Journal of Energy and Power Engineering 18 (2024) 79-87
doi: 10.17265/1934-8975/2023.03.001

The Aharonov-Bohm Effect: An Exploration of Quantum Interference and Electromagnetic Potentials

Bahman Zohuri
Ageno School of Business, Golden Gate University, San Francisco 94105, California, USA

Abstract: The AB (Aharonov-Bohm) effect is a pivotal quantum mechanical phenomenon that illustrates the fundamental role of the electromagnetic vector potential $\vec{A}$ in determining the phase of a charged particle's wave function, even in regions where the magnetic field $\vec{B}$ is zero. This effect demonstrates that quantum particles are influenced not only by the fields directly present but also by the potentials associated with those fields. In the AB effect, an electron beam is split into two paths, with one path encircling a solenoid and the other bypassing it. Despite the absence of a magnetic field in the regions traversed by the beams, the vector potential $\vec{A}$ associated with the magnetic flux $\vec{\Phi}$ through the solenoid induces a phase shift in the electron's wave function. This phase shift, quantified by $\Delta\phi = q\Phi/\hbar c$, manifests as a change in the interference pattern observed in the detection screen. The phenomenon underscores the principle of gauge invariance in QED (quantum electrodynamics), where physical observables remain invariant under local gauge transformations of the vector and scalar potentials. This reinforces the notion that the vector potential $\vec{A}$ has a profound impact on quantum systems, beyond its classical role. This article outlines the AB effect, including its theoretical framework, experimental observations, and implications. The focus on the role of the vector potential in quantum mechanics provides a comprehensive understanding of this important phenomenon.

Key words: AB effect, vector potential ($\vec{A}$), magnetic flux ($\vec{\Phi}$), quantum phase shift, gauge invariance, electromagnetic potentials, interference pattern, scalar fields, longitudinal, waves QED.

1. Introduction

The AB (Aharonov-Bohm) effect is a fascinating quantum mechanical phenomenon that reveals the deep interplay between quantum particles and electromagnetic potentials. First predicted by Yakir Aharonov and David Bohm in 1959, this effect demonstrates that even in regions where the magnetic field is zero, the vector potential can still influence the phase of a charged particle's wave function, leading to observable consequences. This effect underscores the significance of electromagnetic potentials in quantum theory, providing profound insights into the nature of quantum mechanics and gauge invariance.

Standard quantum physics states that electromagnetic fields in areas where a charged particle is strictly forbidden can occasionally affect the particle's motion [1, 2]. We now refer to these phenomena as the AB effect is named for the groundbreaking 1959 publication "Significance of Electromagnetic Potentials in the Quantum Theory", written by Y. Aharonov and D. Bohm [1]. Since then, experts have debated [3-8] what the AB effect tells us about the importance of electromagnetic potentials, assuming that, standard quantum physics accurately describes the natural world.

The opportunity to test quantum mechanics in a new regime, gain new insights into the theory's operation, and some physicists' disbelief in the possibility of observable effects of fields confined to excluded regions have all contributed to the discussion's significant advancement. Over three hundred journal publications about the AB effect have been published in the last thirty years.

Corresponding author: Bahman Zohuri, Ph.D., adjunct professor, research field: electrical and computer engineering.

It has been suggested that calculations ostensibly based on ordinary quantum mechanics demonstrate that the theory does not include the AB effect and that Aharonov and Bohm are merely mistaken [9-12]. The Ehrenfest theorem, which states that a particle or wave packet cannot be deflected in the absence of forces, has been used to demonstrate that something is amiss.

There have been proposed modified versions of quantum mechanics that share the tested predictions of standard theory but do not display the AB effect [13].

According to interpretations based on classical calculations, a particle moving in a field-free region is not truly described by the AB effect; rather, the particle experiences an induced classical force because of its interaction with the source of the fields in the excluded region [14, 15]. Theoretical examinations that validate the conclusions and occasionally the interpretations of Aharonov and Bohm have disproved all these claims [16].

The experimental quantization of the fluxoid in superconducting rings and in Josephson junctions has been interpreted as an experimental confirmation of AB effect [17]. Interference experiments on electron beams have been carried out to provide more direct confirmation, with increasing precision and especially with increasing control of stray fields that might obscure the implications of the experiments [18-20].

The initial concept has also been expanded upon. Theoretically, the AB effect has been explained by substituting a non-Abelian gauge field for the electromagnetic field [7, 8, 21, 22], however there is little likelihood such an experiment could be carried out. It has been proposed that particles with odd spins and presumably unusual statistics, made of electrons bound to magnetic flux lines, might exist in theory [23]. The AB effect is being developed practically to examine the quantum characteristics of mesoscopic normal conductors [24, 25]. Additionally, the charge of the neutron has been measured using the AB effect in a unique experiment [26], and very recently, investigations have shown the structure of flux lines in superconductors [27-28].

A significant number of theoretical disputes have arisen between authors who have said or suggested that they began with similar premises. Others arise when the particle's domain is a multiply connected region, which is always the case in the AB effect, and are caused by the incompleteness of the conventional assumptions. Some authors who rejected the theory's AB effect have questioned the experimenters' purported reduction of inaccuracy from stray fields, casting doubt on the experimental results that show positive results.

The pivotal experiment has now been completed [20]. With remarkable precision and control over the stray field issue, it validates Aharonov and Bohm's predictions.

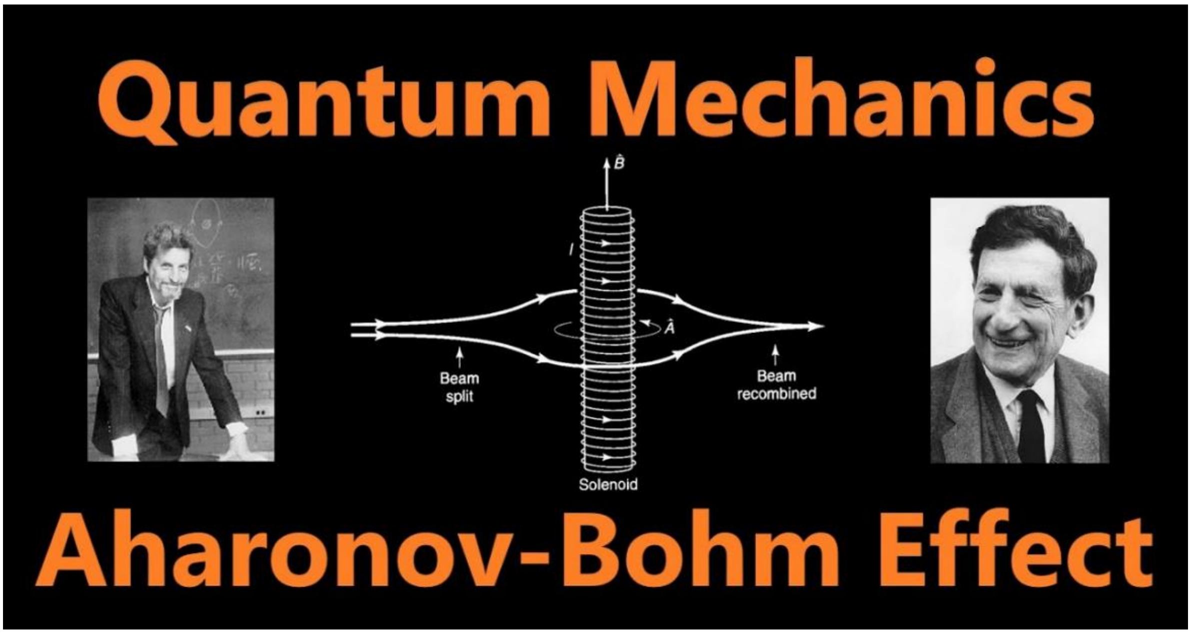
This essay is meant to function as an introduction and critique of the one that follows, written by A. Tonomura, in which he details his own experiments as well as previous attempts at experimentation. The majority of what I share here is not novel. My goals are twofold: first, I will describe the experiment and the theoretical concepts it tests; second, we will talk about the basic problems in physics that the experiment and the theory address.

Nearly all the debate is predicated on nonrelativistic quantum physics, either on the algebraic ramifications of the commutation relations or on the Schrodinger equation. Only a handful of the theory's broad features are necessary for much of it. We think this simple approach—which highlights how closely the AB effect is tied to both the most fundamental and broad aspects of quantum theory—best clarifies all the key difficulties.

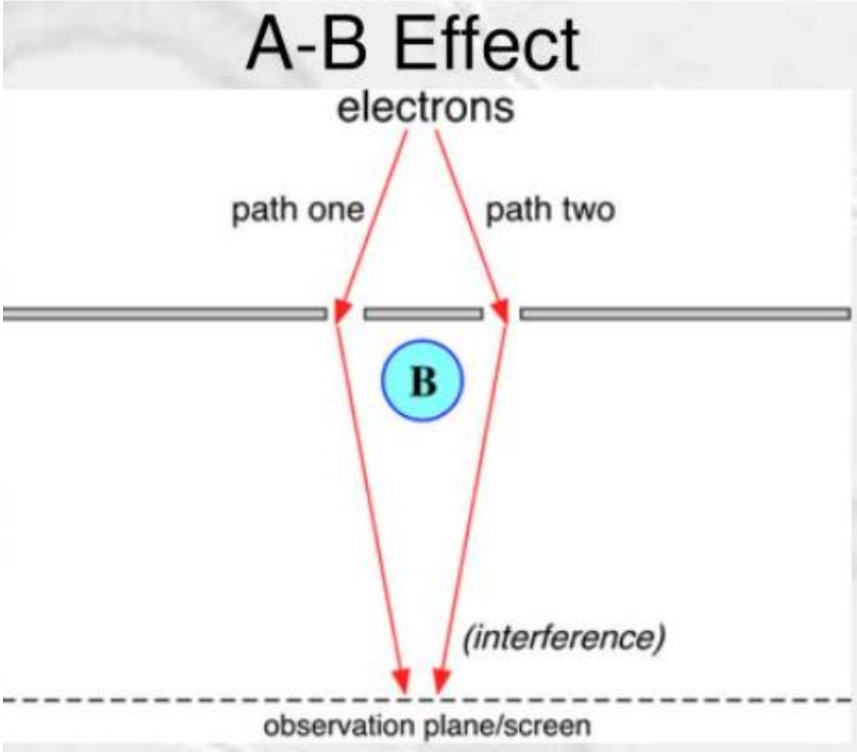
2. What Is the AB Effect?

As we stated in abstract and introductory section of this article above, the following is how the idea was presented in Ref. [2].

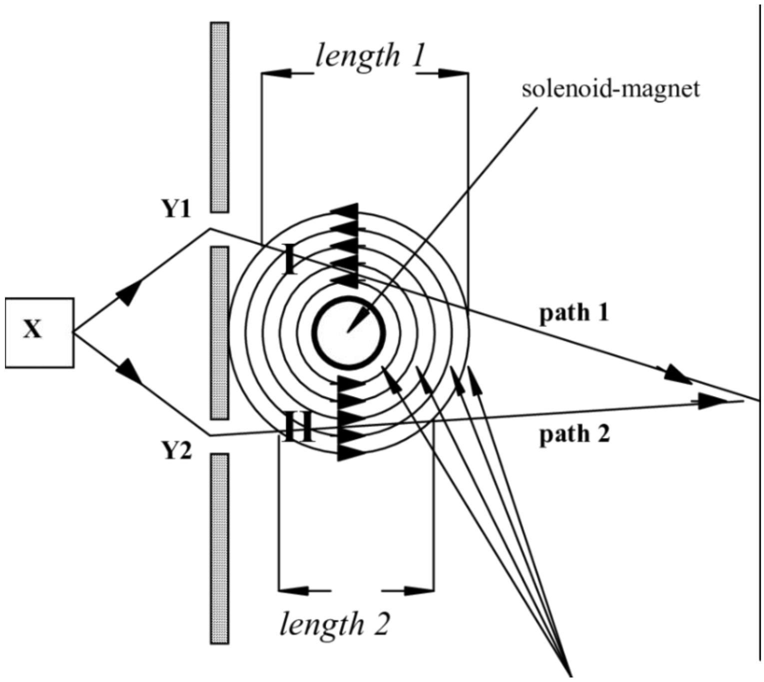
Think about the interference experiment shown in Fig. 1. In a two-arm interferometer, electrons arrive from the left and the beam is split coherently. When the two beams are reunited at the right, any change in the relative phase of the beams in the two arms can, in theory, be seen as a shift in the interference pattern.



(a)

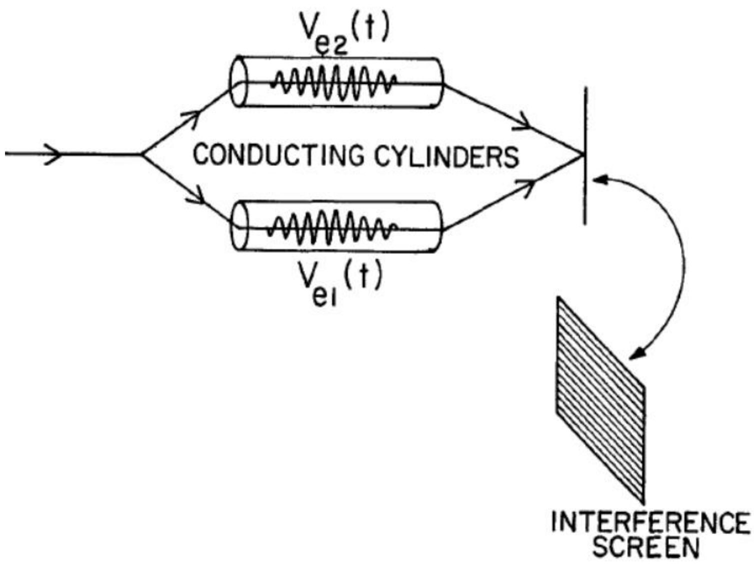


(b)



(c)

Note that: The axis of the solenoid is perpendicular to the page; the wave function is a split plane wave. Also, $Y_{1e} = Y_{2e} = 0$ except, when the wave packet is shielded from the electric field.



(d)

Fig. 1 (a) AB effect; (b) Illustration of interference experiment for AB effect with solenoid inducing magnetic field $\vec{\mathbf{B}}$; (c) Two-slit diffraction experiment of the AB effect; (d) Electric AB effect. $V_{1e} = V_{e2} = 0$ except when the wave packet is shielded from the electric the electric field.

A stationary magnetic field $\vec{\mathbf{B}}$ is introduced in the space between the two beams in the magnetic variant of the AB effect, as shown in Fig. 1a. There are certain barriers that firmly keep the electrons out of that area for all time. The return magnetic flux designed to stay out of the areas where electrons are allowed gives the time-independent wave function $\psi(x)$ and the Hamiltonian H [29]:

$$\mathrm{H} = (1/2m)\left[-i\hbar\nabla + (e/c)A_e(x)\right]^2 - eV_0(x) \qquad (1)$$

$$\psi(x) = \psi_0(x)\exp\{-iS(x)/\hbar\} \tag{2}$$

where $A_e(x)$ is the vector potential due to the excluded magnetic field and $S(x)$ is the line integral as [29]:

$$S(x) = -(e/c)\int^x A_e(x')dx' \tag{3}$$

Note that: deriving these equations the Gaussian units have been used and *e* represents the absolute value of the electron's charge.

And the path of integration is taken along the arm of the interferometer containing the point x, $\phi_0(x)$ is the wave function in the absence of the excluded magnetic field represented by $A_e(x)$, and $V_0(x)$ represents possible electrostatic potentials to steer the beam which do not depend upon the excluded magnetic field.

If the magnetic flux ϕ through the coil is nonvanishing, the vector potential $A_e(x)$ cannot vanish everywhere in the support of $\phi_0(x)$, because $\int^x A_e(x)dx$ on a closed path drawn around the coil through the two arms of the interferometer is equal to ϕ.

In the interference region, the phase shift between the two beams is:

$$\Delta\phi = (S_2 - S_1)/\hbar = (e/\hbar c)\phi \tag{4}$$

where S_2 and S_1 are the action integrals in above equation with that dependency of integral sign calculated along the upper and lower arms of the interferometer.

The phase shift $\Delta\phi$ between the beams in the two arms of the interferometer is gauge invariant, as it must be, depending only upon the magnetic flux through the excluded region. The interference pattern is therefore a periodic function of that magnetic flux, with period equal to London's unit [29],

$$\phi_0 = 2\pi\hbar c/e \tag{5}$$

In the electric version of AB effect, the split beam progresses through ideal conducting pipes that shield the electrons from electric fields as shown in Fig. 1b. In this case, the incident beam must consist of a bunch whose length is much smaller than the length of the conducting pipes. Voltages $V_{e1}(t)$ and $V_{e2}(t)$ are impressed on the two pipes, but only during a limited time interval while the split electron beam is deep inside one pipe or the other, so that an electron never experiences any local electric field. Now the Hamiltonian is given by Peshkin and Tonomura [29]:

$$H = H_0 - eV_e(x,t) \tag{6}$$

where $H_0 = -(\hbar^2/2m)V^2$ and the wave function is:

$$\phi(x,t) = \phi_0(x,t)\exp\left\{-iS_{e\ell}(x,t)/\hbar\right\} \tag{7}$$

where ϕ_0 represents the split wave packet in the absence of external potential $V_e(x,t)$ and

$$S_{e\ell}(x,t) = -e\int_0^t V_e(x,t')dt' \tag{8}$$

When the two packets reach the point *x* in the interference region at some time *t* after $V_e(x,t)$ has returned to zero everywhere, their relative phase is shifted by the amount:

$$\Delta\phi = [S_{2e\ell}(x,t) - S_{1e\ell}(x,t)]/\hbar \tag{9}$$

and that shows up as an observable change in the interference pattern that depends upon the potentials impressed on the two pipes at earlier times *t*' when the electrons were inside the pipes and experienced no local electric field.

Eq. (7) gives a solution of the Schrödinger equation:

$$i\hbar(\partial\phi/\partial t) = [H_0 - eV_e(x,t)]\phi \tag{10}$$

although $\nabla V(x,t)$ vanishes wherever $\phi_0(x,t)$ does not vanish. That is mathematically the essence of the electric AB effect. To achieve that and still get a phase shift between the two beams, we need a region between them where the wave function $\phi_0(x,t)$ vanishes and the electric field, - $\nabla V(x,t)$ $\nabla V(x,t)$, does not vanish. The electron must therefore be confined to a multiply connected region surrounding the excluded electric field, but now that is a space-time region and the periodicity in the external field involves a space-time integral.

3. Classical Physics Context

There is no AB effect in classical physics. AB effect enters quantum mechanics through the appearance of the electromagnetic potentials $V_e(x,t)$ and $A_e(x,t)$ in the Hamiltonian and consequently in the Schrödinger equation.

Potentials occur in the same way as they do in quantum theory when classical theory is presented in the Lagrangian or Hamiltonian formulation. However, we are aware that those classical physics formulas are analogous to Newton's equations, meaning that the local electric and magnetic fields acting on a charged particle control its motion entirely. Newton's second law and the Lorentz force equation, then can be represented as:

$$m\left(d^2r/dt^2\right) = -e\left[E + (\mathrm{v}/c)\times B\right] \qquad (11)$$

and nothing more is needed. As the local conservation of momentum and energy between the particles and fields depends on it, eliminating this aspect of the classical theory in the case of a multiply connected region is not a promising endeavor.

It follows that the peculiarity of quantum theory that the AB effect depends on the flow or the action in units proportional to Planck's constant $\hbar$ should come as no surprise.

Attempts have nevertheless been made to obtain AB effect from classical or semiclassical theory by invoking a reaction on the beam particle which results from its action on the source of the excluded external field [14, 15]. That too is an unpromising way to try to explain an interference pattern or a scattering cross section, because for small e the amplitudes would be proportional to e^2 and cross sections to e^4, while quantum mechanics finds them proportional to e and e^2 respectively.

Moreover, the AB effect primarily arises from quantum mechanics and does not have a classical analog in the same way that classical electromagnetism deals with fields and forces. However, understanding its classical context can help illustrate why this effect is uniquely quantum and not classical.

3.1 Classical Physics Context

3.1.1 Classical Electromagnetism

In classical electromagnetism, the behavior of charged particles is explained by Maxwell's equations, which describe how electric and magnetic fields interact with charges. Classical theory asserts that particles are influenced by the electric and magnetic fields in their immediate vicinity. If a region is devoid of a magnetic field, as in the region around a solenoid in the AB setup, classical electromagnetism predicts that no influence should be felt by particles moving through that region.

3.1.2 Magnetic Field and Vector Potential

In classical theory, the vector potential $\vec{A}$ is a mathematical construct used to simplify calculations, but physical effects are attributed to the magnetic field $\vec{B}$, which is derived from $\vec{A}$ through $\vec{B} = \nabla \times \vec{A}$. In the AB effect, the magnetic field is zero in the region where particles travel, yet the vector potential $\vec{A}$ still influences the particles' quantum phase. This behavior is not predicted by classical theory alone.

3.2 Classical Quantum Mechanical Perspective

3.2.1 Phase Shift and Interference

The AB effect demonstrates that the quantum phase of a particle's wave function is affected by the vector potential $\vec{A}$, even when the magnetic field $\vec{B}$ is zero in the region where the particle travels. This phase shift, which influences interference patterns, is a purely quantum mechanical phenomenon and does not have a direct classical counterpart.

3.2.2 Gauge Invariance

The effect highlights the principle of gauge invariance in quantum mechanics, where physical observables remain unchanged under local gauge transformations. Classical electromagnetism does not account for such gauge invariance in the same way quantum mechanics does, as it deals with fields and forces rather than quantum phases.

3.3 Classical Quantum Mechanical Perspective

3.3.1 Historical Context

The classical theory provided the groundwork for understanding electromagnetism, but quantum mechanics extended these concepts to describe phenomena that classical theory could not. The AB effect emerged from the development of QED (quantum electrodynamics), a quantum field theory that incorporates both the classical electromagnetic fields and quantum principles.

3.3.2 Conceptual Insights

While the AB effect itself is purely quantum, it underscores the limitations of classical theory in fully describing the behavior of particles in electromagnetic potentials. It highlights the need for a quantum mechanical framework to understand and predict the behavior of particles influenced by potentials in ways that classical electromagnetism does not address.

In summary, while the AB effect cannot be fully explained by classical physics alone, understanding the classical context helps clarify why this effect is a distinct and critical phenomenon in quantum mechanics. It illustrates the limitations of classical theories in accounting for quantum effects and the necessity of quantum mechanics for a complete description of particle behavior in electromagnetic fields.

4 Interferometry Coordinates and Interoperability

Interferometry involves measuring the interference patterns created by the superposition of two or more wavefronts. In the context of quantum mechanics and the AB effect, interferometry is used to detect the phase shifts caused by the vector potential.

4.1 Interferometry Coordinates

4.1.1 Spatial Coordinates

In a typical interferometric setup, spatial coordinates define the positions of the beam splitters, mirrors, and detectors. The arrangement of these components determines the paths of the beams and their interference.

4.1.2 Phase Coordinates

Coordinates in phase space describe the phase difference introduced by the vector potential. In the AB effect, this phase shift affects the interference pattern observed.

4.2 Interoperability

4.2.1 Integration with Quantum Mechanics

The AB effect illustrates how quantum mechanical systems can be influenced by potentials that are not directly observable through classical fields. It shows the need for coherence between quantum theoretical predictions and experimental setups.

4.2.2 Compatibility with Other Effects

Interferometry setups can be designed to test various quantum effects, including the AB effect. Ensuring interoperability involves aligning experimental configurations with theoretical models to accurately measure and interpret phase shifts and other quantum phenomena.

In summary, the AB effect reveals the impact of vector potentials on quantum phases, and interferometry is a key technique for observing these effects. Understanding the coordinates and interoperability in these contexts helps in designing experiments and interpreting results in quantum mechanics.

5. About the AB Effect and LSW (Longitudinal Scalar Wave)

The interaction of longitudinal scalar waves with the AB effect is an intriguing topic, but it is important to clarify how these concepts relate within the framework of QED and classical electromagnetism.

5.1 LSW

5.1.1 Definition and Context

In classical electromagnetism, longitudinal waves are those where the oscillations of the field are parallel to the direction of propagation. Scalar waves can be longitudinal if we consider scalar fields that vary in space and time, such as sound waves in a medium. However, in the context of electromagnetism and quantum field theory, scalar waves are generally not longitudinal but are instead described as oscillations in a scalar field.

In quantum field theory, scalar fields describe particles with zero spin, like the Higgs boson. These fields are usually treated as having a scalar nature and are not typically associated with longitudinal wave solutions in the same way as electromagnetic fields.

5.1.2 Longitudinal Components in Electromagnetism

In electromagnetism, electromagnetic waves are typically transverse, meaning the electric and magnetic fields oscillate perpendicular to the direction of propagation. Longitudinal components in electromagnetic theory are usually associated with static fields or specific configurations, such as in electrostatic fields or plasmas, rather than free-space electromagnetic waves.

5.2 Interaction with the AB Effect

5.2.1 AB Effect Overview

The AB effect demonstrates how the vector potential $\vec{A}$ affects the quantum phase of a particle, even in regions where the magnetic field $\vec{B}$ is zero. This effect is primarily concerned with the vector potential's influence on quantum interference patterns.

5.2.2 Scalar Fields and the AB Effect

Scalar fields, including longitudinal scalar waves, do not directly interact with the vector potential in the same manner as the magnetic field does. The AB effect specifically highlights the role of the vector potential in quantum phase shifts. Scalar fields typically affect quantum systems differently and are not directly responsible for the phase shifts observed in the AB effect.

However, if a scalar field were to interact with or modify the electromagnetic potential in a way that affects the vector potential or the charge distribution, it could indirectly influence the AB effect. This interaction would be more complex and would involve the scalar field influencing the effective electromagnetic environment rather than directly causing the AB effect.

6. Conclusions

The AB effect stands as a quintessential demonstration of quantum mechanics, revealing that the vector potential $\vec{A}$, rather than the magnetic field $\vec{B}$, directly influences the quantum phase of particles. This effect, observed through changes in interference patterns of an electron beam encircling a solenoid, underscores the unique principles of quantum mechanics, including gauge invariance and the role of potentials in quantum phase shifts.

Classical electromagnetism, grounded in Maxwell's equations, primarily focuses on the direct influence of electric and magnetic fields on charged particles, without accounting for the quantum phase shifts introduced by vector potentials. The AB effect highlights the limitations of classical theories in describing phenomena that emerge at the quantum level, where potentials have physical significance beyond their classical counterparts.

Interferometry plays a crucial role in experimentally observing the AB effect, providing insights into the quantum behavior of particles and reinforcing the principles of QED. The integration of quantum and classical concepts demonstrates the necessity of quantum mechanics for understanding and predicting the behavior of particles influenced by electromagnetic potentials, marking a significant departure from classical interpretations of field theory [30-34].

References

[1] Aharonov, Y., and Bohm, D. 1959. "Significance of Electromagnetic Potentials in the Quantum Theory." *Phys. Rev.* 115 (3): 485.

[2] Wu, T. T., and Yang, C. N. 1975. "Concept of Nonintegrable Phase Factors and Global Formulation of Gauge Fields." *Phys. Rev. D* 12 (12): 3845-57.

[3] Bocchieri, P., and Loinger, A. 2008. "Nonexistence of the Aharonov-Bohm Effect." *Nuovo Cimento* 47: 475-82.

[4] Ehrenberg, W., and Siday, R. W. 1949. "The Refractive Index in Electron Optics and the Principles of Dynamics." *Proceedings of the Physical Society. Section B* 62 (1): 8.

[5] Lenz, F. 1962. Phys. BI. 18 (1962): 305; see also Lenz, F. 1961. Naturwiss. 48 (1961) 82.

[6] Kobe, D. H. 1979. "Aharonov-Bohm Effect Revisited." *Ann. Phys.* 123 (2): 381-410.

[7] Marton, L. 1953. "Electron Interferometry." *Science* 118: 470-4.

[8] Werner, F. G., and Brill, D. R. 1960. "Significance of Electromagnetic Potentials in the Quantum Theory in the Interpretation of Electron Interferometer Fringe Observations." *Phys. Rev. Lett.* 4 (7): 344.

[9] Chambers, R. G. 1960. "Shift of an Electron Interference Pattern by Enclosed Magnetic Flux." *Phys. Rev. Lett.* 5: 3-5.

[10] Fowler, H. A., Marton, L., Simpson, J. A., and Suddeth, J. A. 1961. "Electron Interferometer Studies of Iron Whiskers." Appl. Phys. 32 (6): 1153-5.

[11] Mollenstedt, G., and Bayh, W. 1962. "Kontinuierliche Phasenschiebung von Elektronenwellen im kraftfeldfreien Raum durch das magnetische Vektorpotential eines Solenoids." *Phys. BI.* 18: 299-305.

[12] Boersch, H., Hamisch, H., and Grohmann, K. 1962. "Experimenteller Nachweis der Phasenverschiebung von Elektronenwellen durch das magnetische Vektorpotential. II." *Z. Phys*. 169: 263-72. (in German)

[13] Jaklevic, R. C., Lambe, J., Mercereau, J. E., and Silver, A. H. 1965. "Macroscopic Quantum Interference in Superconductors." *Phys. Rev. A* 140 (5A): 1628.

[14] Matteucci, G., and Pozzi, G. 1978. "Two Further Experiments on Electron Interference." *Am. J. Phys*. 46: 619-23.

[15] Olariu, S., and Popescu, I. I. 1985. "The Quantum Effects of Electromagnetic Fluxes." *Rev. Mod. Phys*. 57 (2): 339.

[16] Mollenstedt, G., and Docker, H. 1956. "Beobachtungen und Messungen an Biprisma-Interferenzen mit Elektronenwellen." *Z. Phys*. 145: 377-97. (in German)

[17] Bayh, W. 1962. "Messung der kontinuierlichen Phasenschiebung von ElektronenweUen im kraftfeldfreien Raum dutch das magnetische Vektorpotential einer Wolfram-Wendel." *Z. Phys*. 169: 492. (in German)

[18] Mollenstedt, G., and Bayh, W. 1962. "Messung der kontinuierlichen Phasenschiebung von Elektronenwellen im kraftfeldfreien Raum durch das magnetische vektorpotential einer Luftspule." *Naturwiss*. 49: 81-2. (in German)

[19] Aharonov, Y., and Bohm, D. 1961. "Further Considerations on Electromagnetic Potentials in the Quantum Theory." *Phys. Rev*. 123 (4): 1511.

[20] Krimmel, E., Mollenstedt, G., and Rothemund, W. 1964. "Measurement of Contact Potential Differences by Electron Interferometry." *Appl. Physics Letters* 5 (209).

[21] Furray, W. H., and Ramsey, N. F. 1963. *Z. Angew. Phys.* 16; Furray, W. H., and Ramsey, N. F. 1960. "Significance of Potentials in Quantum Theory." *Phys. Rev*. 118 (3): 623-6.

[22] Wegener, H. 1960. *Z. Phys.* 159: 243.

[23] Mitler, H. E. 1961. "Electromagnetic Potentials in Quantum Mechanics." *Phys. Rev*. 124 (3): 940-4.

[24] Feynman, R. P., Leighton, R. B., and Sands, M. 1964. *The Feynman Lectures in Physics* (Vol. 2). Reading, MA: Addison-Wesley, p. 15.

[25] Peshkin, M., Talmi, I., and Tassie, L. J. 1961. "The Quantum Mechanical Effects of Magnetic Fields Confined to Inaccessible Regions." *Ann. Phys.* 12: 426-35.

[26] Tassie, L. J., and Peshkin, M. 1961. "Symmetry Theory of the Aharonov-Bohm Effect: Quantum Mechanics in a Multiply Connected Region." *Ann. Phys.* 16 (2): 177-84.

[27] Noerdlinger, P. D. 1962. "Elimination of the Electromagnetic Potentials." *Nuovo Cimento* 23: 158-67.

[28] DeWitt, B. S. 1962. "Quantum Theory without Electromagnetic Potentials." *Phys. Rev.* 125: 2189.

[29] Peshkin, M., and Tonomura, A. 1998. *Lecture Notes in Physics: The Aharonov-Bohm Effect*. Wien: Springer-Verlag.

[30] Aharonov, Y., and Bohm, D. 1959. "Significance of Electromagnetic Potentials in the Quantum Theory." *Physical Review* 115 (3): 485-91.

[31] Chambers, R. G. 1960. "Shift of an Electron Interference Pattern by Enclosed Magnetic Flux." *Physical Review Letters* 5 (3): 3-5.

[32] Tonomura, S., Endo, J., Matsuda, T., and Kawasaki, T. W. 1986. "Direct Observation of the Aharonov-Bohm Effect." *Physical Review Letters* 56 (8): 792-5.

[33] Aharonov, Y., and Casher, A. 1984. "Topological Quantum Effects for Neutral Particles." *Physical Review Letters* 53 (7): 319-21.

[34] Thouless, D. J., Kohmoto, M., Nightingale, M. P., and den Nijs, M. 1982. "Quantized Hall Conductance as a Topological Distinction between Integer and Fractional Quantum Hall Effects." *Physical Review Letters* 49 (6): 405-8.