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Research on the Systematization of Discrete Mathematics Knowledge Guided by Big Ideas

LI Gaolin, ZHONG Yaoyao
Yancheng Teachers University, Yancheng, China

Discrete Mathematics plays a vital role in formal reasoning and systematic thinking training in computer science education. However, current teaching often faces a structural dilemma of "fragmentation" and "difficulty in establishing a global perspective" due to the differences in symbol systems, proof paradigms, and representation methods across modules. This study introduces the "Big Ideas" and backward instructional design, constructing a knowledge systematization reconstruction paradigm for Discrete Mathematics oriented towards "Five Core Big Ideas - Cross-module Inquiry Units - Two-way Specification Table Evaluation". The effectiveness is further examined through qualitative methods including classroom observation, teacher interviews, and student interviews. The findings show that systematized teaching guided by Big Ideas helps students form cross-module holistic knowledge frameworks, promotes the development of formal reasoning transfer capability, and significantly enhances learning engagement and self-efficacy.

Keywords: Discrete Mathematics, Big Ideas, knowledge systematization, OBE, qualitative research

Introduction and Background

As a core fundamental course for Computer Science and Technology majors, Discrete Mathematics constitutes the theoretical basis for subsequent courses such as Data Structures, Relational Databases, and Compiler Principles. It plays an irreplaceable role in cultivating students' logical reasoning and computational thinking. However, in current teaching practice, Discrete Mathematics generally faces a structural dilemma of highly "fragmented" knowledge. Existing textbooks and syllabi are mostly developed in parallel across four independent branches: Mathematical Logic, Set Theory, Algebraic Systems, and Graph Theory. Because the modules differ significantly in representations, symbol systems, and proof paradigms, students often find it difficult to establish a global perspective of the course during the learning process, resulting in a disconnect between theoretical derivation and subsequent engineering algorithm practice.

With the deeper implementation of Outcomes-Based Education (OBE) in engineering education accreditation, stronger requirements have been placed on students' ability to solve complex engineering problems and think systematically. In response, this study introduces the pedagogical theory of Big Ideas to reveal horizontal connections among branches of discrete mathematics and to explore a knowledge-systematization paradigm with both theoretical consistency and teaching operability.

LI Gaolin, Ph.D., Associate Professor, School of Mathematics and Statistics, Yancheng Teachers University, Yancheng, China.
ZHONG Yaoyao, School of Mathematics and Statistics, Yancheng Teachers University, Yancheng, China.

Analysis of Domestic and International Research Status

This section reviews three contexts: discrete mathematics teaching reform, Big Ideas theory and its extension to higher education, and knowledge systematization under OBE and modular teaching.

Pain Points and Reform Status of Discrete Mathematics Teaching Models

Addressing the issues of extremely scattered knowledge points and the disconnect between theory and practice, early literature generally pointed out that students easily develop cognitive fragmentation during learning (Wang & Li, 2021; Chen & Zhang, 2020). In recent years, some scholars have proposed a reconstruction of discrete mathematics teaching oriented towards systems capability cultivation, emphasizing the vertical connection between theoretical proof and algorithmic practice (Liu & Zhang, 2022; Li & Zhao, 2019). Other scholars have explored modular construction based on knowledge graphs or case series, attempting a spatial combination of discrete mathematics in physical structure (Sun, 2023). However, existing research mostly stays at the superficial level of chapter sequence adjustment, lacking a higher-order logical framework for a deep, substantive integration and systematic reconstruction of these modules (Zhou & Wu, 2021).

Origins of Big Ideas Theory and Extension to Higher Education

The pedagogical origins of Big Ideas theory can be traced to Jerome Bruner's "Fundamental Ideas" theory, which emphasized students' understanding of a discipline's basic structure (Bruner, 1960). Wiggins and McTighe (2005), in Understanding by Design (UbD), further established a backward instructional design paradigm centered on big ideas, defining them as "anchors" that give fragmented facts meaning and connection (pp. 65-71).

Domestic scholars such as Zhong (2021) and Li and Lü (2018) have further developed this line of inquiry, arguing that big ideas connect isolated knowledge nodes with broader disciplinary theories. The approach has moved beyond basic education and has begun to inform foundational science courses in higher education (Zhang, 2022; Wang & Liu, 2021). Literature reconstructing Discrete Mathematics through the lens of Big Ideas remains limited, which provides a key entry point for this study.

Extraction Logic and Connotation Definition of "Big Ideas" in Discrete Mathematics

Logic of Extracting Big Ideas

Extracting big ideas aims to resolve the contradictions of cognitive disconnects and methodology gaps in existing courses. Differing from experiential extraction methods in basic education, this study employs a combination of ontological tracing and computer engineering tracing. Through bottom-up deduction, five progressive core concepts were established. The operational steps are as follows:

(1) Symbol-Level Abstraction: Identify recurring symbolic objects (propositions/predicates, set objects, vertices/edges, etc.) in the course, analyzing their common representational syntax across chapters.

(2) Deduction-Level Modeling: Induce the proof paradigms and reasoning rules of the course (axiomatic deduction, natural deduction, induction, and equivalence calculus) to clarify "how to ensure correct derivation."

(3) Mapping-Level Connection: Analyze structural correspondences from one object to another (relations/functions/algebraic operations/adjacency matrices, etc.) to distill "connection rules."

(4) Structure-Level Invariance: Compare isomorphism/invariance criteria across different physical representations (equivalence, isomorphism, homomorphism, etc.) to extract "how to identify underlying consistency."

(5) Computation-Level Exit: Guide theoretical structures towards computable problem modeling (shortest paths, reachability, state machines, etc.), forming application landing points for learning.

Connotations and Hierarchical Architecture of the Five Core Concepts

Figure 1 presents the progressive relationship among the five big ideas, and Table 1 maps them onto the four traditional modules.

(1) Formal Abstraction and Representation: transforming entities and judgments into rigorous symbolic expressions across logic, sets, and graph topology.

(2) Certainty of Reasoning and Proof: ensuring rigor through natural deduction, equivalence calculus, induction, and axiomatic rules.

(3) Relations and Mapping Rules: explaining dependencies through binary relations, functions, algebraic operations, and graph matrices.

(4) Structure and Invariance: identifying internal consistency across algebraic systems and graph isomorphism through invariant structures.

(5) Discrete Modeling and Algorithm Computability: connecting theory with state machines, cryptography, compilation, and routing algorithms.

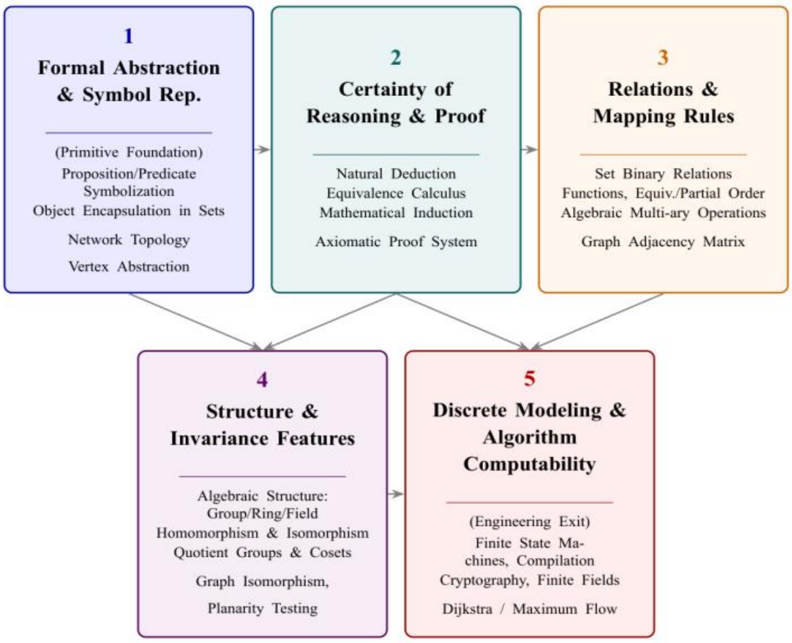

Figure 1. Systematized evolutionary architecture diagram of Discrete Mathematics Big Ideas.

Table 1

Two-Way Mapping Table of Five Core Big Ideas and Traditional Four Modules

(Big Ideas)	Concept Connotation	Logic Module	Set Module	Algebraic Systems Module	Graph Theory Module
I. Formal Abstraction & Symbol Rep.	Everything can be expressed with strict symbols	Prop./Predicate symbolization (WFFs)	Set encapsulation, inclusion, and operations	Formalization of operation rules, identity/inverse defs.	Topological abstraction of nodes and edges
II. Logical Reasoning & Proof System	Deriving conclusions via irrefutable rules	Truth tables, Natural Deduction, CP, Equivalence, Induction	Two-step proof method for subsets	Axiomatic system: deductions in groups/rings/fields	Euler/Hamilton theorems, algorithm correctness proofs
III. Relations & Mapping Rules	Revealing correspondences and constraints	Discrete mapping of logical connectives	Cartesian product, functions; equiv., partial order	Algebraic multi-ary operations (mapping extension)	Adjacency matrices, directed/undirected visualizations

Table 1 to be continued

IV. Structure & Isomorphic Thinking	Revealing underlying rule consistency	Boolean algebra structure equivalence	Structural comparison of set cardinalities, equipotence	Groups, rings, fields; Homomorphism & isomorphism; Quotient groups	Graph isomorphism testing; Planarity & connectivity checking
V. Discrete Modeling & Computability	Theory to algorithm landing, linking to engineering	Bitwise operations foundation and logic instructions	Relational DB constraint modeling and relational algebra	Cryptography (finite fields); FSM & compilation modeling	Dijkstra's shortest path, Graph coloring, Maximum flow

Implementation Path for Knowledge "Systematization" Based on Big Ideas

Based on the five big ideas, teaching should move beyond linear chapter order and build a two-way systematization path.

Vertical Integration: Constructing Cross-Module Inquiry Units

To reduce chapter-level isolation, this study uses big ideas as the main thread to reconstruct comprehensive inquiry units through backward instructional design. For instance, a unit centered on "Structure and Invariance" spans three modules: cardinality and "equipotence" in set theory, homomorphism and isomorphism in algebraic systems, and intuitive judgments about graph isomorphism in topological structures. This process helps students identify structural principles shared across multiple branches.

Each inquiry unit follows a unified chain: problem context, alignment with big ideas, knowledge retrieval, student output, and evaluation through a two-way specification table.

Horizontal Extension: Mapping Integration Oriented Towards Professional Courses

The knowledge network should also clarify how core discrete mathematics concepts connect with later professional courses:

Based on "Reasoning and Rules", connecting with white-box coverage theory and formal verification techniques in Software Engineering;

Relying on "Relations and Mappings", extending to Database Principles, revealing the underlying logic of relational algebra and SQL queries;

Utilizing "Algebraic Structures", transitioning to Cryptography (finite fields and security regimes) and Formal Languages and Automata Theory (compilation basics);

Applying "Modeling and Algorithms", directly transitioning to graph algorithms and connectivity analysis engineering scenarios in Data Structures.

Construction of a System-Level Evaluation System for "Systematized Knowledge"

To verify the teaching reconstruction, the traditional evaluation system that examines mathematical deduction in isolation must be reformed.

Reconstruction of Final Assessments Based on Two-Way Specification Tables

Breaking the convention of allocating fixed scores based on physical modules, the exam paper design is transformed into a two-way specification table mode with "Five Core Concepts" as the vertical axis and "Four Physical Modules" as the horizontal axis. The exam increases cross-module comprehensive questions to assess deep knowledge invocation capabilities. The overall assessment in this study adopts a "ternary structure": the final written exam (two-way specification table) accounts for 50%, the performance-based engineering project accounts for 30%, and process engagement and unit inquiry performance account for 20%.

Introduction of Performance-Based Engineering Project Evaluation

To measure the effectiveness of advanced discrete modeling training, large-scale assignments encompassing engineering contexts are introduced. For example, a project based on “Warehouse Logistics Scheduling and Collision-Free Proof”: requires students to use abstract network graphs for warehouse connectivity routing planning, apply partial order relations to handle concurrent mechanisms of flow operations, and finally complete a deadlock-free proof using predicate logic. The evaluation rubric dimensions are shown in Table 2.

Table 2

Performance-Based Engineering Project Evaluation Rubric

Evaluation Dimension	Description	Weight
Formal Accuracy	Accurate symbol expression/predicate/relation models, clear boundaries	25%
Structure & Invariance Understanding	Identifying and explaining the role of isomorphic/equivalence relations in the system	25%
Verifiable Reasoning Chain	Complete and rigorous proof chain, key steps traceable and verifiable	25%
Engineering Transfer Explanation	Ability to map models and algorithms to application engineering contexts and explain rationality	25%

To align evaluation reform with effectiveness analysis, D1 reflects structural understanding, D2 reflects reasoning and transfer, and D3 reflects task engagement and acceptance of evaluation methods.

Qualitative Study and Effectiveness Analysis of Teaching Reconstruction Based on Big Ideas

To examine teaching effectiveness, this study used classroom observation, in-depth interviews, and instructional text analysis during and after implementation, forming evidence across the three dimensions later defined in Table 3.

Research Design and Research Questions

Research questions (RQs). This study asks whether big ideas-guided systematized teaching can help students form global structural cognition and knowledge connections (RQ1), promote formal reasoning and engineering transfer (RQ2), enhance learning engagement and self-efficacy (RQ3), and provide integrated qualitative evidence of teaching effectiveness (RQ4).

Research field and participants. This study uses discrete mathematics classes taught by the same teacher as the research field, with a teaching intervention period of 8 weeks (16 credit hours in total). The class implementing the big ideas systematization reconstruction plan served as the primary observation subject, with a traditional teaching class retained as a comparative reference. Students participating in in-depth interviews were selected through purposive sampling based on differences in classroom participation and learning performance, covering typical cases from different learning levels. A total of 12 students participated in interviews (8 from the implementation class, 4 from the comparison class). The course teacher was also interviewed twice in semi-structured format, focusing on their subjective assessment of the instructional design execution and student responses.

Data collection and analysis methods. Data sources include three types: (1) Classroom Observation Records: The researcher participated throughout all classes, conducting structured observations focused on students' cross-module knowledge connection behaviors, question-asking patterns, and classroom interaction modes, producing sequential observation notes; (2) Interview Transcripts: Semi-structured interviews were conducted with students and the teacher, with interview guides designed around the three analytical dimensions (D1–D3); audio recordings were transcribed and coded; (3) Student Assignment Texts: Unit inquiry reports and engineering projects were collected and analyzed for cross-module thinking connections and reasoning transfer characteristics.

Data were analyzed through Braun and Clarke's six-step Thematic Analysis, including open coding, category clustering, and theme extraction. Core themes were aligned with D1–D3 and illustrated by quotes and classroom events.

Analytical Dimension Definition

To ensure the qualitative analysis is systematic and traceable, this study explicitly defines the three analytical dimensions, as shown in Table 3.

Table 3

Analytical Dimensions and Definitions

Dim.	Dimension Name	Connotation Description
D1	Global Structural Cognition and Knowledge Connection	Whether students can understand the intrinsic links of discrete math modules from an overall framework of "structure-relation-invariance-algorithm exit".
D2	Formal Reasoning and Engineering Transfer Capability Perception	Students' perception of their formal reasoning/proof capabilities, and mastery of modeling and algorithm transfer methods.
D3	Learning Engagement and Self-Efficacy	Students' interest in the learning style, willingness to actively explore, and sense of self-efficacy.

Big ideas – dimension mapping. D1 covers the five core concepts and focuses on cross-module structural connection; D2 emphasizes reasoning/proof and modeling/computability for engineering transfer; D3 reflects engagement, task control, and acceptance of evaluation methods.

Qualitative Research Findings

RQ1: Global structural cognition and knowledge connection (D1). Classroom observation records show that in the class implementing big ideas systematized teaching, students discussing "functions and relations" spontaneously drew analogies to "proposition truth-value correspondence" from the logic module, demonstrating active cross-module connection awareness. By contrast, students in the traditional teaching class tended to address problems in isolation, rarely positioning current knowledge points within an overall framework.

In interviews, several students expressed similar experiences:

> "Previously, I never felt that graph theory had anything to do with set theory. But now I know an adjacency matrix is essentially a binary relation—thinking this way gives me a 'mental map'." (Student S3)
>
> "The big ideas framework makes me feel that learning a new chapter is not 'starting from scratch,' but adding a new branch to the same tree." (Student S7)

Teacher interviews confirmed this shift: students increasingly annotated the big idea category of knowledge points in unit inquiry reports.

Overall, RQ1 evidence indicates a shift from modular memorization to holistic structural understanding.

RQ2: Formal reasoning and engineering transfer capability perception (D2). Project texts showed that implementation-class reports often formed a complete "modeling – formal expression – reasoning verification" chain, integrating predicate logic, partial orders, and graph reachability.

Representative quotes include:

> "At first I didn't know how to prove that deadlock wouldn't occur. Then I thought of using the 'irreflexivity' of partial order relations—and found it connects perfectly with what we learned in set theory. This made me feel that discrete mathematics is truly 'alive'." (Student S2)
>
> "The natural deduction reasoning I learned in this course—I suddenly realized it uses the same logic as white-box testing in software engineering: both derive conclusions step by step from premises." (Student S5)

By contrast, many comparison-class students still reported uncertainty about the engineering value of proofs.

RQ2 evidence indicates that cross-module inquiry and engineering projects strengthened students' awareness of formal reasoning transfer.

RQ3: Learning engagement and self-efficacy (D3). Observations showed higher voluntary participation and better question quality in the implementation class, shifting from procedural questions to conceptual comparison.

In interviews, students' acceptance of the learning style and changes in self-efficacy were also confirmed:

> "Previously I thought discrete mathematics was just memorizing theorems. Now I've discovered it has a 'skeleton'—learning feels more directional and less anxiety-inducing." (Student S1)
>
> "The project was genuinely difficult, but because big ideas served as 'scaffolding,' I knew which direction to approach from. That feeling was completely different from before." (Student S6)

Teacher interviews also noted more independent reading and proactive questioning in the later teaching stages.

RQ3 evidence indicates improved learning experience and self-efficacy when students faced complex tasks.

RQ4: Integrated evidence across three dimensions. Synthesizing D1–D3, big ideas-guided teaching shows a logical progression: students first build holistic knowledge connections, then translate these connections into transfer capabilities through inquiry tasks and engineering contexts, and finally develop stronger identification with and engagement in the systematized learning approach.

RQ4 evidence suggests that the big ideas framework functions as both a curriculum organizer and a pedagogical scaffold, forming a qualitative evidence loop for the reconstruction.

Qualitative Research Conclusions and Pedagogical Implications

Based on the qualitative analysis, the main pedagogical implications are as follows:

The "Structure-Relation-Invariance-Algorithm Exit" thread shifts learning from fragmented memorization to systematized construction.

Cross-module inquiry units provide an important pathway for capability transfer, especially when students integrate multi-module knowledge in engineering contexts.

Better teaching organization improves engagement and self-efficacy; future courses should strengthen process feedback and stratified task challenges.

Practically, these conclusions can support revision of OBE learning objectives, optimization of the two-way specification table, and follow-up qualitative tracking.

Conclusion and Outlook

Addressing the pervasive dilemma of chapter fragmentation in Discrete Mathematics teaching, this study proposed a full-chain systematization reconstruction covering concept extraction, vertical-horizontal knowledge networking, and teaching evaluation. As routine programming tasks are increasingly automated in the Artificial Intelligence Generated Content (AIGC) era, the formal verification and macroscopic system thinking supported by discrete mathematics become increasingly important. Future research will attempt to integrate intelligent tutoring systems and automated data tracking to support large-scale, personalized exploratory knowledge graph generation.

Synthesizing the RQ1–RQ4 qualitative analysis, classroom observations and interview data indicate positive improvements in structural cognition, reasoning transfer, and learning engagement. The mutually corroborating evidence supports the effectiveness of big ideas-guided systematized teaching. Subsequent research should introduce systematic qualitative tracking and expand the research field to verify external transfer effects.

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