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Comparison of the Statistical Power of Siegel-Tukey and Savage Tests: A Study with Monte Carlo Simulation

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This study presents the results of a Monte Carlo simulation to compare the statistical power of Siegel-Tukey and Savage tests. The main purpose of the study is to evaluate the statistical power of both tests in scenarios involving Normal, Platykurtic and Skewed distributions over different sample sizes and standard deviation values. In the study, standard deviation ratios were set as 2, 3, 4, 1/2, 1/3 and 1/4 and power comparisons were made between small and large sample sizes. For equal sample sizes, small sample sizes of 5, 8, 10, 12, 16 and 20 and large sample sizes of 25, 50, 75 and 100 were used. For different sample sizes, the combinations of (4, 16), (8, 16), (10, 20), (16, 4), (16, 8) and (20, 10) small sample sizes and (10, 30), (30, 10), (50, 75), (50, 100), (75, 50), (75, 100), (100, 50) and (100, 75) large sample sizes were examined in detail. According to the findings, the power analysis under variance heterogeneity conditions shows that the Siegel-Tukey test has a higher statistical power than the other nonparametric Savage test at small and large sample sizes. In particular, the Siegel-Tukey test was reported to offer higher precision and power under variance heterogeneity, regardless of having equal or different sample sizes.

Keywords: nonparametric test, statistical power, Siegel-Tukey test, Savage test, Monte Carlo simulation

Introduction

The importance of statistics in contemporary scientific research has increased with a variety of statistical analysis methods, especially common in the social and health sciences. Statistics aims to make reliable predictions with samples in difficult situations of population data. Applied statistics, such as biostatistics, biometrics, econometrics, sociometrics, archaeometrics, etc., has spread over a wide area including many disciplines. The results obtained by using statistical methods frequently in medical research are widely accepted in the scientific community.

Most statistical methods generally share two key features. The first is the assumption that the underlying

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density function is known; the second focuses on hypothesis testing on the parameters of the density function or their estimates. These tests are called parametric tests based on certain population assumptions. These assumptions usually relate to population normality and random selection of samples. If these assumptions are not met, nonparametric techniques have to be used. Nonparametric methods were mostly proposed in the late 1940s and many studies on their properties have been conducted since then (Gibbons, 1993).

While robust and distribution-independent rank tests are widely available to assess differences between population locations, the same is not true for scale differences. One of the most common distribution-independent tests used to examine scale differences is the Siegel-Tukey test. This test is a linear rank statistic based on score functions symmetric around 1/2 (Clifford Blair, & Thompson, 1992).

The Siegel-Tukey test can be considered a multivariate generalization of its function for testing equality of variance in the univariate context (Pawar, & Shirke, 2019). The Siegel-Tukey test is effective in identifying distribution or scale differences and is a modification of the Wilcoxon test. Instead of ranking the observations of the combined sample, observations are ranked based on their distance from the median of the combined sample (Savage, 1978). Developed in 1960, the Siegel-Tukey test has a similar procedure to the Wilcoxon Rank Sum test that examines differences in location. The Siegel-Tukey test is a nonparametric statistical test used to determine whether one of two populations has a wider distribution than the other (Chen et al., 2019). This test is based on the logic that if two samples come from the same median value, the sample with higher variability will have more extreme scores. The Siegel-Tukey statistic has the advantage that it can be transformed into a U statistic for use with the Mann-Whitney U critical values table as well as using Wilcoxon's critical values table (Fahoome, & Sawilowsky, 2000).

The hypotheses of a two-way test are as follows:

H_0: There is no difference in variance between the two populations.

H_1: There is a significant difference in variance between the two populations.

Unlike the Siegel-Tukey test, the Savage test does not assume that location remains constant. Here, differences in scale are assumed to cause a change in location. Samples are considered to be drawn from continuous distributions (Fahoome, & Sawilowsky, 2000).

H_0: There is no difference in dispersion against the two-way alternative, but there is a difference.

Theoretically, the Savage test is considered to be the locally most powerful ranking test for the left-skewed Gumbel distribution (Funato et al., 2024). It is a generalized version of the Savage Scores test. The exponential ordinal score test is a method for rank-based analysis of data under a given distribution. The generalized version extends this basic method, allowing for a wider range of applications, thus providing a more appropriate and flexible test for different distribution types and data characteristics (Arboretti et al., 2018).

The statistical formula of the Savage test is as follows:

$$S = \sum_{i=m}^{m} a(R_i) \tag{1}$$

here

$$a(i) = \sum_{j=N+1-i}^{N} \frac{1}{j} \tag{2}$$

so that

$$a(1) = \frac{1}{N}, a(2) = \frac{1}{N-1} + \frac{1}{N}, \ldots, a(N) = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{N-1} + \frac{1}{N}$$

For large sample sizes, the following normal-like approach can be used.

$$S^* = \frac{S - n}{\sqrt{\frac{nm}{N-1}\left(1 - \frac{1}{N}\sum_{j=1}^{N}\frac{1}{j}\right)}} \tag{3}$$

Siegel and Tukey, assuming that the two medians are equal and unknown, define the test statistics as follows. The observations in the pooled samples are ordered and then replaced by the following orders:

$$1\;4\;5\;8\;9\;\cdots\;7\;6\;3\;2 \tag{4}$$

T test statistic, x-refers to the sum of the orders of observations in the sample. H_0 Under (hypothesis) TThe mean and variance of is as follows:

$$\mu_0 = m\frac{N+1}{2}$$
$$\sigma_0^2 = \frac{1}{12}mn(N+1) \tag{5}$$

The sum of the ranks for a sample is calculated. This rank sum can be used with a table of critical values, or it can be converted into a U statistic with the following formula.

$$U^* = R_n - \frac{1}{2}n(n+1) \tag{6}$$

$$U^* = R_m - \frac{1}{2}m(m+1) \tag{7}$$

A simpler method than using positive integer weights symmetrically with respect to the mean is the first N may be the use of an arrangement of integers. Because these integers are the weights used in position for the Wilcoxon rank sum test WN, the tables of the null probability distribution are easily accessible. Siegel and Tukey were first used to provide a sensitive statistic to scale differences. N proposed the use of an arrangement involving positive integers (Miramontes, & Medina, 2014).

In the Siegel-Tukey test, methods with a low standard deviation identify true trends more reliably than methods with a high standard deviation (Genz et al., 2007). $H_0: \lambda = 1$against $H_a: \lambda > 1$The most effective rank test is the Savage test (Randles, & Wolfe, 1979).

In the study, the statistical powers of the nonparametric Siegel-Tukey and Savage tests used to analyze the data obtained from two independent samples were compared. In order to compare the statistical power of Siegel-Tukey and Savage tests, 20000 Monte Carlo simulations were used. In this way, each case is evaluated from a broader perspective and the results are based on a more solid foundation. The use of nonparametric tests is of great importance when the data are not normally distributed. However, there is limited information on the performance of these tests in the literature. This study aims to evaluate the statistical power of Siegel-Tukey and Savage tests under different sample sizes and standard deviations, and to provide researchers with important guidance on test selection.

Nonparametric tests are preferred especially when the data are not normally distributed and have an important place in statistical analysis. However, there are limited number of studies in the literature on comparative power analysis of Siegel-Tukey and Savage tests. In this context, comparing the performance of the two tests under different distribution types and standard deviation values will provide researchers with important information on test selection. Using Monte Carlo simulation, this study aims to evaluate the statistical power of Siegel-Tukey and Savage tests under Normal, Platykurtic and Skewed distributions with different sample sizes and standard deviation values.

Materials and Methods

Simulation Study

Monte Carlo simulation studies can be used for a variety of purposes, including evaluating statistical models in challenging scenarios such as limited sample sizes and non-normal data distributions, investigating new statistical questions, and estimating the empirical distribution of a statistic by bootstrapping (Feinberg, & Rubright, 2016).

In the Monte Carlo simulation, the SAS 9.00 computer program was used to obtain different distributions. In this process, Fleishman's (1978) power function was applied. The RANNOR procedure of SAS was used to generate population distributions. This procedure generated random numbers from a standard normal distribution with zero mean and one standard deviation, according to Fleishman's power transformation method. The transformation was performed through Fleishman's power function equation (Fleishman, 1978).

$$Y= a+ [(d\times X + c)\times X + b]\times X \quad (8)$$

The equation used for Fleishman's power function is based on certain constants and includes the distribution variable Y and the normally distributed random variable X with zero mean and one standard deviation. These constants include the values of a, b, c and d, with samples generated using the RANNOR procedure in the SAS program. a is a constant value and includes the variable values a = -c, b, c and d. Once the sample populations are generated, power simulations are performed using the PROC NPAR1WAY procedure (Ramazanov, & Senger, 2023).

The study used Algina, Olejnik, and Ocanto's (1989) three different population distributions: Normal, Platykurtic, and Skewed distributions (Algina et al., 1989). The aim of the study was to examine how statistical power changes under different conditions such as small and equal, small and different, large and equal, large and different sample sizes for each distribution.

In the study, Siegel-Tukey test and Savage test for heterogeneous variances were evaluated to calculate and apply appropriate test statistics for small and large samples. In this planning, 3 different main population distributions, 6 different standard deviation ratios (2, 3, 4, 1/2, 1/3 and 1/4) and 24 different sample size combinations were used, 12 for large sample sizes and 12 for small sample sizes. Two nonparametric statistical tests, Siegel-Tukey and Savage tests, were evaluated to compare the data from the two samples. This information (n_1, n_2) expressed as ordered pairs, where n_1 and n_2 represent the respective sample sizes of the first and second samples, respectively. For example, the pair (5, 5) indicates that both samples consist of 5 elements.

Table 1

Fleishman's Power Function for μ = 0 and σ = 1

Distribution	Skewness ( ) γ_1	Kurtosis ( ) γ_2	*a*	*b*	*c*	*d*
Normal	0.00	0.00	0.00	1.0000000	0.00	0.00
Platykurtic	0.00	-0.50	0.00	1.0767327	0.00	-0.0262683
Skewed	0.75	0.00	-0.1736300	1.1125146	0.1736300	-0.0503344

Sources: Lee (2007), and Senger (2011).

Combinations with small and equal sample sizes include 5, 8, 10, 12, 16 and 20, while combinations with small and different sample sizes include (4, 16), (8, 16), (10, 20), (16, 4), (16, 8) and (20, 10). In the program, combinations with large, equal sample sizes are 25, 50, 75 and 100, while combinations with large and different sample sizes include (10, 30), (30, 10), (50, 75), (50, 100), (75, 50), (75, 100), (100, 50) and (100, 75). The study ran 20 000 Monte Carlo simulations for each case separately through the SAS package. The distributions analyzed include the Normal distribution, the Platykurtic distribution and the Skewed distribution.

Results and Discussion

Findings Obtained in Small Sample

In the study, results were obtained using Monte Carlo simulation software to model the conditions of two samples with 3 distributions and 6 standard deviations but using different sample sizes. In the analysis on small sample size, a total of 12 different samples were analyzed. Half of these samples have exactly equal sample sizes 5, 8, 10, 12, 16, 20, while the other half have different sample sizes (4, 16), (8, 16), (10, 20), (16, 4), (16, 8) and (20, 10) and these conditions were created through simulation.

The results obtained using Monte Carlo simulations showed how the statistical power of the Siegel-Tukey and Savage tests varied for different standard deviation values. For standard deviation 2, the power of the Siegel-Tukey and Savage tests was 0.087 and 0.079, respectively, for small and equal sample sizes. For standard deviation 3, these power values were 0.111 for Siegel-Tukey test and 0.101 for Savage test; for standard deviation 4, these values were 0.122 for Siegel-Tukey test and 0.112 for Savage test. For standard deviation 1/2 and 1/3, the power values of the Siegel-Tukey test were 0.088 and 0.108, and the power values of the Savage test were 0.079 and 0.097, respectively. In the case of standard deviation 1/4, the power of the Siegel-Tukey test was 0.121 and the power of the Savage test was 0.111.

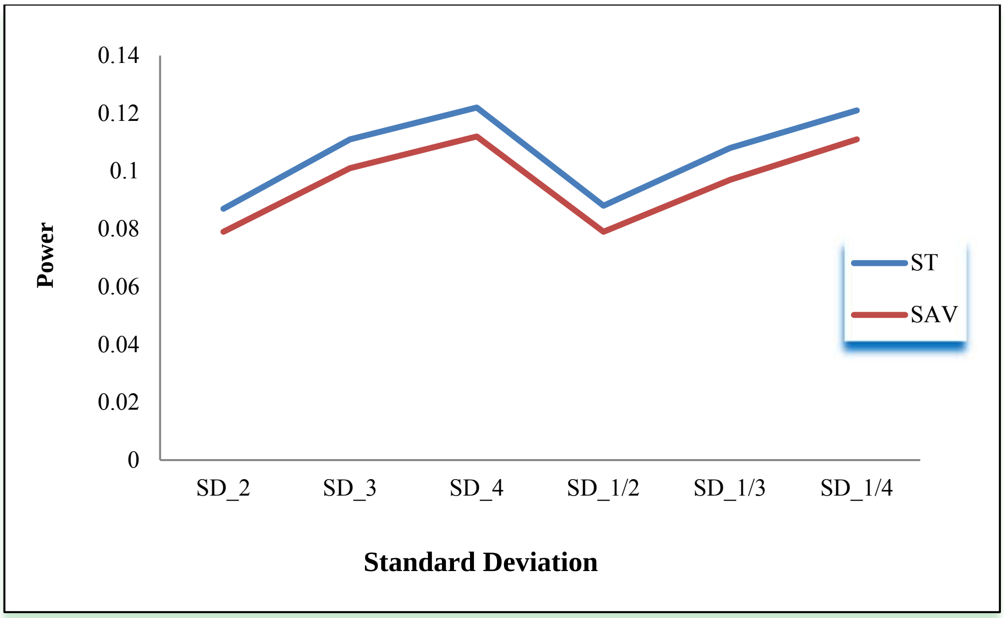

Figure 1. Comparison of the statistical power of Siegel-Tukey and Savage tests for Normal, Platykurtic and Skewed distributions with standard deviation ratios of 2, 3, 4, 1/2, 1/3 and 1/4 in small samples, when sample sizes are equal and variances are heterogeneous.

Small and equal sample size at Standard Deviation 2: This statement refers to the case where the standard deviation is 2 and the sample sizes used in this case are small and equal. That is, the sample sizes used for both tests are equal and small. Test Ratio: The ratios given refer to the power values of the tests. Test power indicates the ability of a hypothesis test to correctly detect the alternative hypothesis. The power values indicated here (between 0 and 1) are low, indicating that the tests generally show low power.

Analyses using Monte Carlo simulations evaluated the statistical power of the Siegel-Tukey and Savage tests at small and different sample sizes and at different standard deviation values. The results obtained are as follows:

For standard deviation 2, the power of Siegel-Tukey test is 0.132 and the power of Savage test is 0.098 for Normal, Platykurtic and Skewed distributions. For standard deviation 3, the power of Siegel-Tukey test is 0.148 and the power of Savage test is 0.119. For standard deviation 4, the power of Siegel-Tukey test is 0.151 and the power of Savage test is 0.131. For standard deviation 1/2, the power of Siegel-Tukey test is 0.129 and the power of Savage test is 0.097. For standard deviation 1/3, the power of Siegel-Tukey test is 0.147 and the power of Savage test is 0.121. For standard deviation 1/4, the power of Siegel-Tukey test is 0.152 and the power of Savage test is 0.132.

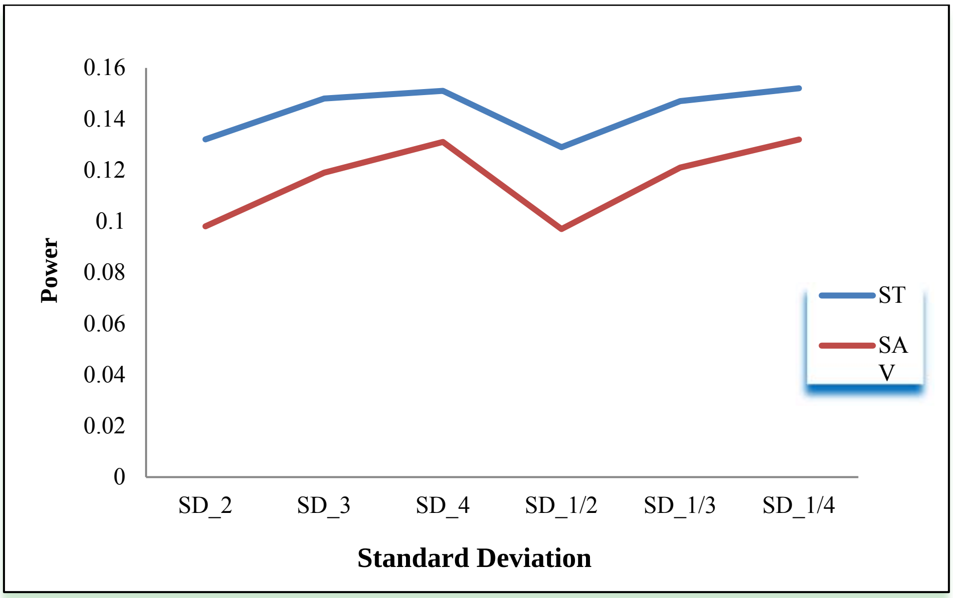

Figure 2. Comparison of the statistical power of Siegel-Tukey and Savage tests for Normal, Platykurtic and Skewed distributions when standard deviation ratios are set to 2, 3, 4, 1/2, 1/3 and 1/4 in small samples, when sample sizes are different and variances are heterogeneous.

Findings Obtained in Large Sample

In this study, a total of 12 different sample sizes were used, focusing on large sample groups. These sample sizes were obtained using Monte Carlo simulation. Each sample size was applied to 6 different population distributions and the results were evaluated by considering the standard deviation rates in each population. In the context of large samples, 4 of the 12 sample sizes are large and equal, while the other 8 are large and different. Large and equal sample groups were determined as 25, 50, 75 and 100, respectively. On the other hand, large and different sample groups are: (10, 30), (30, 10), (50, 75), (50, 100), (75, 50), (75, 100), (100, 50) and (100, 75).

Based on the Monte Carlo simulation results, the statistical power of the Siegel-Tukey and Savage tests was evaluated. The analysis is based on standard deviation values and covers Normal, Platykurtic and Skewed distributions with large and equal sample sizes.

First, when the standard deviation is fixed at 2, the statistical power for the Siegel-Tukey test is 0.205 and for the Savage test is 0.201. This shows a low statistical power for distributions with a certain regularity. When the standard deviation values were increased to 3 and 4, the statistical power of the Siegel-Tukey test was 0.267 and 0.301, and the statistical power of the Savage test was 0.263 and 0.297, respectively. This reveals that the statistical power of the tests increases with the increase in the standard deviation.

On the other hand, when the standard deviation values are considered as 1/2, 1/3 and 1/4, the statistical power of Siegel-Tukey test is 0.205, 0.276 and 0.302, and the statistical power of Savage test is 0.202, 0.272 and 0.298, respectively.

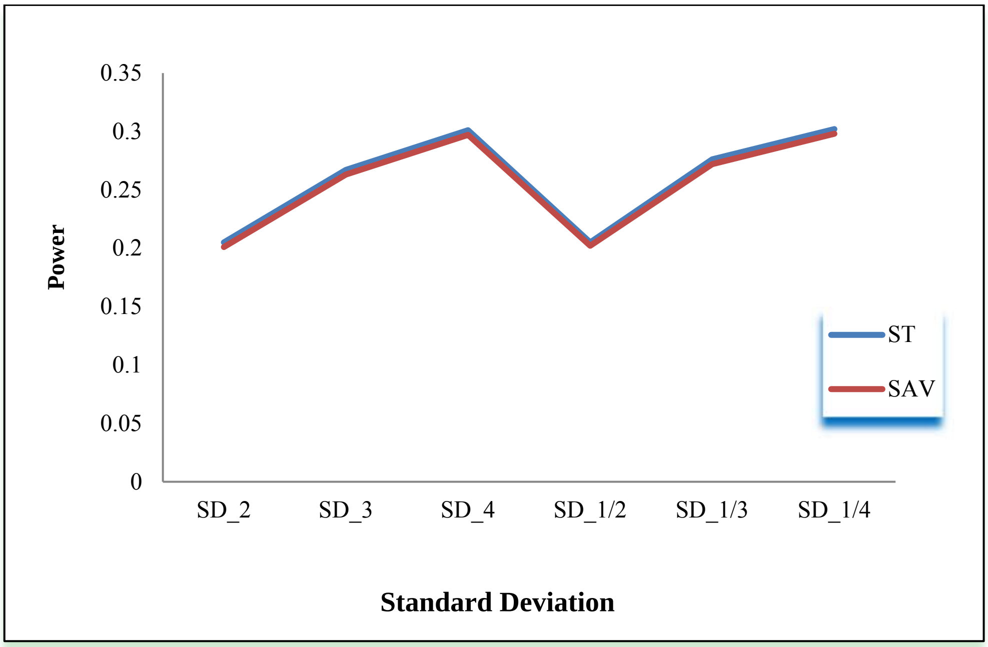

Figure 3. Comparison of the statistical power of Siegel-Tukey and Savage tests for Normal, Platykurtic and Skewed distributions with standard deviation ratios of 2, 3, 4, 1/2, 1/3 and 1/4 for large samples, when sample sizes are equal and variances are heterogeneous.

Based on Monte Carlo simulation results, this study evaluates the performance of Siegel-Tukey and Savage statistical tests in terms of standard deviation values. The study includes Normal, Platykurtic and Skewed distributions with large and different sample sizes. This analysis was carried out to understand the statistical power of the tests and to identify the factors affecting this power.

First, when the standard deviation value was set as 2, the statistical power for the Siegel-Tukey test was 0.199 and the statistical power for the Savage test was 0.196.

When the standard deviation values were increased to 3 and 4, the statistical power of the Siegel-Tukey test was calculated as 0.262 and 0.291, and the statistical power of the Savage test as 0.259 and 0.287, respectively. This shows that the statistical power of the tests increases with increasing standard deviation. In other words, it was observed that the tests were more effective in a wider distribution range.

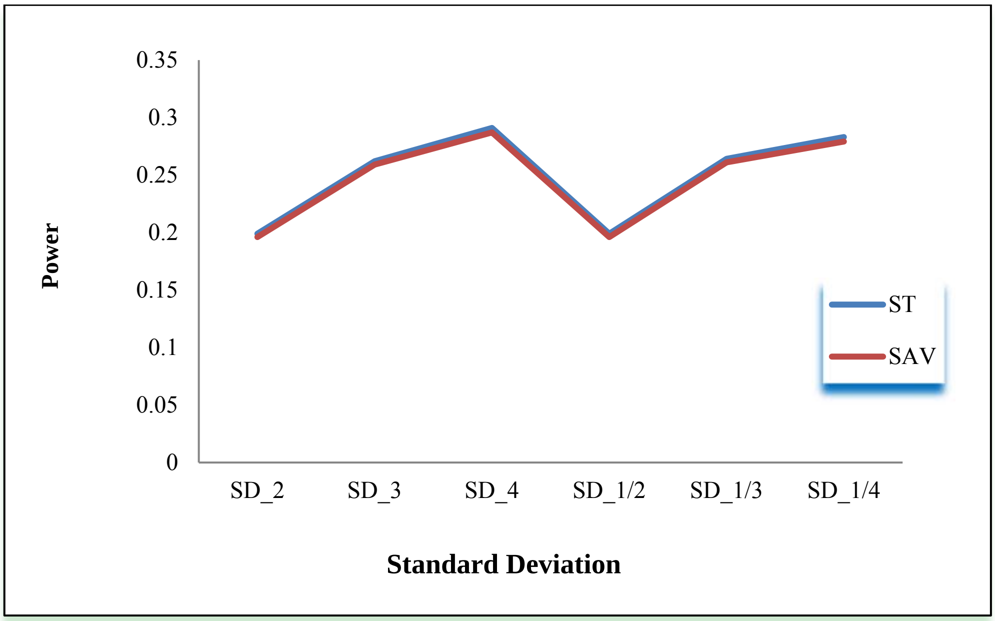

Figure 4. Comparison of the statistical power of Siegel-Tukey and Savage tests for Normal, Platykurtic and Skewed distributions when standard deviation ratios are set to 2, 3, 4, 1/2, 1/3 and 1/4 for large samples, when sample sizes are different and variances are heterogeneous.

Table 2

Powers of ST and SAV Tests for Normal, Platykurtic and Skewed Distributions When the Variances of Sample Sizes Are Heterogeneous Between Two Samples in Small, Equal and Large Samples (Ratio of Standard Deviations = 2, 3, 4, 1/2, 1/3, 1/4)

Population distribution	$\frac{\sigma_1}{\sigma_2}$	Statistical power	
		ST	SAV
Normal	2	0.101	0.089
	3	0.125	0.113
	4	0.144	0.129
	1/2	0.112	0.090
	1/3	0.137	0.111
	1/4	0.141	0.128
Platykurtic	2	0.114	0.087
	3	0.131	0.118
	4	0.145	0.133
	1/2	0.146	0.094
	1/3	0.138	0.118
	1/4	0.141	0.129
Skewed	2	0.205	0.201
	3	0.267	0.259
	4	0.301	0.297
	1/2	0.205	0.202
	1/3	0.276	0.272
	1/4	0.302	0.298

On the other hand, when the standard deviation values are considered as 1/2, 1/3 and 1/4, the statistical power of the Siegel-Tukey test is 0.199, 0.264 and 0.283, and the statistical power of the Savage test is 0.196, 0.261 and 0.279, respectively.

Equal, small and large sample sizes were applied to three different main population distributions, with a total of 24 samples. While determining the sample sizes, standard deviation ratios were calculated over the values of 2, 3, 4, 1/2, 1/3 and 1/4 with special care. The findings are visualized in graphs and these graphs express in detail the distinction made according to sample sizes, main mass distributions and standard deviations. In the graphs presented in Figures 1-4, statistical power values are categorized by equal, small and large sample sizes. In addition, the results of the Siegel-Tukey and Savage tests are expressed graphically and the statistical power ranges of these nonparametric tests are summarized in Table 2.

Conclusion

In this study, the statistical power of the nonparametric Siegel-Tukey and Savage tests used to test data from two independent samples were compared. The findings of this study on the use of these tests have important implications that will guide future scientific studies.

Monte Carlo simulation was used to evaluate the statistical power of Siegel-Tukey and Savage tests on Normal, Platykurtic and Skewed distributions with different sample sizes and standard deviation values. Based on the results of the analysis, the following important conclusions were drawn: First, when the standard deviation is fixed at 2, the Siegel-Tukey test is 0.205 and the Savage test is 0.201. This indicates a low statistical power for distributions within a certain order. However, it was observed that the statistical power of the tests increased as the standard deviation value increased. In particular, when the standard deviation value increases to 3 and 4, the Siegel-Tukey test is 0.267 and 0.301, and the Savage test is 0.263 and 0.297.

On the other hand, when the standard deviation values are considered as 1/2, 1/3 and 1/4, the Siegel-Tukey test is 0.205, 0.276 and 0.302, and the Savage test is 0.201, 0.272 and 0.298, respectively. This shows that the statistical power of the tests decreases as the standard deviation decreases.

One of the most important findings of the study is that power analyses conducted under variance heterogeneity conditions reveal that the Siegel-Tukey test has a higher statistical power than the other nonparametric Savage test at both small and large sample sizes. This result suggests that the Siegel-Tukey test offers higher precision and power under variance heterogeneity, even when sample sizes are equal or different. This finding emphasizes that if researchers prefer the Siegel-Tukey test over the Savage test, they are more likely to identify differences between two samples with the same population distribution.

The superior performance of the Siegel-Tukey test, especially under variance heterogeneity conditions, paves the way for recommending the use of this test. The results suggest that the Siegel-Tukey test should be preferred in studies with both small and large sample sizes, equal or different sample sizes, as well as larger or smaller standard deviation rates. Given that the Siegel-Tukey test tends to offer higher sensitivity and power under variance heterogeneity, this provides important guidance to researchers in test selection.

This study provides important contributions to guide researchers in selecting statistical tests and determining sample sizes. Filling the gap in the literature in this area will enable researchers to make more informed choices among nonparametric tests. Thus, the accuracy and reliability of statistical analyses will be increased. The study provides guiding findings for future scientific studies and provides a comprehensive perspective on the issues to be considered in test selection.

In addition, this study evaluates the performance of nonparametric tests while also revealing the factors that affect the power of the tests. For example, the effect of standard deviation values and sample sizes on the statistical power of the tests is analyzed in detail. It was found that the statistical power of both tests increased when the standard deviation increased, but the Siegel-Tukey test performed better. On the other hand, when the standard deviation decreases, the statistical power of both tests decreases, but the Siegel-Tukey test still offers higher power.

In conclusion, this study contributes to making more informed and accurate choices in the use of nonparametric tests and provides researchers with valuable information on test selection. In this context, it is important to consider conditions such as variance heterogeneity to improve the performance of the tests. The superior performance of the Siegel-Tukey test will enable researchers to obtain more reliable and accurate results if they prefer this test. This study supports the reliability and validity of scientific research by improving the quality of statistical analysis.

As can be seen, the power values of Siegel-Tukey and Savage tests do not exceed 30% in any case. This indicates that the tests are generally low-powered and show limited effectiveness in detecting alternative hypotheses. Low power values are usually caused by small sample sizes and high standard deviations.

However, tests with low power values can be advantageous under certain conditions. In particular, thanks to their non-parametric properties, these tests can also be used when the assumption of normal distribution is not met. Therefore, researchers should carefully evaluate the power and relevance of the tests and select the most appropriate tests for the characteristics of their data sets.

References

Algina, J., Olejnik, S., & Ocanto, R. (1989). Type I error rates and power estimates for selected two-sample tests of scale. *Journal of Educational Statistics*, *14*(4), 373-384.

Arboretti, R., Fontana, R., Pesarin, F., & Salmaso, L. (2018). Nonparametric combination tests for comparing two survival curves with informative and non-informative censoring. *Statistical Methods in Medical Research*, *27*(12), 3739-3769.

Chen, N., Valdes, D., Marlin, C., Blanchoud, H., Guerin, R., Rouelle, M., & Ribstein, P. (2019). Water, nitrate and atrazine transfer through the unsaturated zone of the Chalk aquifer in northern France. *Science of the Total Environment*, *652*, 927-938.

Clifford B. R., & Thompson, G. L. (1992). A distribution-free rank-like test for scale with unequal population locations. *Communications in Statistics-Simulation and Computation*, *21*(2), 353-371.

Fahoome, G., & Sawilowsky, S. S. (2000). *Review of Twenty Nonparametric Statistics and Their Large Sample Approximations.*

Feinberg, R. A., & Rubright, J. D. (2016). Conducting simulation studies in psychometrics. *Educational Measurement: Issues and Practice*, *35*(2), 36-49.

Fleishman, A. I. (1978). A method for simulating non-normal distributions. *Psychometrika*, *43*(4), 521-532.

Funato, M., Murakami, H., Kössler, W., & Mukherjee, A. (2024). Nonparametric tests for combined location-scale and Lehmann alternatives using adaptive approach and max-type metric. *Journal of the Korean Statistical Society*, 1-38.

Genz, A. S., Fletcher, C. H., Dunn, R. A., Frazer, L. N., & Rooney, J. J. (2007). The predictive accuracy of shoreline change rate methods and alongshore beach variation on Maui, Hawaii. *Journal of Coastal Research*, *23*(1), 87-105.

Gibbons, J. D. (1993). *Nonparametric Statistics: An Introduction* (No. 90). Sage.

Lee, C. H. (2007). A Monte Carlo study of two nonparametric statistics with comparisons of type I error rates and power. Oklahoma State University.

Miramontes B. S., & Medina E. R. J. (2014). Análisis de la evolución temporal dela actividad vegetal en áreas de manglar (1990-2010): deltas de los ríos Yaqui y Mayo en Sonora, yrío Fuerte en Sinaloa, México. *Licenciatura en Geoinformática.*

Pawar, S. D., & Shirke, D. T. (2019). Nonparametric tests for multivariate multi-sample locations based on data depth. *Journal of Statistical Computation and Simulation*, *89*(9), 1574-1591.

Ramazanov, S., & Senger, Ö. (2023). For small, equal and large sample sizes, Wald- Type I error rates of Wolfowitz, Kolmogorov-Smirnov and Mann-Whitney tests comparison. *Journal of Ardahan University Faculty of Economics and Administrative Sciences*, *5*(2), 137-144.

Randles, R. H. & Wolfe, D. A. (1979). *Introduction to the Theory of Nonparametric Statistics*. John W iley & Sons, New York.
Savage, I. R. (1978). Nonparametric statistics: The field. *In International Encyclopedia of Statistics*, Vol. 1, ed. V. V. H. Kruskal and J. M. Tanur. New York: The Free Press.
Senger, Ö. (2011). Mann-Whitney, Kolmogorov-Smirnov and Wald-Wolfowitz tests type I Error with Monte Carlo Simulation study in terms of their rates and statistical power. karşılaştırılması. (Published Doctoral Dissertation). Erzurum: Atatürk University Social Institute of Sciences.