

Node Degree Distribution and Interrelationships of China High-speed Railway Network, Aviation Network and Composite Network

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Abstract: This study examines China high-speed railway network, aviation network and the composite network. Using stations as nodes and the lines between them as edges, the node degree probability distribution of the resulting complex networks is calculated. Through regression analysis, it is found that: the node degree probability distribution of China high-speed railway network is negative exponential function, aviation network and composite network are power function. The probability distribution curve of the high-speed railway network is at the left, the distribution curve of the composite network is in the middle, and the distribution curve of the aviation network is at the right. The node degree probability distribution curve of the high-speed railway network is the steepest. The probability distribution curve of the aviation network and composite network have similar shapes. For all three, the occurrence probability of node degrees decreases as the degree increases.

Key word: High-speed railway network, aviation network, composite network, regression analysis, node degree distribution.

1. Introduction

The air transportation network is a typical complex network and exhibits small-world characteristics [1-4]. As a type of transportation network, the air network uses cities with airports as nodes and flight routes as edges. The node degree probability distributions of aviation network have power function [3, 4]. The previous studies found that: the node degree probability distributions of Beijing subway, bus and composite network follow a negative exponential function [5]. The China high-speed railway as a type of land public transportation method, may has similar character with subway network and bus network. But the feature of composite network consists of high-speed railway network and aviation network is need to study.

Here, using the data of China high-speed railway in 2025 [6] as in Fig. 1, the node degree probability distribution of China high-speed railway network was

calculated. Through statistic data [7], the node degree probability distribution of China aviation network in 2020 was obtained. The former is negative exponential function and the latter is power function. The node degree probability distribution of composite network consists of China high-speed railway network and aviation network follows power function. The probability distribution curve of the high-speed railway network is at the left, the distribution curve of the composite network is in the middle and the distribution curve of the aviation network is at the right. For all three, the occurrence probability of node degrees decreases as the degree increases.

2. Node Degree Probability Distribution of China High-speed Railway Network

Using China high-speed railway data in 2025, as shown in Fig. 1, there are a total of 1,054 stations. Each station serves as a node, and the lines serve as edges to

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construct a complex network.

Let the adjacent matrix of China high-speed railway network be A_R . The node quantity of China high-speed railway network is $N_R = 1054$. For network $G_R = (V_R, E_R)$, where $v_i \in V_R$ is the node of G_R . V_R is the set of nodes. E_R is the set of edges, $(v_i, v_j) \in E_R$. Matrix $A_R = (a_{i,j})_{N_S \times N_S}$ was constructed, where:

$$a_{i,j} = \begin{cases} 1, & (v_i, v_j) \in E_R \\ 0, & \text{otherwise} \end{cases}$$

Let the node degree be x axis and the probability be y axis in Fig. 2. Let $v = \ln y$ in Fig. 3. The points in Fig. 3 were calculated by the points in Fig. 2. The correlation coefficient r of the points in Fig. 3 was calculated by Eq. (1).

$$r = \frac{L_{xy}}{\sqrt{L_{xx}L_{yy}}} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2 \sum_{i=1}^n (y_i - \bar{y})^2}} \quad (1)$$



Fig. 1 China high-speed rail network map.

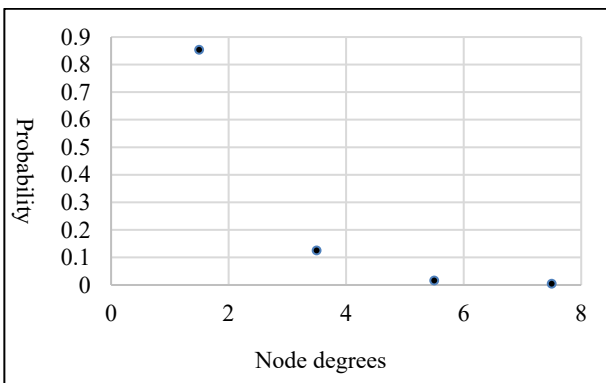


Fig. 2 Probability distribution of node degrees of China high-speed railway network.

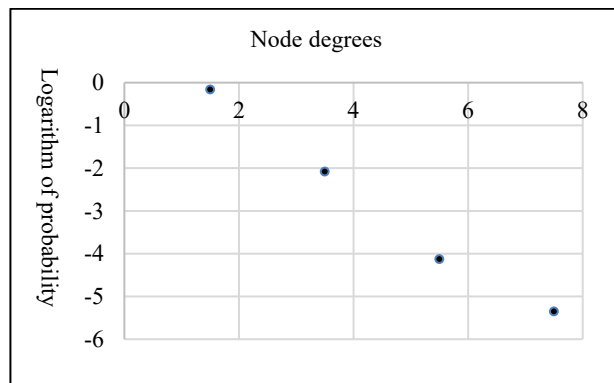


Fig. 3 Relationship between node degree and the logarithm of probability of China high-speed railway network.

Table 1 Node degree distribution frequency and probability of China high-speed railway Network.

Interval of node degrees	[1,2]	[3,4]	[5,6]	[7,8]
Median	1.5	3.5	5.5	7.5
Times	900	132	17	5
Probability	0.854	0.125	0.016	0.005

Here, $n = 4$. The value of correlation coefficient r was calculated, $r = -0.995$. The critical value of $r_{\alpha=1\%}^f$ was 0.990 found in critical value table [8] at degree of freedom $f = n - 2 = 2$ and level of significant α of 1%. Since $|r| = 0.995 > r_{\alpha=1\%}^f = 0.990$, the scattered

points in Fig. 3 had relatively significant linear correlation. Least square method [8] was used as an approach in Eq. (2) to fit the line with points in Fig. 3.

$$\begin{cases} \hat{\beta}_0 = \bar{v} - \hat{\beta}_1 \bar{x} = 1.041 \\ \hat{\beta}_1 = \frac{L_{xv}}{L_{xx}} = -0.882 \end{cases} \quad (2)$$

The linear equation:

$$\hat{v} = 1.041 - 0.882x \quad (3)$$

The fitting line Eq. (3) was drawn with the sample points in Fig. 4 with good fitting effect.

To take t test [8] of Eq. (3), test hypotheses is: $H_0: \beta_1 = 0$,

When the hypotheses was true, there is:

$$\hat{\beta}_1 \sim N\left(0, \frac{\sigma^2}{L_{xx}}\right) \quad (4)$$

Here, $\hat{\beta}_1$ fluctuate near zero, statistic t is build.

$$t = \frac{\hat{\beta}_1}{\sqrt{\frac{\hat{\sigma}^2}{L_{xx}}}} = \frac{\hat{\beta}_1 \sqrt{L_{xx}}}{\hat{\sigma}} \quad (5)$$

Where:

$$\hat{\sigma}^2 = \frac{1}{n-2} \sum_{i=1}^n (v_i - \hat{v}_i)^2 \quad (6)$$

Calculated by statistic data: $t = -55.5$

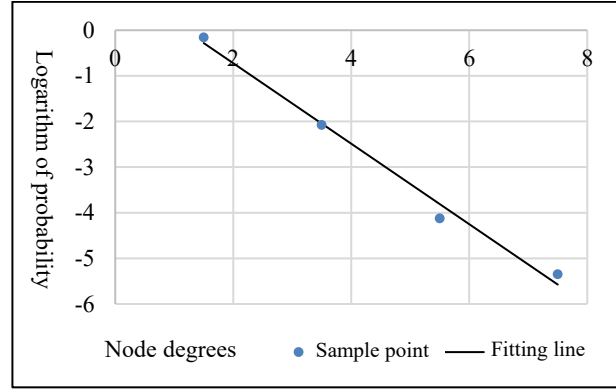


Fig. 4 Fitting linear relationship between node degree and logarithm of probability of China high-speed railway network.

To check the t distribution table, at significant level α of 0.01 and degree of freedom $f = n - 2 = 2$, the value of $t_{\alpha=0.01}^f$ in table is 6.965. So,

$|t| = 55.5 > t_{\alpha=0.01}^f = 6.965$, null hypotheses H_0 is

refused. The linear correlation of Eq. (3) is relatively significant. The fitting curve Eq. (7) of nonlinear relationship between node degree and probability was deduced from Eq. (3):

$$\hat{y} = 2.832e^{-0.882x} \quad (7)$$

The points of the fitting curve Eq. (7) and the sample points were drawn in Fig. 5 with good fitting effect. It showed that the node degree of China high-speed railway network had a negative exponential relationship with the probability.

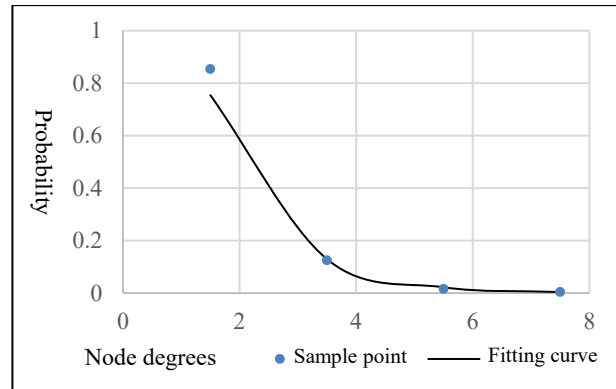


Fig. 5 Fitting nonlinear relationship between node degree and probability of China high-speed railway network.

3. Node Degree Probability Distribution of China Aviation Network

Using data from China aviation network in 2020 [7]. There are 235 cities with airports. Each city is treated as a node, and the routes between cities are treated as edges to construct a complex network. The median node degree against the probability as scatter points were drawn in Fig. 6. Let the China aviation network be G_A . Let the adjacent matrix of China aviation network be A_A . The quantity of node degree of China aviation network is $N_A = 235$. For network

$G_A = (V_A, E_A)$, where $v_i \in V_A$ is the node of G_A . V_A is the set of nodes. E_A is the set of edges, $(v_i, v_j) \in E_A$.

Matrix $A_A = (a_{i,j})_{N_A \times N_A}$ was constructed, where:

$$a_{i,j} = \begin{cases} 1, & (v_i, v_j) \in E_A \\ 0, & \text{otherwise} \end{cases}$$

Let the node degree be x axis and the probability be y axis in Fig. 6. Let $\mu = \ln x$ and $\nu = \ln y$ in Fig. 7. The points in Fig. 7 were calculated by the points in Fig. 6.

Table 2 Node degree distribution frequency and probability of China aviation network.

Interval of node degree	[1,30]	[31,60]	[61,90]	[91,120]	[121,150]	[151,180]
Median	15.5	45.5	75.5	105.5	135.5	165.5
Times	158	35	18	14	5	5
Probability	0.672	0.149	0.077	0.06	0.021	0.021

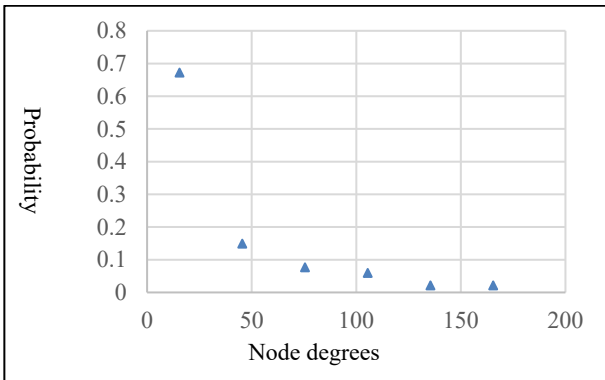


Fig. 6 Probability distribution of node degrees of China aviation network.

$$r = \frac{L_{uv}}{\sqrt{L_{uu}L_{vv}}} = \frac{\sum_{i=1}^n (u_i - \bar{u})(v_i - \bar{v})}{\sqrt{\sum_{i=1}^n (u_i - \bar{u})^2 \sum_{i=1}^n (v_i - \bar{v})^2}} \quad (8)$$

Here, $n = 6$. Using the data in Table 2, the value of correlation coefficient r was calculated by Eq. (8), $r = -0.986$. The critical value of $r_{\alpha=1\%, f=4}$ was 0.947 found in critical value table at degree of freedom $f = n - 2 = 2$ and level of significant α of 1%. Since $|r| = 0.986 > r_{\alpha=1\%, f=4} = 0.947$, the scattered points in

Fig. 7 had significant linear correlation. Least square method was used as an approach in Eq. (9) to fit the line with points in Fig. 7.

$$\begin{cases} \hat{\beta}_0 = \bar{v} - \hat{\beta}_1 \bar{u} = 3.714 \\ \hat{\beta}_1 = \frac{L_{uv}}{L_{uu}} = -1.474 \end{cases} \quad (9)$$

The linear equation:

$$\hat{v} = 3.714 - 1.474u \quad (10)$$

The fitting line Eq. (10) were drawn with the sample points in one diagram of Fig. 8 with good fitting effect.

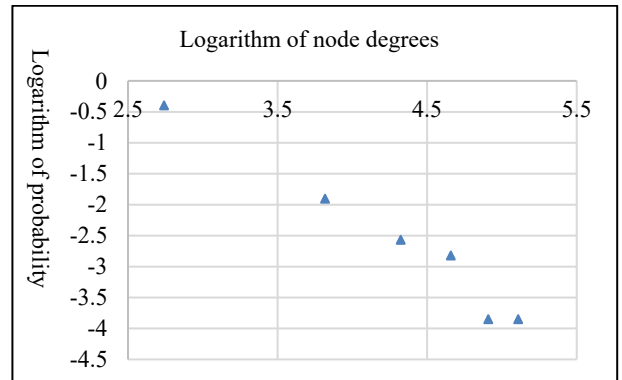


Fig. 7 Relationship between the logarithm of node degree and the logarithm of probability of China aviation network.

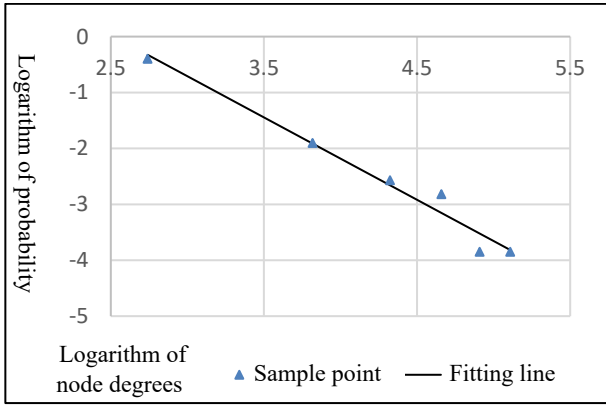


Fig. 8 Fitting linear relationship between logarithm of node degree and logarithm of probability of China aviation network.

To take t test of Eq. (10), test hypotheses is: $H_0: \beta_1 = 0$.

When the hypotheses was true, there is:

$$\hat{\beta}_1 \sim N\left(0, \frac{\sigma^2}{L_{uu}}\right) \quad (11)$$

Here, $\hat{\beta}_1$ fluctuate near zero, statistic t is build.

$$t = \frac{\hat{\beta}_1}{\frac{\hat{\sigma}^2}{\sqrt{L_{uu}}}} = \frac{\hat{\beta}_1 \sqrt{L_{uu}}}{\hat{\sigma}} \quad (12)$$

Where:

$$\hat{\sigma}^2 = \frac{1}{n-2} \sum_{i=1}^n (v_i - \hat{v}_i)^2 \quad (13)$$

Calculated by statistic data: $t = -11.94$

To check the t distribution table, at significant level α of 0.01 and degree of freedom $f = n - 2 = 4$ the value of $t_{\alpha=0.01, f=4}$ in table is 3.747. So,

$|t| = 11.94 > t_{\alpha=0.01, f=4} = 3.747$, null hypotheses H_0 is

refused. The linear correlation of Eq. (10) is significant. The fitting curve Eq. (14) of nonlinear relationship between node degree and probability was deduced from Eq. (10):

$$\hat{y} = 41.02x^{-1.474} \quad (14)$$

The points of the fitting curve Eq. (14) and the sample points were drawn in Fig. 9 with good fitting

effect. It showed that the probability distribution of node degree of China aviation network is power function.

4. Node Degree Probability Distribution of China Composite Network

The total number of stations in the composite network equals the sum of high-speed rail way network and aviation network stations, which is 1,289. There are 159 cities that have both high-speed railway stations and airports. In a city, the high-speed railway station and airport are considered two separate nodes, and there are connections between the nodes of high-speed railway stations and airports in these 159 cities.

China composite network consist of high-speed rail network and aviation network is summarized in Table 3. The median node degree against the probability as scatter points were drawn in Fig. 10. Let the China composite network be G_C . Let the adjacent matrix of Beijing composite network be A_C . The quantity of node degree of China composite network is $N_C = N_R + N_A = 1289$. For network $G_C = (V_C, E_C)$, where $v_i \in V_C$ is the node of G_C . V_C is the set of nodes. E_C is the set of edges, $(v_i, v_j) \in E_C$. Matrix $A_C = (a_{i,j})_{N_C \times N_C}$ was constructed,

where: $a_{i,j} = \begin{cases} 1, & (v_i, v_j) \in E_C \\ 0, & \text{otherwise} \end{cases}$.

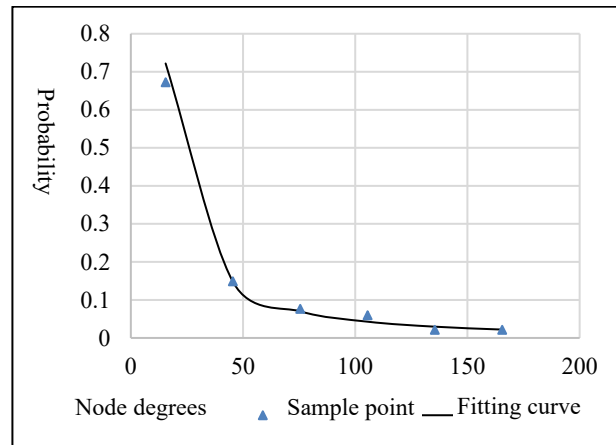


Fig. 9 Fitting nonlinear relationship between node degree and probability of China aviation network.

Table 3 Node degree distribution frequency and probability of China composite network.

Interval of node degree	[1,30]	[31,60]	[61,90]	[91,120]	[121,150]	[151,180]
Median	15.5	45.5	75.5	105.5	135.5	165.5
Times	1212	35	18	14	5	5
Probability	0.94	0.027	0.014	0.011	0.004	0.004

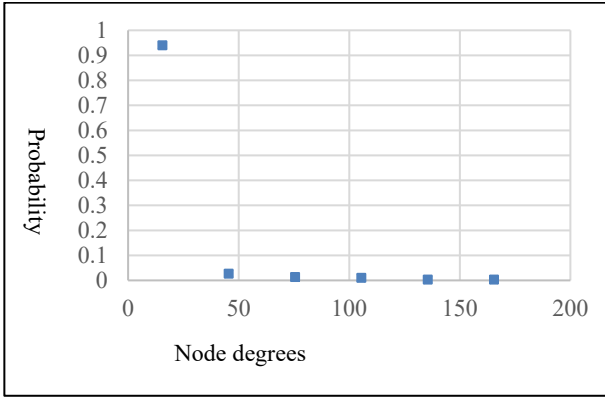


Fig. 10 Probability distribution of node degrees of China composite network.

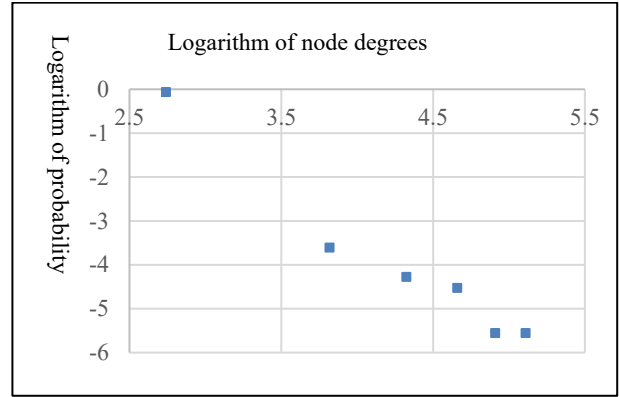


Fig. 11 Relationship between logarithm of node degree and the logarithm of probability of China composite network.

Let the node degree be x axis and the probability be y axis in Fig. 10. Let $u = \ln x$ and $v = \ln y$ in Fig. 11. The points in Fig. 11 were calculated by the points in Fig. 10.

Here, $n = 6$. Using the data in Table 3, the value of correlation coefficient r was calculated by Eq. (8), $r = -0.978$. The critical value of $r_{\alpha=1\%, f=4}$ was 0.947 found in critical value table at degree of freedom $f = n - 2 = 4$ and level of significant α of 1%. Since $|r| = 0.978 > r_{\alpha=1\%, f=4} = 0.947$, the scattered points in

Fig. 11 had significant linear correlation. Least square method was used as an approach in Eq. (15) to fit the line with points in Fig. 11.

$$\begin{cases} \hat{\beta}_0 = \bar{v} - \hat{\beta}_1 \bar{u} = 5.823 \\ \hat{\beta}_1 = \frac{L_{uv}}{L_{uu}} = -2.289 \end{cases} \quad (15)$$

The linear equation:

$$\hat{v} = 5.823 - 2.289u \quad (16)$$

The fitting line Eq. (16) were drawn with the sample points in one diagram of Fig. 12 with good fitting effect.

To take t test of Eq. (16), test hypotheses is: $H_0: \beta_1 = 0$.

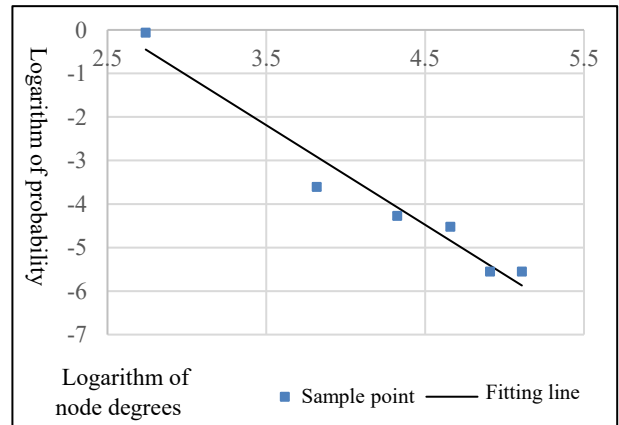


Fig. 12 Fitting linear relationship between logarithm of node degree and logarithm of probability of China composite network.

When the hypotheses were true, there are: Eq. (11), Eq. (12) and Eq. (13).

Statistic t was calculated by data in Table 3 and Eq. (11), Eq. (12), Eq. (13): $t = -9.48$

To check the t distribution table, at significant level α of 0.01 and degree of freedom $f = n - 2 = 4$ the value of $t_{\alpha=0.01, f=4}$ in table is 3.747. So,

$|t| = 9.48 > t_{\alpha=0.01, f=4} = 3.747$, null hypotheses H_0 is

refused. The linear correlation of Eq. (16) is significant.

The fitting curve Eq. (17) of nonlinear relationship between node degree and probability was deduced from Eq. (16):

$$\hat{y} = 337.98x^{-2.289} \quad (17)$$

The points of the fitting curve Eq. (17) and the sample points were drawn in Fig. 13 with good fitting effect. It showed that the node degree probability distribution of China composite network consists of high-speed railway network and aviation network is power function.

5. Interrelationship Among the Node Degree Distribution in China High-speed Railway Network, Aviation Network, and Composite Network

The fitting curve equations of the node degree probability distributions and some complex network parameters of China high-speed railway network, aviation network, and composite networks are listed in Table 4. The probability distributions of China high-speed railway network is negative exponential function and the other two are power function.

By plotting the curves from Table 4 together in Fig. 14, their relative positions can be observed. The probability distribution curve of the high-speed railway network is at the left, the distribution curve of the composite network is in the middle and the distribution curve of the aviation network is at the right. The node

degree probability distribution curve of the high-speed railway network is the steepest, indicating that the interval of node degree is concentrated on smaller value zone than the other two. Since the average node degree of the high-speed railway network is less than the other two, the probability distribution curve of the high-speed railway network is at the left. The probability distribution curve of the aviation network and composite network have similar shapes. They are both power function distribution. The average node degree of aviation network is more than the composite network, so the probability distribution curve of aviation network is at the right of the composite network. For all three, the occurrence probability of node degrees decreases as the degree increases.

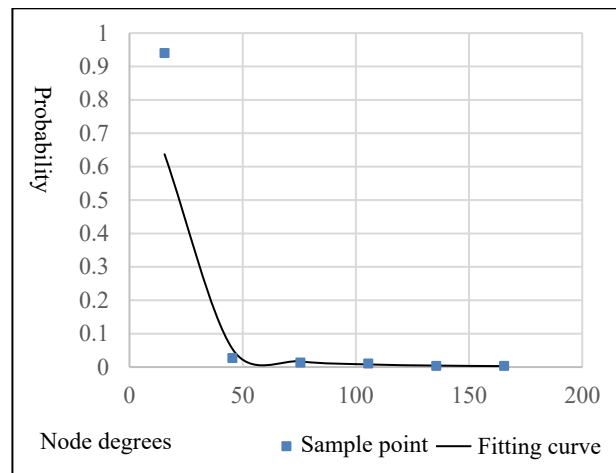


Fig. 13 Fitting nonlinear relationship between node degree and probability of China composite network.

Table 4 The fitting curve equations of node degree distributions and some complex network parameters of China high-speed railway network, aviation network and composite network.

	China high-speed railway	Aviation network	Composite network
Node number	1054	235	1289
Edge number	1159	3816	5130
Average node degree	2.20	32.48	7.96
Fitting curve equation	$\hat{y} = 2.832e^{-0.882x}$	$\hat{y} = 41.02x^{-1.474}$	$\hat{y} = 337.98x^{-2.289}$

Node Degree Distribution and Interrelationships of China High-speed Railway Network, Aviation Network and Composite Network

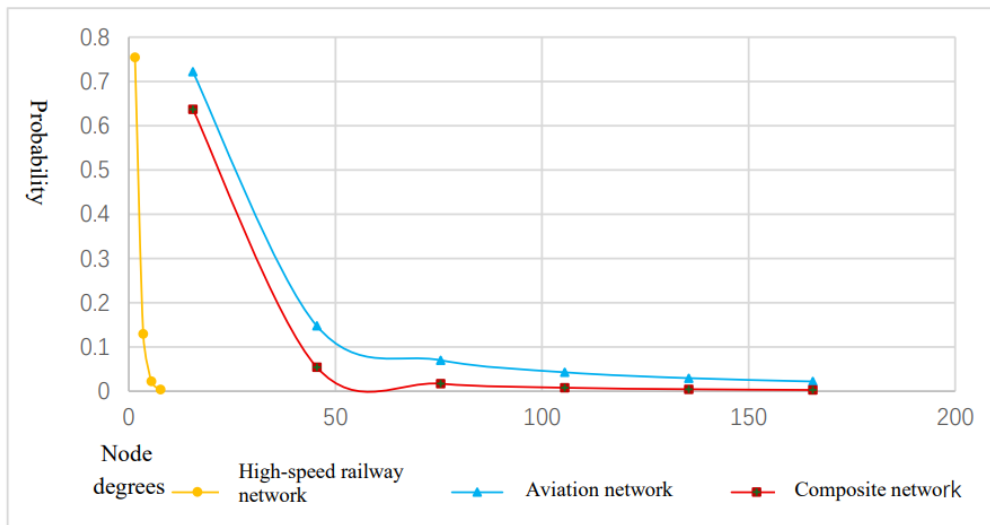


Fig. 14 Interrelationship among the node degree distribution in China high-speed railway network, aviation network and composite networks.

6. Conclusion

After analysis of data, it is found that: the node degree probability distribution of China high-speed railway network is negative exponential function, aviation network and composite network are power function. The probability distribution curve of the high-speed railway network is at the left, the distribution curve of the composite network is in the middle, and the distribution curve of the aviation network is at the right. The node degree probability distribution curve of the high-speed railway network is the steepest, indicating that the interval of node degree is concentrated on smaller value zone than the other two. Since the average node degree of the high-speed railway network is less than the other two, the probability distribution curve of the high-speed railway network is at the left. The probability distribution curve of the aviation network and composite network have similar shapes. They are both power function distribution. The average node degree of aviation network is more than the composite network, so the probability distribution curve of aviation network is at the right of the composite network. For all three, the

occurrence probability of node degrees decreases as the degree increases.

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