the ambient ($\text{W} \cdot \text{m}^{-2} \cdot ^{\circ}\text{K}^{-1}$)
u	Fin thickness (m)
V	Flow speed (m/s)
W	Fin spacing (m)

Greek Letters

α	Absorptance coefficient
β	Tilt angle of the solar PV-Thermal collector
δ	Reflectance coefficient
ε	Emittance coefficient
η	Efficiency of PV cells
θ	Coefficient of thermal expansion ($^{\circ}\text{K}^{-1}$)
λ_{η}	PV cells efficiency correction coefficient for temperature ($^{\circ}\text{C}^{-1}$)
μ	Dynamic viscosity ($\text{Pa} \cdot \text{s}$)
ν	Kinematic viscosity (m^2/s)
ρ	Mass density (kg/m^3)
σ	Stefan-Boltzmann constant ($\sigma = 5,6697 \cdot 10^{-8}\text{ W} \cdot \text{m}^{-2} \cdot ^{\circ}\text{K}^{-4}$)
τ	Transmittance coefficient
τ^*	Thermal radiation transmittance coefficient
φ	Thermal diffusivity (m^2/s)

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Subscripts

a	Ambient
b	Bottom of ventilation duct
c	Cover (glass)
f	Cooling air at the mean temperature
fi	Cooling air at the inlet temperature
i	Thermal insulation
l	Fin
p	PV-Thermal absorber plate
s	Sky
w	Wind

1. Introduction

The use of standard solar PV collectors (PV-C) in regions with a hot climate for electricity production is facing issues such as a decrease in energy efficiency and a less durability of the solar cells. Indeed, most solar PV collectors are negatively affected by the heat which lessens the efficiency of the solar cells and shortens their lifetime [1-3].

These negative effects of the heat on solar PV panels are some of the reasons that leded, for the past few years, many study and design projects on hybrid solar PV-Thermal collectors [4-7]. The main objectives of most of those projects on solar PV-Thermal panels

were to evacuated the useless heat, to maintain the operating temperature of the PV cells at a quite low level and to increase the electrical energy produced by the photovoltaic cells.

This work aims at first to do the mathematical modelling of the steady-state heat transfer within a glazed flat-plate PV-Thermal air collector. Next, with the mathematical model developed, the operating temperature of the PV cells inside the flat-plate PV-Thermal collector is estimated. Finally, the best configuration of the PV-Thermal collector (optimal spacing between the glass and the PV-Thermal absorber plate as well as the appropriate thickness of the ventilation duct) and the proper properties of the cooling air that will allow for an optimal cooling of the PV cells are assessed.

2. Mathematical Modelling Method

2.1 Description of the PV-Thermal Collector

The basic parts of the flat-plate PV-Thermal collector as well as the several heat transfer mechanisms occurring within the collector are schematically represented in Fig. 1.

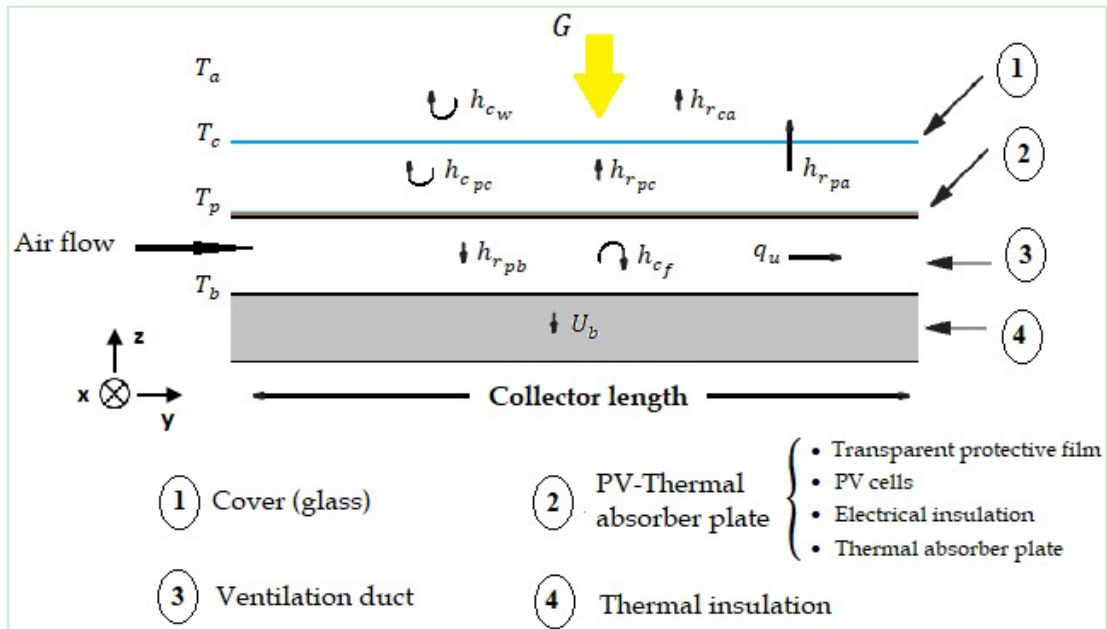


Fig. 1 2D scheme representing the different parts of the glazed flat-plate PV-Thermal collector and the several heat transfer mechanisms.

2.2 Modelling Assumptions

The mathematical modelling of steady-state heat transfer within the glazed flat-plate PV-Thermal collector is simplified by the following assumptions:

- The temperature of the glass, that of the PV-Thermal absorber plate and that of the bottom of the ventilation duct are supposed uniform and named T_c , T_p and T_b respectively.
- The heat transfer mechanisms between the different parts of the glazed flat-plate PV-Thermal collector and through the thermal insulation are one-dimensional (i.e. along the z direction).
- The sky is considered as a blackbody at the temperature T_s , in regard to thermal radiation.
- Heat transfers through the top and bottom of the GFPVT-C are to the same ambient temperature.

2.3 Heat Transfer Coefficients

The expressions of the different heat transfer coefficients within the glazed flat-plate PV-Thermal collector are given as follows:

2.3.1 Coefficient of Radiation $h_{r_{pc}}$ between the PV-Thermal Absorber Plate and the Glass

It is deduced from the equation of Stefan-Boltzmann on radiation heat transfer between 2 parallel surfaces [8]:

$$h_{r_{pc}} = \frac{(1 - \tau_c^*)}{1 - \delta_p \delta_c} \times \frac{\sigma(T_p^2 + T_c^2)(T_p + T_c)}{\frac{1}{\varepsilon_p} + \frac{1}{\varepsilon_c} - 1} \quad (1)$$

2.3.2 Coefficient of Natural Convection $h_{c_{pc}}$ between the PV-Thermal Absorber Plate and the Glass

The coefficient $h_{c_{pc}}$ is calculated using Eq. (2) by estimating first the Nusselt number for natural convection between 2 parallel plates, which is given by Eq. (3), established by Hollands et al. [9]:

$$h = \frac{Nu}{D} k \quad (2)$$

$$Nu = 1 + 1,44 \left[1 - \frac{1708(\sin(1,8\beta))^{5/3}}{Ra \cdot \cos(\beta)} \right] \times$$

$$\left[1 - \frac{1708}{Ra \cdot \cos(\beta)} \right]^+ + \left[\left(\frac{Ra \cdot \cos(\beta)}{5830} \right)^{1/3} - 1 \right]^+ \quad (3)$$

The hydraulic diameter or the characteristic length in this case is equal to the spacing between the glass and the PV-Thermal absorber plate ($D = e_{pc}$).

The Rayleigh number Ra is calculated as follows:

$$Ra = \frac{g\theta\Delta T e_{pc}^3}{\nu\varphi} \quad (4)$$

where $\theta = 1/T$ ($\text{in}^\circ\text{K}^{-1}$) and $\Delta T = (T_p - T_c)$ (in°K)

2.3.3 Coefficient of Radiation $h_{r_{ca}}$ between the Glass and the Sky

It is deduced from the equation of Stefan-Boltzmann on radiation heat transfer between a surface and the sky:

$$h_{r_{ca}} = \frac{\sigma\varepsilon_c(T_c^4 - T_s^4)}{(T_c - T_a)} \quad (5)$$

The sky temperature T_s is estimated from the ambient temperature T_a using Swinbank equation [10]:

$$T_s = 0,0552T_a^{1,5} \quad (6)$$

2.3.4 Coefficient of Wind Convection h_{c_w} between the Glass and the Ambient

The coefficient h_{c_w} is calculated using Eq. (2) by estimating first the Nusselt number for wind convection over a plane surface given by Eq. (7), which is established by Sparrow et al. [8, 11]:

$$Nu = 0,86Re^{1/2}Pr^{1/3} \quad (7)$$

Re is the Reynolds number and Pr , the Prandtl number, both given as follows:

$$Re = \rho V D / \mu \quad (8)$$

$$Pr = \nu / \varphi \quad (9)$$

The characteristic length D in this case is equal to four times the collector area A divided by the collector perimeter P ($D = 4A/P$).

2.3.5 Coefficient of Radiation $h_{r_{pa}}$ between the PV-Thermal Absorber Plate and the Sky

It is deduced from the equation of Stefan-Boltzmann on radiation heat transfer between 2 parallel surfaces:

$$h_{r_{pa}} = \frac{\tau_c^* \sigma \varepsilon_p (T_p^4 - T_s^4)}{(T_p - T_a)} \quad (10)$$

2.3.6 Forced Convection Coefficient h_{c_f} for the Cooling Air

The coefficient h_{cf} is calculated using Eq. (2) by estimating first the Nusselt number for air forced convection between parallel plates given by Eq. (11), which is derived from the data of Kays and Crawford [8, 12]:

$$Nu = 0,0158Re^{0,8} \quad (11)$$

The characteristic length or the hydraulic diameter in this case is equal to twice the thickness of the ventilation duct ($D = 2e_{pb}$).

2.3.7 Coefficient of Radiation $h_{r_{pb}}$ between the PV-Thermal Absorber Plate and the Bottom of the Ventilation Duct

It is deduced from the equation of Stefan-Boltzmann on thermal radiation between 2 parallel surfaces [8]:

$$h_{r_{pb}} = \frac{4\sigma\bar{T}^3}{\left(\frac{1}{\varepsilon_p} + \frac{1}{\varepsilon_b} - 1\right)} \quad (12)$$

where $\bar{T}$ is the mean temperature between the two surfaces so that:

$$\bar{T}^3 = 1/4 (T_p^2 + T_b^2)(T_p + T_b) \quad (13)$$

2.3.8 Coefficient of Thermal Conduction U_b through the Thermal Insulation

It is calculated considering the thermal conductivity k_i and the thickness e_i of the thermal insulation:

$$U_b = \frac{k_i}{e_i} \quad (14)$$

2.4 Modelling Equations

The mathematical modelling of the steady-state heat transfer within the glazed flat-plate PV-Thermal collector is performed using the basic steady-state energy balance equation for flat-plate solar collectors [8] which indicates how the incident solar radiation reaching the top area of the collector is distributed between the heat transferred to the cooling air on the one hand, and the optical and thermal losses on the other hand:

$$Q_u = A[S - U_L(T_p - T_a)] \quad (15)$$

The solar radiation absorbed by the flat-plate PV-Thermal collector S is related to the transmittance-

absorptance product ($\tau\alpha$) of the collector and to the efficiency of the PV cells η , both given as follows [8]:

$$(\tau\alpha) = \frac{\tau_c\alpha_p}{1 - \delta_p\delta_c} \quad (16)$$

$$\eta = \eta_0[1 - \lambda_\eta(T_p - T_0)] \quad (17)$$

where T_0 is the reference temperature (25 °C) and η_0 the PV cells conversion efficiency at the reference temperature T_0 .

Eq. (15) can be rewritten as a function of the cooling air mean temperature T_f or as a function of the cooling air inlet temperature T_{fi} :

$$Q_u = AF'[S - U_L(T_f - T_a)] \quad (18)$$

$$Q_u = AF_R[S - U_L(T_{fi} - T_a)] \quad (19)$$

In Eq. (18), F' is the thermal efficiency factor and U_L the overall heat loss coefficient to the ambient; the procedure for deriving these two parameters will be explained later.

In Eq. (19), the heat removal factor F_R is calculated using the following expression [8]:

$$F_R = \frac{\dot{m}C}{AU_L} \left[1 - \exp\left(-\frac{AU_L F'}{\dot{m}C}\right) \right] \quad (20)$$

Finally, the equations used for the calculation of the cooling air mean temperature T_f and the PV-Thermal absorber plate mean temperature T_p are given as follows:

$$T_f = T_{fi} + \frac{Q_u/A}{F_R U_L} (1 - F'') \quad (21)$$

$$T_p = T_{fi} + \frac{Q_u/A}{F_R U_L} (1 - F_R) \quad (22)$$

In Eq. (21), F'' is the flow factor, which is defined as the ratio of the heat removal factor F_R to the thermal efficiency factor F' :

$$F'' = \frac{F_R}{F'} = \frac{\dot{m}C}{AU_L F'} \left[1 - \exp\left(-\frac{AU_L F'}{\dot{m}C}\right) \right] \quad (23)$$

2.5 Modelling Procedure

2.5.1 PV-Thermal Collector without Fins

The mathematical modelling of the steady-state heat transfer within the flat-plate PV-Thermal collector begins with the writing of the steady-state energy balance equation for four different parts of the collector (i.e. the glass, the PV-Thermal absorber plate, the

cooling air and the bottom of the ventilation duct).

A system of 4 linear equations is obtained which is reported in Table 1 and where:

$$\begin{aligned} U_{ca} &= (h_{cw} + h_{r_{ca}}) \\ U_{pc} &= (h_{c_{pc}} + h_{r_{pc}}) \\ q_u &= Q_u/A \end{aligned}$$

In a second step, by analytical solving or hand solving, the heat transferred to the cooling air per unit of collector area q_u is expressed as a function of T_f and T_a only. An equation of the following form is obtained:

$$q_u = X_1[X_2 - X_3(T_f - T_a)]$$

Then, by matching the resulting equation with Eq. (18), the parameters F' , S and U_L are derived:

$$\begin{aligned} X_1 = F' &= \frac{h_{cf} \left(1 + \frac{h_{r_{pb}}}{U_0}\right)}{\left(U_2 - \frac{U_{pc}^2}{U_{pc} + U_{ca}} - \frac{h_{r_{pb}}^2}{U_0}\right)} \\ X_2 = S &= (\tau\alpha)G(1 - \eta) + \frac{\alpha_c G U_{pc}}{U_{pc} + U_{ca}} \\ X_3 = U_L &= \left(U_1 - \frac{U_{pc}^2}{U_{pc} + U_{ca}} + \frac{h_{r_{pb}} U_b}{U_0}\right) + \\ &\quad \frac{U_b \left(U_2 - \frac{U_{pc}^2}{U_{pc} + U_{ca}} - \frac{h_{r_{pb}}^2}{U_0}\right)}{U_0 \left(1 + \frac{h_{r_{pb}}}{U_0}\right)} \end{aligned}$$

where:

$$\begin{aligned} U_0 &= (h_{cf} + h_{r_{pb}} + U_b) \\ U_1 &= (U_{pc} + h_{r_{pa}}) \\ U_2 &= (U_{pc} + h_{r_{pa}} + h_{cf} + h_{r_{pb}}) \end{aligned}$$

Finally, the parameters F_R and F'' are estimated using Eq. (20) and Eq. (23) respectively and the mean temperatures T_f and T_p calculated using Eq. (21) and

Eq. (22) respectively.

2.5.2 PV-Thermal Collector with Fins

To intensify the heat exchange between the cooling air and the PV-Thermal absorber plate, longitudinal fins of thickness u and height e_{pb} , and separated from each other by a distance W are added to the ventilation duct (Fig. 2).

Regarding heat transfer between the PV-Thermal absorber plate and the fins, only the thermal conduction is considered. Thermal radiation is only considered in heat transfer between the PV-Thermal absorber plate and the bottom of the ventilation duct.

The mathematical modelling of the steady-state heat transfer within the flat-plate PV-Thermal collector with fins begins with the solving of the steady-state energy balance equation for a single fin of length L , thickness u and height dz , which is written as follows:

$$\underbrace{-k_l(Lu) \frac{\partial T_l}{\partial z} \Big|_z}_{q_{lf} \text{ at } z} = \underbrace{-k_l(Lu) \frac{\partial T_l}{\partial z} \Big|_{z+dz}}_{q_{lf} \text{ at } z+dz} + \underbrace{2h_{cf}(Ldz)(T_l - T_f)}_{q_{lf}} \quad (24)$$

The necessary boundaries conditions required for the solution of Eq. (24) are given as follows:

$$T_l|_{z=0} = T_p \text{ and } T_l|_{z=e_{pb}} = T_b$$

After solving Eq. (24), the expression for the temperature distribution T_l along the height of the fins is obtained, which takes the following form:

$$T_l(z) = T_f + \left[\frac{(T_b - T_f)}{\sinh(me_{pb})} - \frac{(T_p - T_f)}{\tanh(me_{pb})} \right] \times \sinh(mz) + (T_p - T_f) \cosh(mz) \quad (25)$$

Table 1 System of linear equations for steady-state heat transfer within the PV-Thermal collector without fins.

Steady-state energy balance equation for the cover
$\alpha_c G + U_{pc}(T_p - T_c) = U_{ca}(T_c - T_a)$
Steady-state energy balance equation for the absorber plate
$(\tau\alpha)G(1 - \eta) = U_{pc}(T_p - T_c) + h_{r_{pa}}(T_p - T_a) + h_{cf}(T_p - T_f) + h_{r_{pb}}(T_p - T_b)$
Steady-state energy balance equation for the cooling air
$h_{cf}(T_p - T_f) = h_{cf}(T_f - T_b) + q_u$
Steady-state energy balance equation for the bottom of ventilation duct
$h_{cf}(T_f - T_b) + h_{r_{pb}}(T_p - T_b) = U_b(T_b - T_a)$

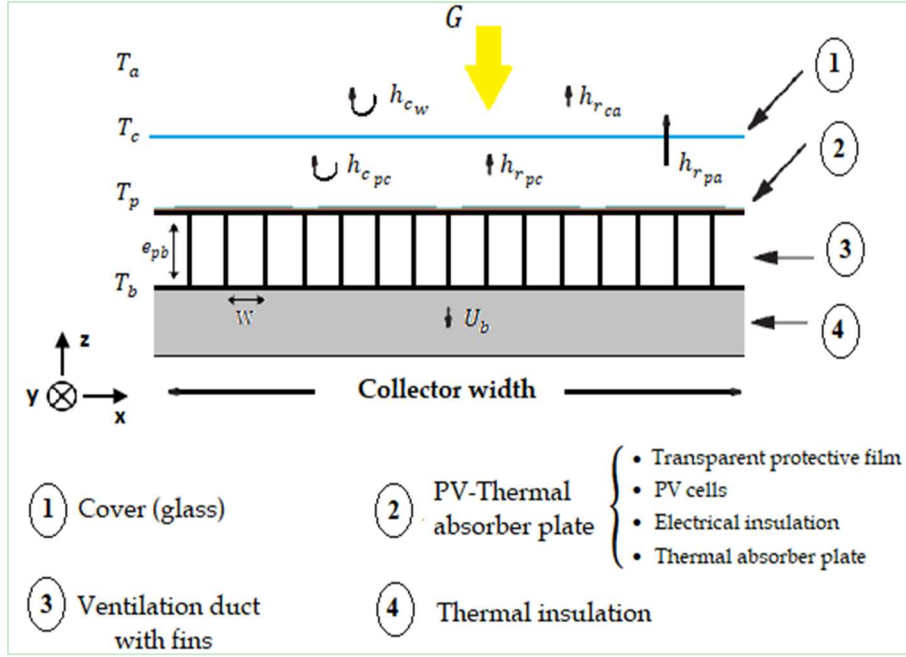


Fig. 2 2D scheme representing the glazed flat-plate PV-Thermal collector with fins.

where $m = 2h_{cf}/k_l u$ and $z \in [0 \ e_{pb}]$

Using Eq. (25), the different heat flux densities exchanged by the set of $n = l/W$ fins are calculated, which expressions are as follows:

$$nq_l|_{z=0} = -r_0[r_5(T_b - T_f) - r_6(T_p - T_f)] \quad (26)$$

$$nq_l|_{z=e_{pb}} = -r_0[r_3(T_p - T_f) + r_4(T_b - T_f)] \quad (27)$$

$$nq_{lf} = r_0[r_1(T_p - T_f) + r_2(T_b - T_f)] \quad (28)$$

The expressions for the several coefficients $r_0, r_1, r_2, r_3, r_4, r_5$ and r_6 are given in Table 2.

Moreover, the heat exchange area A' between the PV-Thermal absorber plate and the cooling air or between the bottom of the ventilation duct and the cooling air is reduced by the area occupied by the fins:

$$A' = L(l - nu) = A \left(1 - \frac{u}{W}\right) = Ax_0 \quad (29)$$

Taking these aspects into account when writing the steady-state energy balance equation for the four parts of the glazed flat-plate PV-Thermal collector with fins, a new system of 4 linear equations is obtained which is reported in Table 3.

Afterwards, by analytical solving, the heat transferred to the cooling air per unit of collector area q_u is expressed as a function of T_f and T_a only. An

equation of the following form is obtained:

$$q_u = X'_1[X'_2 - X'_3(T_f - T_a)]$$

Then, by matching the obtained equation with Eq. (18), the parameters F' , S and U_L are derived:

$$X'_1 = F' = \left(\frac{K_0}{K_1}\right)$$

$$X'_2 = S = (\tau\alpha)G(1 - \eta) + \frac{\alpha_c G U_{pc}}{U_{pc} + U_{ca}}$$

$$X'_3 = U_L = \left(K_2 + \frac{K_1 K_3}{K_0}\right)$$

Table 2 Coefficients $r_0, r_1, r_2, r_3, r_4, r_5$ and r_6 .

$r_0 = \left(\frac{k_l m u}{W}\right)$
$r_1 = \left[\sinh(me_{pb}) - \frac{\cosh(me_{pb}) - 1}{\tanh(me_{pb})} \right]$
$r_2 = \left[\frac{\cosh(me_{pb}) - 1}{\sinh(me_{pb})} \right]$
$r_3 = \left[\sinh(me_{pb}) - \frac{\cosh(me_{pb})}{\tanh(me_{pb})} \right]$
$r_4 = \left[\frac{\cosh(me_{pb})}{\sinh(me_{pb})} \right]$
$r_5 = \left[\frac{1}{\sinh(me_{pb})} \right]$
$r_6 = \left[\frac{1}{\tanh(me_{pb})} \right]$

Table 3 System of linear equations for steady-state heat transfer within the PV-Thermal collector with fins.

Equation for the cover
$\alpha_c G + U_{pc}(T_p - T_c) - U_{ca}(T_c - T_a) = 0$
Equation for the absorber plate
$(\tau\alpha)G(1 - \eta) - U_{pc}(T_p - T_c) - h_{r_{pa}}(T_p - T_a) - h_{cf}x_0(T_p - T_f) - h_{r_{pb}}x_0(T_p - T_b) - nq_l _{z=0} = 0$
Equation for the cooling air
$h_{cf}x_0(T_p - T_f) + nq_{lf} - h_{cf}x_0(T_f - T_b) - q_u = 0$
Equation for the bottom of ventilation duct
$h_{cf}x_0(T_f - T_b) + h_{r_{pb}}x_0(T_p - T_b) + nq_l _{z=e_{pb}} - U_b(T_b - T_a) = 0$

The expressions for the several coefficients K_0, K_1, K_2 and K_3 are reported in Table 4.

Finally, the parameters F_R and F'' are estimated using Eq. (20) and Eq. (23) respectively and the mean temperatures T_f and T_p calculated using Eq. (21) and Eq. (22) respectively.

2.6 Calculation Algorithm

Since the calculation of the temperatures T_c , T_p

and T_f of the glass, the PV-Thermal absorber plate and cooling air respectively is implicit (because some of the parameters needed for the calculation of the temperatures T_c , T_p and T_f such as thermal radiation coefficients are determined using these temperatures), an iterative solution method is employed.

The diagram in Fig. 3 briefly summarizes the main steps of the calculation algorithm.

Table 4 Coefficients K_0, K_1, K_2 and K_3 .

$K_0 = \left((h_{cf}x_0 + r_0r_1) - \frac{(h_{cf}x_0 + r_0r_2)(r_0r_3 - h_{r_{pb}}x_0)}{(h_{cf}x_0 + r_0r_4 + h_{r_{pb}}x_0 + U_b)} \right)$
$K_1 = (U_{pc} + h_{r_{pa}} + h_{cf}x_0 + r_0r_6 + h_{r_{pb}}x_0) - \frac{U_{pc}^2}{U_{ca}} + \frac{(r_0r_5 + h_{r_{pb}}x_0)(r_0r_3 - h_{r_{pb}}x_0)}{(h_{cf}x_0 + r_0r_4 + h_{r_{pb}}x_0 + U_b)}$
$K_2 = \left(U_{pc} + h_{r_{pa}} - \frac{U_{pc}^2}{U_{ca}} + \frac{(r_0r_5 + h_{r_{pb}}x_0)U_b}{(h_{cf}x_0 + r_0r_4 + h_{r_{pb}}x_0 + U_b)} \right)$
$K_3 = \left(\frac{(h_{cf}x_0 + r_0r_2)U_b}{(h_{cf}x_0 + r_0r_4 + h_{r_{pb}}x_0 + U_b)} \right)$

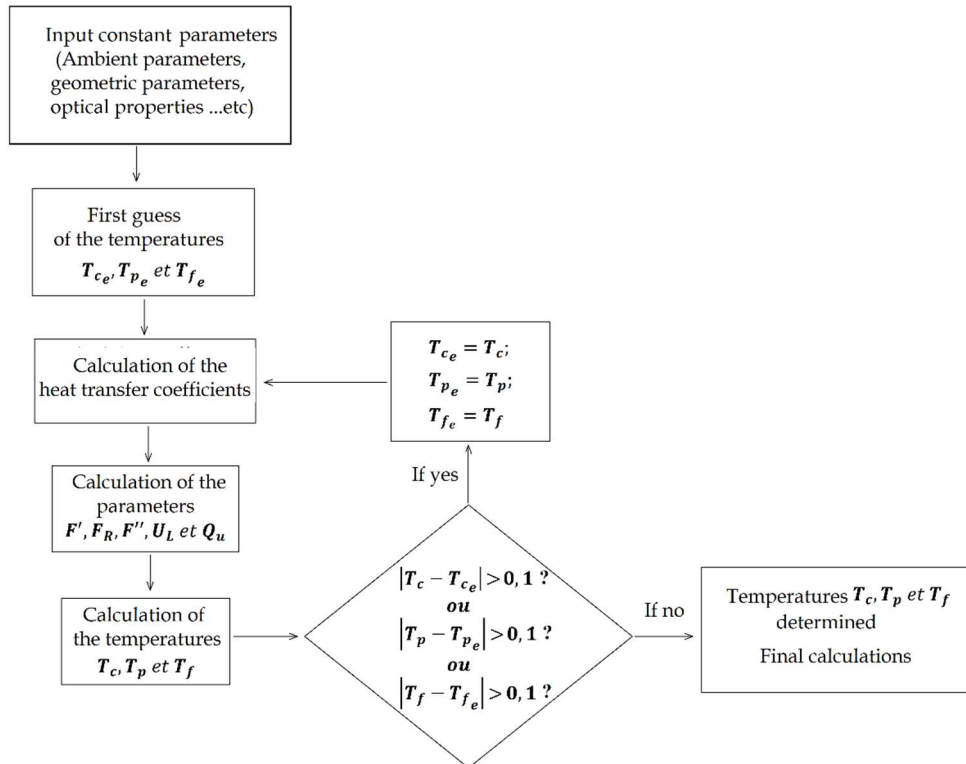


Fig. 3 Main steps of the algorithm for the calculation of the temperatures T_c , T_p and T_f .

2.7 Numerical Parameters

The different parameters considered in the calculations are given as follows:

- Ambient parameters:

Conditions of NTE (Nominal Terrestrial Environment):

$$G = 800 \text{ W/m}^2; T_a = 20 \text{ }^\circ\text{C}; V_a = 1 \text{ m/s}$$

Standard environmental conditions:

$$G = 1000 \text{ W/m}^2; T_a = 25 \text{ }^\circ\text{C}; V_a = 1 \text{ m/s}$$

Extreme environmental conditions:

$$G = 1000 \text{ W/m}^2; T_a = 40 \text{ }^\circ\text{C}; V_a = 1 \text{ m/s}$$

- Geometric parameters of the glazed flat-plate PV-Thermal collector:

Size: 1,400 x 600 x 80 mm.

Spacing between glass and PV-Thermal absorber plate: $e_{pc} = 15 \text{ mm}$.

Ventilation duct thickness: $e_{pb} = 20 \text{ mm}$.

- Optical properties of the glazed flat-plate PV-Thermal collector:

$$\alpha_c = 0.05; \tau_c = 0.85; \delta_c = 0.1; \varepsilon_c = 0.88; \tau_c^* = 0.05;$$

$$\alpha_p = 0.9; \tau_p = 0.0; \delta_p = 0.1; \varepsilon_p = \varepsilon_b = 0.8$$

- Cooling air properties:

$$T_{fi} = 10 \text{ }^\circ\text{C}; V_f = 7 \text{ m/s}$$

- Thermal insulation properties:

$$k_i = 0.045 \text{ W/(m}\cdot^\circ\text{C)}; e_i = 20 \text{ mm}$$

- PV cells properties:

$$\eta_0 = 12 \text{ } \%; \lambda_\eta = -0.45 \text{ } \%/^\circ\text{C}$$

3. Results and Discussion

3.1 Influence of the Ambient Conditions on the PV-Thermal Absorber Plate Temperature T_p

Using the developed mathematical model of steady-state heat transfer within the flat-plate PV-Thermal collector, the temperature T_p of the PV-Thermal absorber plate (and thus of the PV cells) is calculated for various ambient conditions: for nominal terrestrial environment conditions (NET conditions), for standard environmental conditions and for extreme environmental conditions. The estimations of the temperature T_p are

then compared to the operating temperature T_{PV} of the same type PV cells within standard PV collectors subjected to the same ambient conditions (Fig. 4). The temperature T_{PV} is calculated using the Nominal Operating Cell Temperature model or NOCT model (Eq. (30)) which estimates the operating temperature of standard PV collectors under outdoor conditions:

$$T_{PV} = T_a + (T_{NOCT} - 20) \left(\frac{G}{800} \right) \quad (30)$$

It is noted that the operating temperature of PV cells within standard PV collectors under NTE conditions, named T_{NOCT} is approximately $47 \text{ }^\circ\text{C}$.

The operating temperature of PV cells can be as high as $58 \text{ }^\circ\text{C}$ under standard ambient conditions and exceed $70 \text{ }^\circ\text{C}$ under extreme ambient conditions.

But with the glazed flat-plate PV-Thermal collector, it is observed that the operating temperature T_p of the PV cells varies very little depending on the ambient conditions. In addition, for the appropriate cooling air properties (i.e. inlet temperature $T_{fi} = 10 \text{ }^\circ\text{C}$ and flow speed $V_f = 7 \text{ m/s}$), the operating temperature of the PV cells can be maintained, regardless of the ambient conditions, at $25 \text{ }^\circ\text{C} \pm 5 \text{ }^\circ\text{C}$, which is the reference PV cell temperature for standard test conditions.

Fig. 5 shows the corresponding PV cells conversion efficiency for the different operating temperature T_p and T_{PV} calculated. The considered PV cells have an efficiency of 12% at $25 \text{ }^\circ\text{C}$ and an efficiency correction coefficient for temperature λ_η of $-0.45 \text{ } \%/^\circ\text{C}$, which are literature average values for crystalline silicon [13].

It is estimated that for the chosen cooling air properties, the flat-plate PV-Thermal collector makes it possible to increase by more than 2%, the PV cells efficiency under hot ambient conditions, compared to standard PV collectors.

Fig. 6 shows the impact of reducing the operating cell temperature on the peak power supplied by the PV cells. It compares the peak power for two 20 ha PV power plants, one using standard PV collectors and the other flat-plate PV-Thermal collectors. Both kind of collectors are made of the same type of PV cells.

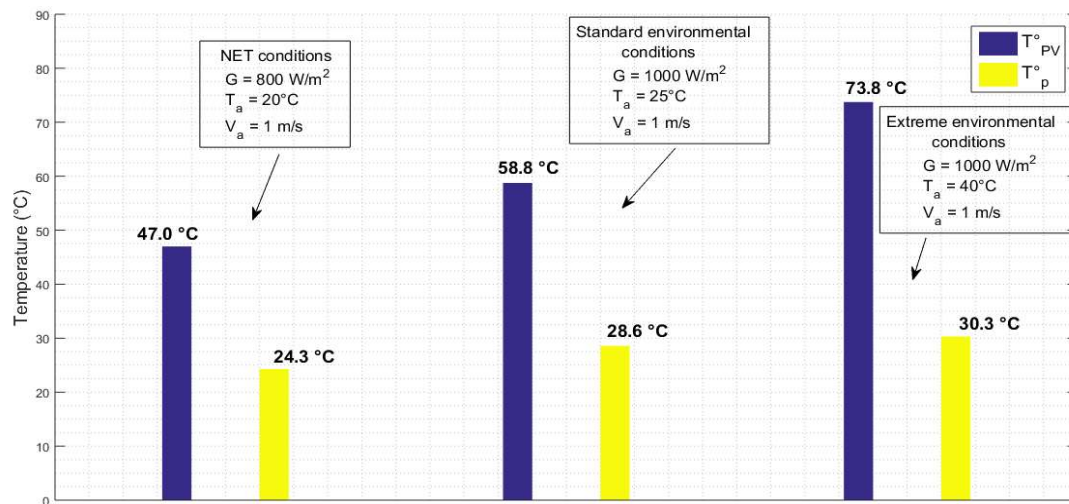


Fig. 4 Comparison of temperatures T_{PV} and T_p for various ambient conditions.

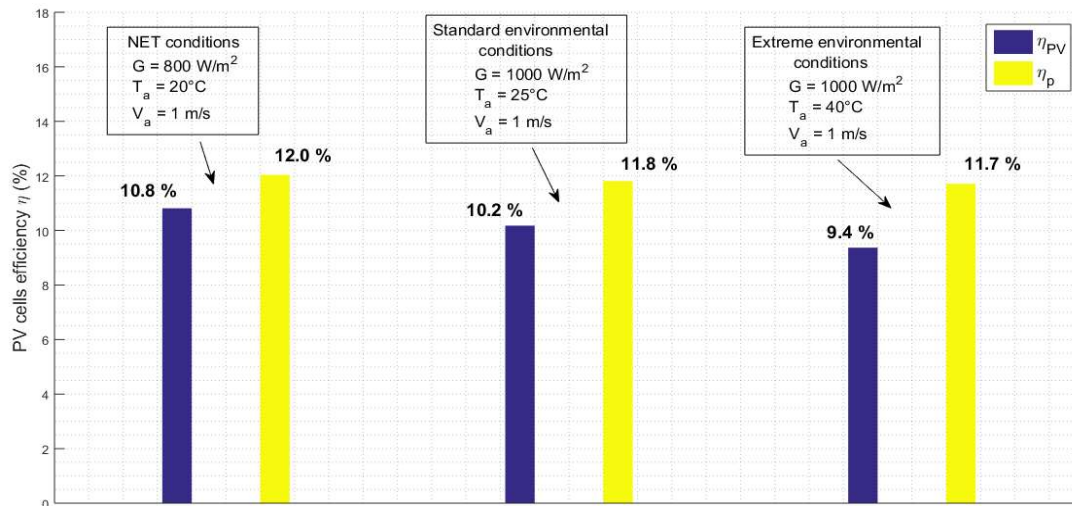


Fig. 5 Comparison of the efficiencies η_{PV} and η_p for various ambient conditions.

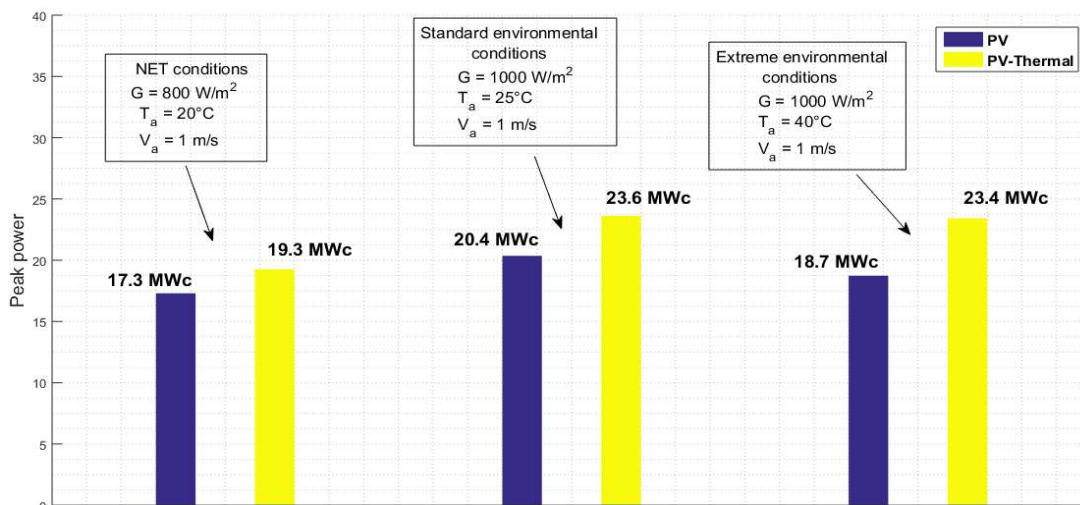


Fig. 6 Peak power for two 20-ha PV power plants: one using standard PV collectors and the other flat-plate PV-Thermal collectors.

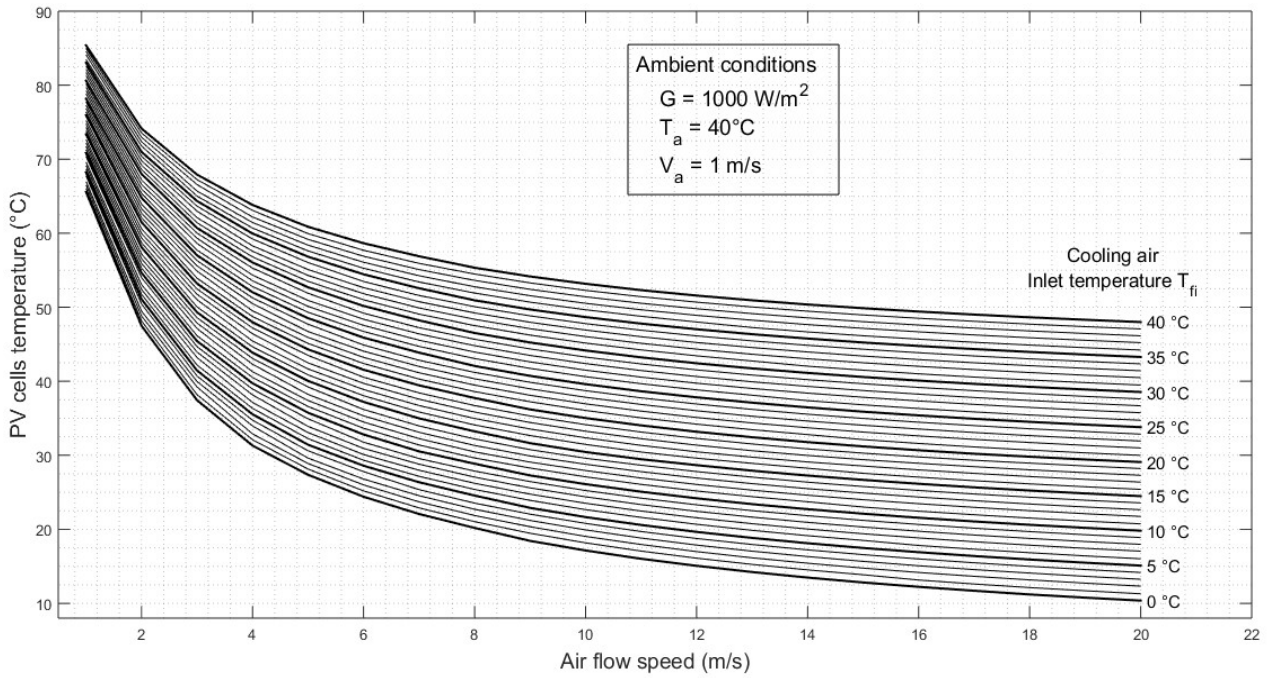


Fig. 7 Diagram estimating the temperature T_p as a function of the inlet temperature and flow speed of the cooling air.

It is estimated that for the chosen cooling air properties, the use of flat-plate PV-Thermal collectors increases by more than 25%, the peak power under extreme ambient conditions.

3.2 Influence of the Cooling Air Properties on the PV-Thermal Absorber Plate Temperature T_p

Fig. 7 represents a diagram estimating the operating temperature of PV cells within the glazed flat-plate PV-Thermal collector based on the inlet temperature T_{fi} and the flow speed V_f of the cooling air.

It is noticed that the cooling of the PV-Thermal absorber plate is best at moderate to high flow speeds (i.e. $V_f \geq 7 \text{ m/s}$ (25.2 km/h)). For low air flow velocities, the forced convection coefficient h_{cf} is small and the heat transfer between the cooling air and the PV-Thermal absorber plate inefficient.

It is also noted that the inlet temperature of the cooling air is a key parameter in reducing the temperature of the PV-Thermal absorber plate. The higher the cooling air inlet temperature, the greater the required air flow speed to reduce significantly the operating temperature of PV cells.

3.3 Influence of the Ventilation Duct Thickness e_{pb} on the PV-Thermal Absorber Plate Temperature T_p

Fig. 8 shows how the thickness of the ventilation duct e_{pb} affects the cooling of the PV-Thermal absorber plate. To assess the impact of the thickness e_{pb} on the temperature T_p , the cooling air flow speed is set to $V_f = 7 \text{ m/s}$; then the air mass flow rate $\dot{m}_f$ and the temperature of the PV-Thermal absorber plate are calculated for various values of the ventilation duct thickness e_{pb} .

It is observed, as expected, that the air mass flow rate $\dot{m}_f$ increases in proportion to the thickness of the ventilation duct e_{pb} . But for the temperature of the PV-Thermal absorber plate T_p , it is noted that there is an optimal thickness of the ventilation duct (between approximately 15 mm and 20 mm) at which the cooling of the PV-Thermal absorber plate is better (i.e. the forced convection coefficient h_{cf} is at its maximum and the temperature T_p is at its minimum). Increasing the ventilation duct thickness beyond that optimal value doesn't reduce further the temperature of the PV-Thermal absorber plate, even if the air mass flow rate is greater.

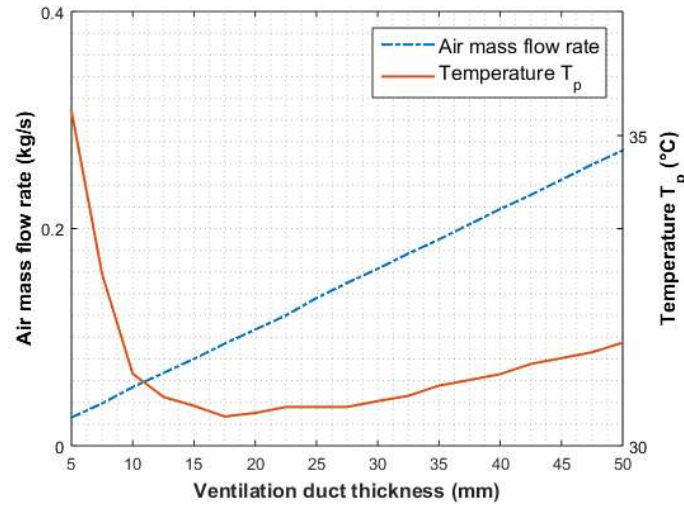


Fig. 8 Air mass flow rate $\dot{m}_f$ and temperature T_p as a function of the ventilation duct thickness e_{ph} .

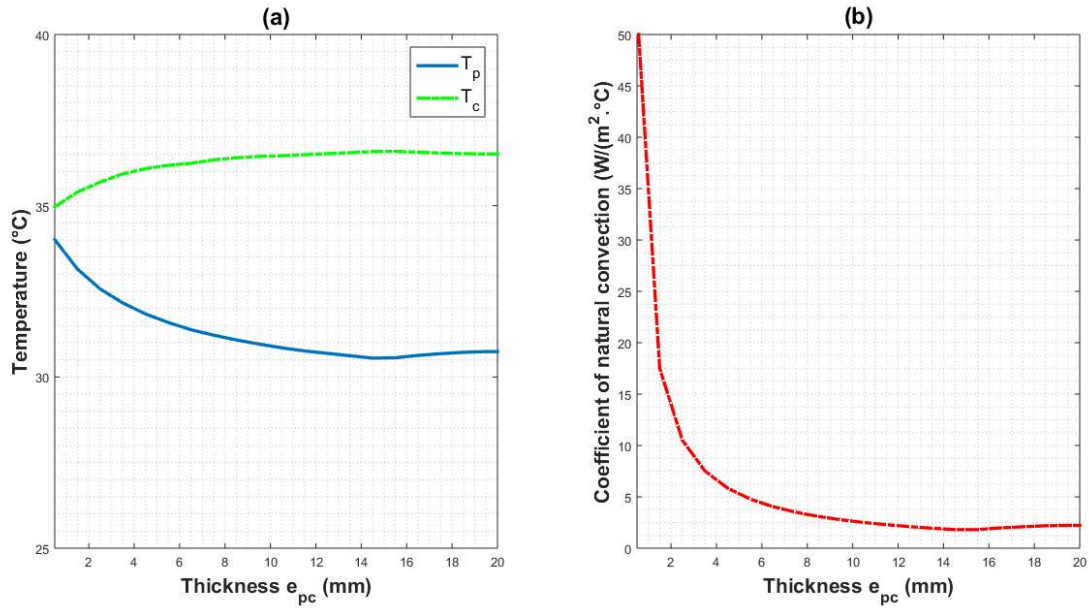


Fig. 9 Temperatures T_c and T_p and coefficient of natural convection $h_{c_{pc}}$ as a function of the thickness e_{pc} .

Fig. 9(a) shows how the spacing e_{pc} between the glass (cover) and the PV-Thermal absorber plate affects the temperature T_c of the glass and T_p of PV cells while Fig. 9(b) exhibits the impact of e_{pc} on the coefficient of natural convection $h_{c_{pc}}$ of the air layer contained within that space.

It is observed that as the thickness e_{pc} increases, the coefficient of natural convection $h_{c_{pc}}$ drops rapidly and is at its minimum for a thickness between 10 mm and 15 mm. Increasing the spacing beyond 15 mm has no significant effect on the coefficient $h_{c_{pc}}$.

This 15 mm spacing between the glass and the PV-Thermal absorber plate allows for further cooling of the PV cells, as shown in Fig. 9(a).

3.4 Influence of the Transmittance-Absorptance Product ($\tau\alpha$) on the Temperature T_p

Fig. 10 shows how the lack of precision in determining the transmittance-absorptance product ($\tau\alpha$) can affect the temperature T_p calculated.

It is noted that, for moderate to high air flow velocities, a 20% difference in the value of the

transmittance-absorptance product ($\tau\alpha$) has a negligible effect on the calculated temperature T_p . But for low air flow speeds, a 20% difference in the value of the transmittance-absorptance product ($\tau\alpha$) can cause a deviation of slightly more than 9 °C in the calculated temperature T_p .

For glazed flat-plate solar thermal collectors, a commonly used value for the ($\tau\alpha$) product is 0.9 [8]. The value of ($\tau\alpha$) considered for the glazed flat-plate PV-Thermal collector is approximately 0.8.

3.5 Influence of the Emittance ϵ_p on the Temperature of the PV-Thermal Absorber Plate T_p

Fig. 11 shows the variation of the temperature T_p as a function of the air flow velocity and for several values of the emittance of the PV-Thermal absorber plate ϵ_p .

It is observed that a 100% difference in the value of the emittance ϵ_p has virtually no effect on the calculated temperature T_p .

Therefore, even if the emittance ϵ_p is determined with low precision, the effect on the calculated temperature T_p is negligible.

The value of ϵ_p considered for the modelling of the steady-state heat transfer within the glazed flat-plate solar PV-Thermal collector is 0.8.

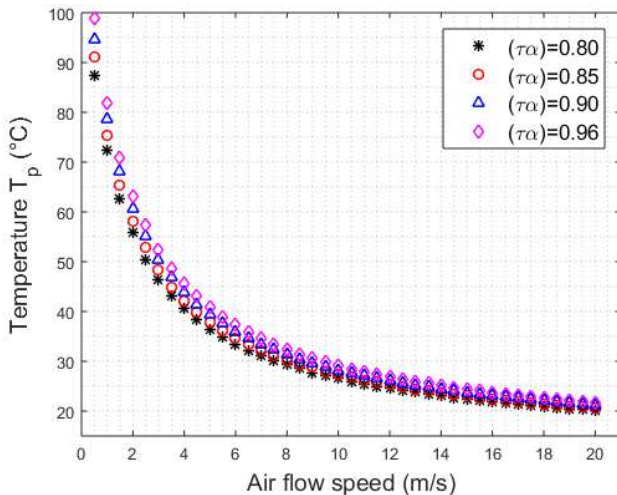


Fig. 10 Temperature T_p as a function of the air flow speed and for different values of ($\tau\alpha$)

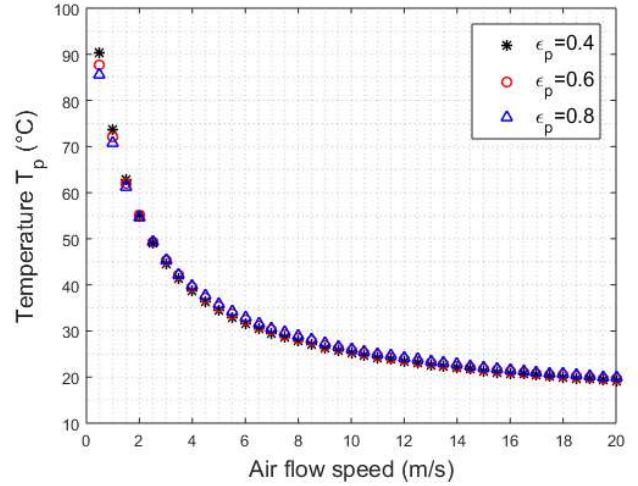


Fig. 11 Temperature T_p as a function of air flow speed and for different values of the emittance ϵ_p

3.6 Comparison of the PV-Thermal Absorber Plate Temperatures T_p with and without Fins

Fig. 12 allows for the estimation of the fins' contribution to enhancing the heat exchange between the PV-Thermal absorber plate and the cooling air and thus to reducing the temperature T_p .

It is noted that for low air flow velocities, the contribution of the fins to reducing the temperature of the PV-Thermal absorber plate T_p is more significant.

In other words, the higher the air flow speed, the less significant the additional cooling of the PV-Thermal absorber plate resulting from the adding of fins.

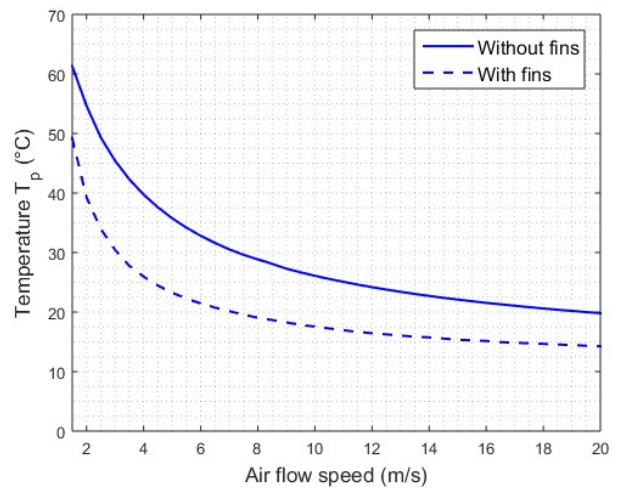


Fig. 12 Temperature T_p (with and without fins) as a function of the air flow speed V_f

4. Conclusion

This work presents a method for the mathematical modelling of steady-state heat transfer within a glazed flat-plate solar PV-Thermal collector using air as a cooling fluid. This device is designed to reduce and stabilize the operating temperature of the PV cells in order to avoid the detrimental effects of cyclic temperature variations in standard solar photovoltaic collectors, variations that can range from ambient temperature to a maximum of 90 °C under certain conditions [3].

The actual study assessed also the best configuration for the designed device (optimal spacing between the glass and the PV-Thermal absorber plate and the appropriate thickness of the ventilation duct) and the proper properties of the cooling air that will allow for an optimal cooling of the PV cells.

The results show that, with the glazed flat-plate PV-Thermal collector, the operating temperature of the PV cells can be maintained at $25\text{ °C} \pm 5\text{ °C}$ (regardless of the ambient conditions), which results in an increase of more than 25% in the peak power output of the PV cells under hot ambient conditions, compared to the peak power output of standard solar PV collectors.

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