

# From Hands-on Practice to Mathematical Inquiry: Task-Chain Design in Junior High School Mathematics Comprehensive and Practical Activities—A Case Study of Geometric Relationships in Paper Folding

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Hands-on work in junior high school mathematics has instructional value when it draws students into mathematical questions rather than stopping at finished products. Taking the lesson “Geometric Relationships in Paper Folding” as a case, this paper discusses how a sequence of folding tasks can be organized around creases. Students begin with familiar paper shapes, make and mark creases, compare the relationships they see, and then explain parallelism, symmetry, equal sides, and transfer under changed conditions. The chain is arranged as situational activation, operational generation, relationship discovery, mathematical explanation, and transfer application. The crease becomes the hinge between action and reasoning: It starts as a physical trace, is read as a geometric line, axis, bisector, or boundary relation, and finally serves as evidence in explanation. When students are required to record their folds, name relationships, and check their reasons, paper folding can remain open-ended while keeping a clear mathematical focus.

*Keywords:* junior high school mathematics, comprehensive and practical activities, paper folding, task chain, mathematization

## Problem Statement and Design Orientation

The *Mathematics Curriculum Standards for Compulsory Education (2022 Edition)* states that junior high school Comprehensive and Practical Activities may use project-based learning to help students experience mathematization and explore relationships and patterns in authentic contexts (Ministry of Education of the People’s Republic of China, 2022a). The *Curriculum Plan for Compulsory Education (2022 Edition)* also calls for learning by doing, using, and creating (Ministry of Education of the People’s Republic of China, 2022b). In this context, Guo and Cao (2022) discussed theme selection, task setting, teacher guidance, and presentation-based evaluation. Wang (2022) stressed the “three abilities” of seeing the world with mathematical eyes, thinking with mathematical thinking, and expressing with mathematical language, together with mathematized outcomes.

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Liu (2023; 2024) examined tasks, processes, and evaluation from the perspectives of performance assessment and interdisciplinary thematic learning. For lesson design, these studies raise a concrete question: How can students' activity traces be turned into mathematical expression, and how can that expression become evidence of learning?

Paper folding is a familiar carrier for geometric Comprehensive and Practical Activities because every fold leaves a line that can be pointed to, named, and discussed. In the activity "folding a regular pentagon with paper", for example, Shi (2022) used a problem chain to connect textbook paper-folding content with the development of core competencies. Catman-Aksoy and Isiksal-Bostan (2024) found that paper-folding activities can improve students' definitions and mental images of parallelism and perpendicularity. These studies give paper folding both a classroom basis and a research basis. The present case narrows the focus to creases and asks how they can help students discover relationships, justify them, and use them in new folding situations.

Two classroom risks are worth naming. In some paper-folding lessons, paper, tools, and finished products attract most of the attention, while the mathematical relationships remain faint. In others, the activity becomes only a livelier version of a geometry exercise: Students follow a familiar conclusion or a fixed procedure, but do not explain why the folding works. The lesson discussed here tries to hold material experience and mathematical explanation together. Paper gives students something to see and manipulate; mathematics asks them to turn creases into relationships that can be explained, checked, and used again. The guiding question is therefore: How can a task chain centered on creases help students move from operation to relationship discovery, reasoned explanation, and method transfer?

### **From Creases to Mathematization: Rationale for Task-Chain Design**

By task chain, this paper does not mean a schedule of activities placed one after another. A task chain places several problems around the same mathematical object, so that what students do in one task becomes useful in the next task, and the next task asks for a clearer explanation or a broader use. Here, "mathematization" means that students turn actions such as folding, superposing, unfolding, and comparing into geometric objects, positional relationships, and reasoned statements. In this lesson, the shared object is the crease. A crease is first a trace made by students' hands. It can then be treated as a line, an axis of symmetry, an angle bisector, or a boundary relationship in a figure. After making a crease, students need to ask what it has to do with the known sides, angles, and points; whether the relationship can be justified with prior knowledge; and whether the method still works after the paper shape changes. These questions carry the work from operation toward inquiry.

Working with creases does several things for the lesson. It makes inquiry accessible, because students can produce something visible before they begin to reason. It also opens a useful gap between experience and proof. A crease may look parallel, but students still have to say why it is parallel; a folded triangle may look isosceles, but equal sides need a basis. Creases also make transfer possible. When the paper changes from a triangle to a quadrilateral, or from a complete figure to a damaged one, students can look for relationships that remain stable. These comparisons allow students to observe geometric relationships in paper folding, judge the grounds on which those relationships hold, and express the reasons for their folding methods. The curriculum's three abilities become visible in the activity rather than remaining slogans outside it.

The task chain is guided by several design choices. Students begin with familiar materials, such as right triangles and rectangles, before working with general triangles and irregular paper pieces. Folding is always

followed by marking and explanation, so an operation leaves a visible trace and a reason. Knowledge that students often learn separately, including parallel lines, axial symmetry, angle bisectors, special triangles, and special quadrilaterals, is brought together through consecutive tasks. The aim is to help students build an evidence chain for observing, expressing, and justifying geometric relationships.

Table 1

*Task-Chain Design for “Geometric Relationships in Paper Folding”*

Phase	Learning task	Focus of mathematization	Evidence of learning
Situational activation	Start from paper-folding products and the creases left on them	Treat paper pieces and creases as objects for mathematical questions	Problem records; initial conjectures
Operational generation	Fold parallel lines or special figures in triangles, quadrilaterals, and rectangles	Produce operations that can be repeated and compared	Crease diagrams; step explanations
Relationship discovery	Compare relationships among creases, sides, angles, and points	Identify stable geometric relationships in different folding methods	Figure markings; peer discussion
Mathematical explanation	Explain why each folding method works	Use parallel lines, axial symmetry, angle bisectors, and properties of figures to give reasons	Oral explanation; revised presentation
Transfer application	Try variations: damaged paper, largest square, and isosceles/equilateral triangles	Check where a method can be used and where its conditions change	Variation products; reflection records

Table 1 is a map of the classroom progression rather than a rigid teaching script. In actual teaching, the five phases may appear more than once. When students struggle with a transfer task, the class may return to relationship discovery and mathematical explanation. When students propose a new folding method, they also need time to verify and revise their reasoning.

### **Lesson Design: Moving Paper-Folding Activities Toward Mathematical Inquiry**

The lesson is designed as a teaching case. It may be used in geometric Comprehensive and Practical Activities, or as review and extension from the second semester of Grade 7 to the first semester of Grade 8. Two class periods are recommended. Students should already know criteria for parallel lines, axial symmetry, angle bisectors, special triangles, and special quadrilaterals. The recurring classroom prompt is simple: “Can you fold it and explain why?” The lesson then proceeds through operation, marking, explanation, and transfer. In the accompanying task sheet, the five activities gradually move from special cases to general cases, from complete paper to damaged paper, and from single relationships to integrated construction.

#### **Activity 1: Folding Parallel Lines in Triangular Paper**

Activity Task: Folding Parallel Lines in Triangular Paper:

- (1) Fold the right-triangular paper as shown so that the crease is parallel to one leg of the triangle, and explain why the two lines are parallel.
- (2) Fold the general triangular paper as shown so that the crease is parallel to one side of the triangle, and explain the reason.
- (3) Choose any point P inside the triangular paper. Can you fold a crease through P that is parallel to one side of the triangle? See Figure 1.

Activity 1 begins with a right triangle, where a parallel crease is easier to obtain. It then turns to a general triangle and finally asks students to fold a crease through an interior point P that is parallel to one side (see Figure 1). In Figure 1, the left diagram gives the triangular paper and point P; the right diagram adds the target crease

through P and an auxiliary crease. The classroom demand is to build a relationship before explaining parallelism. Students may use perpendicularity, superposition, and unfolding to form auxiliary creases, and then justify the result with the fact that two lines perpendicular to the same line are parallel.

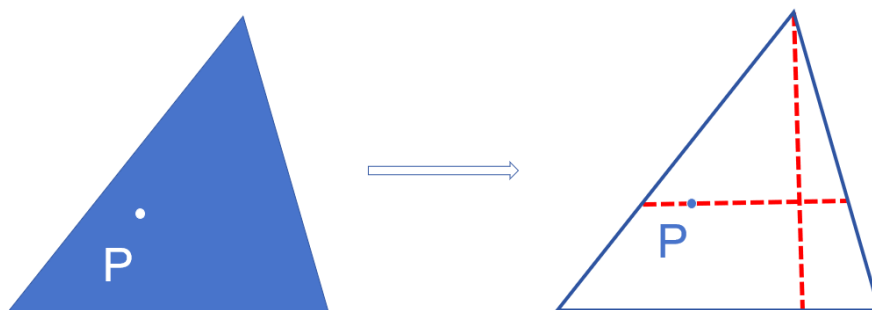


Figure 1. Folding a parallel crease through point P in triangular paper.

### Activity 2: Folding Parallel Lines in Quadrilateral and Damaged Paper

Activity Task: Folding Parallel Lines in Quadrilateral Paper:

(1) Fold the quadrilateral paper as shown so that the crease is parallel to one side of the quadrilateral. See Figure 2.

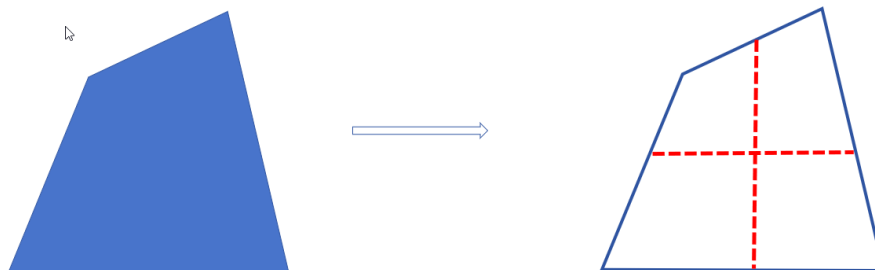


Figure 2. Folding a crease parallel to one side in quadrilateral paper.

(2) If the quadrilateral paper is damaged into the shape shown, can you fold two parallel creases? See Figure 3.

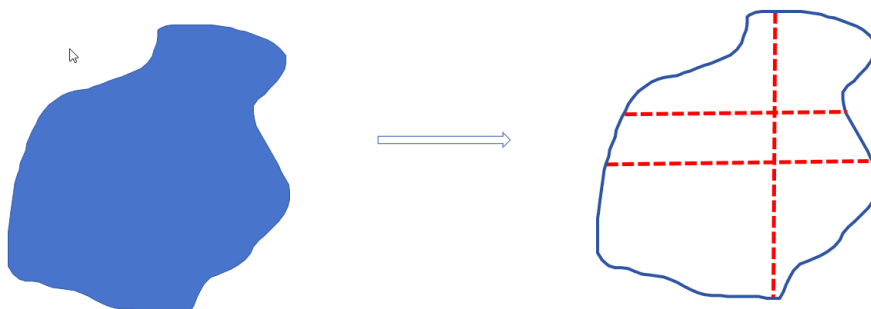


Figure 3. Folding two parallel creases in damaged paper.

Activity 2 moves from a complete quadrilateral (Figure 2) to damaged paper (Figure 3). The damaged shape deliberately reduces the hints provided by a neat outer boundary. Students have to decide whether a parallel relationship depends on the visible outline of the paper or on relationships that can be constructed and checked during folding. The teacher can ask students to compare methods: Why are the two creases parallel? Would the

method still work on another damaged piece? The task therefore moves from trial to condition analysis and makes the range of a folding method clearer.

**Activity 3: Folding Mutually Parallel Creases in Rectangular Paper**

Activity Task: Folding Mutually Parallel Creases in Rectangular Paper:

(1) As shown in Figure 4, crease AB bisects one right angle of the rectangular paper. Can another crease parallel to AB be folded?

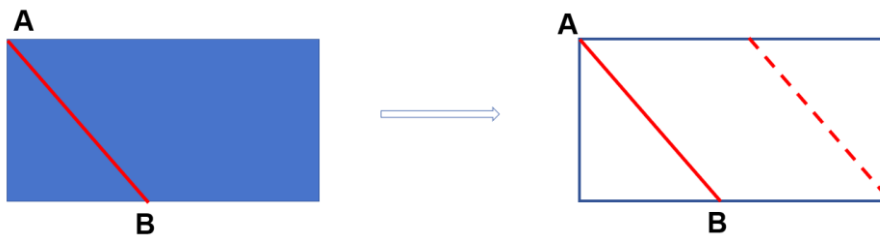


Figure 4. Angle-bisecting crease AB and its parallel crease.

(2) As shown in Figure 5, points A and B lie on opposite sides of the rectangular paper. Can two mutually parallel creases be folded through A and B respectively?

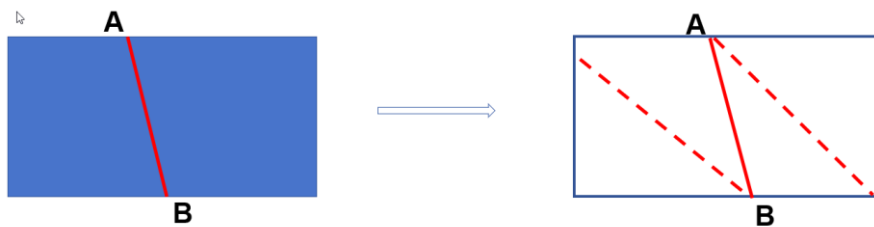


Figure 5. Folding mutually parallel creases through points A and B.

Activity 3 uses rectangular paper. Students use familiar facts about rectangles: Opposite sides are parallel, the four angles are equal, and some parts can coincide through folding. Starting from the angle-bisecting crease AB, they try to make another crease parallel to it, or to make two mutually parallel creases through points A and B (see Figures 4 and 5). Copying a line would be too thin a response here. Students need to connect angle bisectors, symmetry, and criteria for parallel lines when they explain the relationship between the creases.

**Activity 4: Folding the Largest Square in Rectangular Paper**

Activity Task: Folding the Largest Square in Rectangular Paper:

Using the rectangular paper shown, can you fold a square with the largest possible area? See Figure 6.

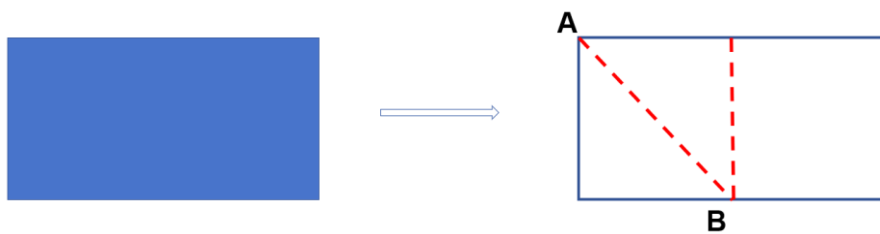


Figure 6. Folding the largest square in rectangular paper.

Activity 4 asks students to fold the largest possible square in a rectangle (see Figure 6). Here the crease is no longer used only to express a position; it becomes part of a construction. Students first check the defining conditions of a square: four equal sides and four right angles. They can then discuss why the folded square is the largest one allowed by the rectangle. The task can be layered. Most students focus on how to obtain a square, while students who are ready for extension discuss why the rectangle's shorter side limits the square's side length and area.

### Activity 5: Folding Special Triangles in Rectangular Paper

Activity Task: Folding Special Triangles in Rectangular Paper:

- (1) Using the rectangular paper shown, can you fold an isosceles triangle? See Figure 7.

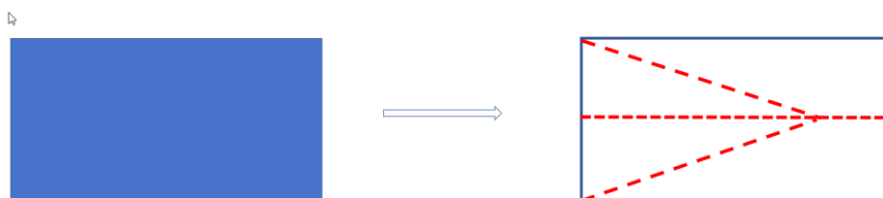


Figure 7. Folding an isosceles triangle in rectangular paper.

- (2) Using the rectangular paper shown, can you fold an equilateral triangle? See Figure 8.

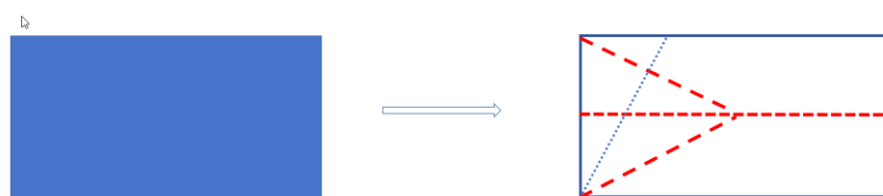


Figure 8. Folding an equilateral triangle in rectangular paper.

Activity 5 turns to isosceles and equilateral triangles (see Figures 7 and 8). Students often obtain these figures through superposition, but the mathematical work is to name the relationships produced by that superposition: equal sides, equal angles, and axial symmetry. They also need to distinguish the conditions for “isosceles” and “equilateral”. The teacher can ask students to mark vertices, creases, and equality relationships on their products before explaining the basis in words. The folded triangles are then treated as geometric objects with reasons, not just shapes that look right.

In the classroom summary, the learning route is expressed as a move from paper-folding problems to mathematical problems and then to properties of figures. The process is further summarized as transformation, inquiry, explanation, summarization, and reflection. This summary fits the main line of the lesson. Students first turn folding problems into mathematical problems, then inquire into and explain properties of figures, and finally form methods that can be transferred. The five activities are not separate craft tasks. They form a task chain from crease generation to relationship discovery, reasoned expression, and method transfer.

### Implementation Scaffolds: From Personal Experience to Shared Mathematical Expression

Open tasks still need visible supports. In paper-folding inquiry, students record their work on a task sheet by folding, drawing, marking, and explaining. These records keep the crease diagrams available for later

discussion. Teacher prompts also help students say what they did, what relationship they found, and why the relationship holds. During communication, the class can check whether a method can be repeated, whether the relationships are marked clearly, and whether the reasons are sufficient. Such supports leave thinking with students while making their personal experience available for public mathematical discussion.

Errors and differences deserve attention in this lesson. A folded line may deviate slightly. An explanation may stay at the level of “it looks like this”. A method may work on one sheet of paper but fail on another. These moments should not be closed too quickly by giving the standard method. They can become discussion materials. Students can compare “looking parallel” with “being able to justify parallelism”, and “working on this sheet” with “working for a class of figures”. Such comparison is closer to the learning value of Comprehensive and Practical Activities than a neat answer supplied by the teacher.

One practical classroom rhythm is individual trial, group comparison, whole-class argumentation, and second-round revision. Individual trial gives each student direct operational experience. Group comparison brings different strategies into view. Whole-class argumentation pushes students to make their mathematical language more precise. Second-round revision asks them to adjust their products and task sheets after discussion. Paper folding then becomes more than hands-on work; it becomes a cycle of revising and deepening mathematical understanding.

### **Evaluation Design: Using Process Evidence to Improve Learning**

Evaluation in this lesson should look at more than whether the folded product is finished or whether follow-up exercises are correct. Better evidence appears during the task chain: Can students propose a folding strategy? Can they mark key geometric relationships? Can they explain the reason in mathematical language? Can they adjust the method when the conditions change? Teachers can collect task sheets, crease diagrams, classroom talk, peer comments, and second-round revisions. Products, explanations, and revisions then check one another. A student’s marking, reasoning, and revision under changed conditions become the main evidence of learning.

For diagnostic evaluation, the teacher can keep photos and written explanations of different folding methods, because they show how students approach the same problem in different ways. Typical errors are also worth saving, especially creases that appear parallel but lack reasons; they help the class discuss what counts as evidence. After transfer tasks, short student reflections can reveal whether students have moved from operation toward inquiry. The same materials give students a basis for improving their expression and give teachers clues for refining later tasks.

Future teaching can make the case stronger by analyzing classroom episodes and student products more closely. Two or three task sheets, for example, may be selected to compare changes in folding methods, markings, and explanations. A group discussion may also be recorded to show how students move from intuitive judgment to evidence-based explanation. These materials can test more directly whether the task chain supports the intended move from operation to inquiry.

### **Conclusion**

Overall, “Geometric Relationships in Paper Folding” suggests that junior high school mathematics Comprehensive and Practical Activities should be designed as inquiry through doing. The quality of the task chain matters. When consecutive and open-ended tasks are organized around creases, students can discover relationships through operation, refine reasons through communication, and test methods through variations. In

this case, the crease works as the stable mathematical object that holds the tasks together. Each folding operation is expected to leave visible and explanatory evidence, and variation tasks are used to examine where a method applies. Paper operation, geometric representation, reasoned expression, and process-based evaluation are therefore linked in one lesson structure. Since this is a design-based lesson case, the task chain still needs to be examined with student products and classroom discussion records in future teaching.

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