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Cumulative Effect of Debt and Tax on Firm Value: Optimal Capital Structure Theories in the Light of EMM

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We have shown that cornerstone articles considering effects of corporate debt on the firm value and constituting the basis of the trade-off theory of capital structure are wrong. Their main mistake is in ignoring the business securing expenses (BSEs). In the framework of the extended Merton model (EMM), we consider the cumulative effect of debt and corporate taxes on the firm value and its survival, in other words, we revisit Modigliani-Miller Proposition 3 (MMP3). We show that (1) debt affects the firm value and its survival, (2) this effect is negative, diminishing the firm value and its chances to survive, (3) the pressure increases as the debt grows provoking the firm's default, (4) the main factors depressing the levered firm are its debt payments added to the BSEs of the identical unlevered firm and the length of debt maturity, (5) corporate taxes cause development of positive skewness in the asset distribution, but do not affect the location of this distribution in the asset axis. The presented model helps estimate the consequences of choosing this or that level of debt in the presence of corporate taxes and can make a useful instrument for practicing financial managers.

Keywords: geometric Brownian motion (GBM), extended Merton model, business securing expenses, corporate debt, corporate taxes, default probability

Introduction and Literature Review

Modern theories of capital structure and dividend policy decisions start with the celebrated series of articles by Modigliani and Miller (1958; 1961; 1963) widely known as Modigliani-Miller Propositions (hereafter MMPs). The MMP1 or the theorem of irrelevance (1958) states that under conditions of the perfect market, the capital structure does not affect the expected firm value. In other words, there is no optimal debt leverage. The MMP2 (1961) argues that dividend policy in the same conditions has no influence on the expected firm value. At the time of publication, the MMP1 and MMP2 have shocked economists because a good choice of the capital structure and of the dividend policy was considered crucial for the firm success. The MMP3 (1963) considers two firms: an all-equity (unlevered) firm and the levered firm identical to the unlevered one in all respects but the capital structure. The theorem states that in the perfect market with corporate income taxes, the after-tax expected value of the levered firm equals the after-tax value of the unlevered firm plus the present value of tax deductions. Thus, the MMP3 gives theoretical underpinning to the trade-off theory suggested later by Kraus and Litzenberger (1973). They claim that in the presence of corporate taxes with deductions and bankruptcy costs, there is an optimal debt leverage maximizing the firm value.

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Trade-off Theory

Brennan and Schwartz (1978) and Bradley, Jarrell, and Kim (1984) present the first quantitative models build in the framework of the *static trade-off theory*, but by general opinion, the crucial contribution to this theory is made by Leland (1994) and Leland and Toft (1996). Leland's model (1994) follows the ideas of MMP1 (1958), Merton (1974), and Brennan and Schwartz (1978) in assuming that (i) the firm activities do not depend on its financial structure, and (ii) capital structure decisions, once made, do not change anymore. The author considers the firm and a security representing a claim on the firm which continuously pays for a nonnegative coupon per instant of time when the firm is solvent. Leland assumes that the firm finances the net cost of this coupon by selling additional equity. In these conditions, the asset value is described by the geometric Brownian motion (GBM) equation admitting no firm's payments. The author argues that the security value depending on the firm value and time follows Merton's general equation (Merton, 1974) with boundary conditions determined by payments at debt maturity and by payments in bankruptcy, should it happen before the maturity. Because a solution of the general time-dependent problem is unknown, Leland looks for a stationary solution to this problem. He comes to a closed-form solution for this marginal case writing explicit equations for the firm's debt, equity, and the firm value equal to the asset value, plus the tax deduction of coupon payments, less the value of bankruptcy costs. The last relation makes a quantitative basis for the static trade-off theory, which Leland applies to determine the optimal asset structure for the exogenous default line. If firm's management can choose the moment of default, they can do it, maximizing firm's equity at that moment (the endogenous default). Leland presents a multilateral analysis of various debt covenants and their influence on debt variables. However, his optimal debt level occurs extremely high (75%-90%) while a typical firm has a debt ratio in the 20-30% range (Beattie, Goodacre, & Thomson, 2006). This contradiction definitely indicates that the author has missed important phenomena in his analysis of debt. The problem setting of 1994 has a hard restriction: The debt maturity is supposed to be infinite. Leland and Toft (1996) revisit the problem relaxing this restriction and considering the same problem with a debt of fixed maturity.

There are four main consequences from this theory. First, the firm must have a target debt ratio specific for each firm and a state of its business environment, and the firm must adjust towards the target debt ratio (Myers, 1984). Second, the higher the firm risk, the lesser the debt ratio must be (Myers, 1984). Collateral: All other things equal, firms with relatively safe tangible assets are expected to borrow more than firms with risky intangible assets. Third, firms with higher marginal tax rates must have higher debt ratios (MacKie-Mason, 1990). Fourth, firms with more taxable income and few non-debt tax shields (investment tax credits, depreciation, etc.) must have higher debt ratios (DeAngelo & Masulis, 1980). However, numerous attempts to provide empirical support to these predictions meet with mixed success. Myers (1984) concludes that the static trade-off theory possibly works to some extent, but empirical models trying to prove existence of the target debt ratio, the effects of tax on the debt ratio, etc., have an unacceptably low coefficient of determination R^2 that casts deep doubts on the reliability of those models.

Inability of static trade-off models to explain why levered firms do not follow closely their target debt ratios gives scholars an incentive for developing *dynamic trade-off models* which look for the trade-off between debt benefits and bankruptcy costs in multi-period time intervals in the presence of adjustment costs and other subtle factors affecting financing decisions. Here we give a brief review of those models and their results, sending the reader interested in further details to the comprehensive review on dynamic models by Strebulaev and White

(2012). It is important to mark that all dynamic models in both literature reviews are based on the discrete- or continuous-time geometric Brownian motion (GBM) equation.

Concerning the first prediction, Myers (1984) remarks that empirical data show that actual debt ratios vary widely across apparently similar firms because either the firms take extended excursions from their targets, or the targets themselves depend on factors not yet recognized or understood. Kane, Marcus, and McDonald (1984) and Brennan and Schwartz (1984) are the two pioneer papers considering multi-period continuous-time stochastic models with debts, taxes, and bankruptcy costs, but no transaction costs. An immediate firm reaction to the deviation of debt ratio from its target value revealed in these models proves the importance of adjusting costs for getting a realistic speed of reaction. Fisher, Heinkel, and Zechner (1989) are the first who take into account transaction costs in a dynamic trade-off model. This model confirms that the higher the costs, the farther the debt ratio deviates from the target value, the longer time it takes of the debt ratio to return back to the target value. Strebulaev (2007) confirms this result. Those models show that there is a spacious inaction region in which the firm does not actively adjust its debt ratio, but when at last the firm refinance, it issues a lot of debt.

At the same time, dynamic models give a hope that the dynamic debt adjustment can reconciliate the trade-off theory with the facts observed in business practice. Goldstein, Ju, and Leland (2001) show that if the firm can vary its debt over time, it will opt for a lower initial leverage increasing the leverage step by step as firm's assets grow. Dynamic trade-off models provide lesser optimal debt ratios than static trade-off models, though still higher than those observed in practice. Investigating persistent difference between dynamic model leverage and empirical data, Strebulaev (2007) puts forward an idea that findings about "optimal" capital structure relate only to the moments when financial decisions are made, and debt leverage is actively adjusted. In practice, empirical data on leverage come mostly from the cross-section of firms that spend substantial time in their inaction zones. Thus, it is incorrect to compare empirical cross-sectional debt ratios with theoretical debt ratios because they refer to different time moments. To support this idea, Strebulaev (2007) simulates leverage dynamics for the cross-section of firms implied by a dynamic trade-off model. He simulates a stationary cross-section distribution of debt leverage which occurs to be very asymmetric, with the optimal leverage close to the distribution mode rather than to the distribution mean.

An empirical study by Titman and Wessels (1988) tries to identify factors that affect corporate financial policies and estimate their explanatory power and sign. They find that debt leverage robustly correlates only with size, profitability, and the market-to-book debt ratio. Fama and French (2002) put forward a hypothesis that the effect of profitability is the only really outstanding difference between the trade-off theory and the pecking-order theory suggested by Myers and Majluf (1984). In the pecking-order model, higher profitability leads to less reliance on debt, because internal funds reduce the need in external debt financing, and thus lower the leverage. The trade-off calls for higher leverage in those conditions, because higher profitability increases potential tax benefits of debt. Using regression analysis, Fama and French (2002) find, consistent with most prior research, that leverage is negatively related to profitability which they consider as a strong argument for pecking order. However, Strebulaev (2007) argues that Fama and French (2002) confuse the predictions of the dynamic trade-off model at the times of refinancing with "true dynamics". When all firms reach the refinancing date, the regression of leverage on profitability shows that leverage and profitability are *positively* related in the world described by the trade-off model. Yet the same regression performed on both actively rebalancing and passive firms produces a *negative* relation between leverage and profitability.

Firms can borrow either in the market issuing public bonds, or take loans in banks. When a firm is unable to cover its debt obligations, the default leads either to reorganization or bankruptcy of the firm. For public bonds, any change in features of a debt contract, such as principal, interest, or maturity, requires a unanimous consent of all bondholders. As a consequence, it is very hard to change bond contract parameters. On the contrary, firms can attempt to renegotiate a private debt and, if the renegotiation is successful, the firms change the structure of payments to debtholders to avoid default. In the strategic renegotiation, shareholders refuse to honor debt obligations in full and threaten default, even though they have sufficient resources to fulfill their promises. If debtholders face the bankruptcy costs they prefer to avoid, this threat can be effective. Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997), Fan and Sundaresan (2000), and Hackbarth, Hennessy, and Leland (2007) consider capital structure models with strategic renegotiation. They present closed-form formulas for the renegotiation threshold, optimal debt coupon rate, and optimal levered firm value. They also show that the bank debt coupon is higher than the coupon on public debt. The leverage ratio can be lower in the bank debt case, because bank debt leads to lower ex-ante inefficiency and to a higher total value of the firm. If the firm issues both bank and public debt at the same time, it is optimal to give a strict seniority to bank debt, which is consistent with empirical evidence (Hackbarth et al., 2007).

The separation of investment and financing means that cash flow generating machines are assumed to work independently of financing considerations. Boyle and Guthrie (2003) have started exploring dynamic interactions of financing and investment policies within the framework of dynamic contingent claims. Campello and Hackbarth (2012) incorporate financial considerations into the standard real option setup by adding costly equity financing and a renegotiable bank debt (with a strategic renegotiation structure). A number of further studies explore the impact of agency conflicts on the interaction of investment and financing (Childs, Mauer, & Ott, 2005), growth options and priority structure of debt (Hackbarth & Mauer, 2012), undiversified entrepreneurial finance and capital structure in incomplete markets (Chen, Miao, & Wang, 2010), and the interaction between time-to-build investment lags and capital structure (Tsyplakov, 2008). Even this brief review of applications of static and dynamic trade-off models to various problems of capital structure shows that it is a powerful literature stream involving many talented scholars.

Myers (1984) concludes his article with a remark:

> People feel comfortable with the static trade-off story because it sounds plausible and rationalizes moderate borrowing. Well, the story may be moderate and plausible, but that does not make it right. We have to ask whether it explains firms' financing behavior. If it does, fine. If it does not, then we need a better theory before offering advice to managers. (p. 588)

We show in the next section that all trade-off theories both static and dynamic are wrong because they are founded on the GBM model which does not take account of firm's payments. Any model that fails to include such an intrinsic part of borrowing as debt redemption is absolutely unrealistic with a very high risk of jumping to false conclusions. So, all results of the reviewed papers must be checked with a model taking account of firm's payments.

Pecking-Order Theory of Capital Structure

Pecking order suggests that firms do not have an optimal capital structure, but the mix of funds is determined by the cost of each capital source—internal (retained earnings), or external. If a firm must use external funds, the preference is to use the following order of financing sources: debt, convertible securities, preferred stock, common stock. Myers (1984) argues (and we agree with him) that adverse selection makes firms prefer internal to external financing. When internal funds occur insufficient, firms prefer debt to equity because of lower information costs associated with debt issuing. These line of reasoning leads to a testable prediction that if firms follow pecking order, then in a regression of net debt issues on the financing deficit, a slope coefficient of one must be observed (Shyam-Sunder & Myers, 1999). Their empirical study supports this prediction using a sample of firms that had traded continuously over the period 1971-1989. The authors present the pecking-order model as a parsimonious empirical model of capital structure. However, later empirical evidence is mixed. Frank and Goyal (2003) show that greatest support for pecking order is among large firms. On the contrary, Chirinko and Singha (2000) and Leary and Roberts (2010) find no support for the theory. However, even if the theory is not strictly correct, it still does a robust job. Pecking order is a competitor to other mainstream empirical models of corporate leverage like the trade-off theory (Kraus & Litzenberger, 1976; Leland, 1994), signaling theory (Ross, 1977), and market timing theory (Lucas & McDonald, 1990; Korajczyk, Lucas, & McDonald, 1992). In recent years, papers appear whose authors try to prove that trade-off and pecking order are complementary theories (Chatzinas & Papadopoulos, 2018; Banga & Gupta, 2017; McNamara, Murro, & O'Donohoe, 2017; Pacheco & Tavares, 2017; Serrasqueiro & Caetano, 2015). Another group of researchers reject both trade-off and pecking-order theories (Ohman & Yazdanfar, 2017; Dasilas & Papasyriopoulos, 2015; Wellalage & Locke, 2015). However, the majority of scholars support either the trade-off or pecking-order theory.

The key idea of pecking order is that owners and managers of the firm know the true value of the firm's assets and growth opportunities, while outside investors can only estimate those values with some error (the information asymmetry). The reason why the firm's managers prefer internal funding to external funding can be seen in the fact that external funding requires rather high flotation costs and disclosure of financial information about firm's perspectives, which the firm's managers prefer not to disclose publicly. Since common stocks hold the right in the management, reluctance issuing common stocks reflects managers' incentive to retain control over the firm and their striving to avoid a negative market reaction to an announcement of a new equity issue. Myers and Majluf (1984) explain this hierarchy of funding methods in terms of risk. The firms anxious for lesser risks prefer internal funds to external and debt to equity. The pecking-order theory is vividly discussed till now, and we hope that our arguments on the subject will be of use.

Capital Structure Theories in the Light of EMM

Modigliani-Miller Propositions and trade-off theory. Before we start analysis of papers investigating the problem of capital structure and its dynamics, let us briefly revisit MMP1 (1958) and MMP3 (1963). Shemetov (2023) using Merton's model (Merton, 1974) shows that the only way to obtain all MMP results in one framework is to have absolutely no payments for the levered and unlevered firms. Because there are no dividend payments for both firms, the dividend policy does not affect the firm value (MMP2, 1961). Because there are no debt payments, the capital structure has no influence on the firm value (MMP1, 1958). Because the levered firm does not pay for its debt but enjoys the tax shield, its mean after-tax value is higher than the mean after-tax value of the unlevered firm due to tax deductions (MMP3, 1963). However, the MMP3 is a logical error. To enjoy tax deductions, the levered firm must make no debt payments but receive tax deductions for the nominal debt. In other words, the levered firm in this construction becomes an arbitraging machine making money out of thin air. To improve the situation, one must note that because the levered firm does not pay for its debt, it is indistinguishable from the identical unlevered firm, and, therefore, there is no tax deductions and the tax shield must be zero. So, in the perfect market with corporate income taxes, the after-tax value of the levered firm equals the after-tax value of the unlevered firm (MMP1 with corporate taxes). In all other cases taking account of firm's payments, MMP1 and MMP3 are wrong (Shemetov, 2023). Although MMP1 and MMP3 are false, they have set up the basis for capital structure theories and determined the direction of their development for decades. No wonder that all cornerstone articles investigating financial risks such as Black and Scholes (1973), Merton (1974), Leland (1994), and many others use the MMPs as the absolute truth with sad consequences for their results.

Two most important articles in the trade-off theory are Leland (1994) and Leland and Toft (1996). Leland's objective (1994) is to estimate the corporate debt value and determine the optimal capital structure of the firm paying corporate taxes and bankruptcy costs. The author follows the capital structure theory suggested by the MMP1 (1958) and MMP3 (1963), expanding it with arguments by Kraus and Litzenberger (1973) that as the debt leverage increases, the tax advantage of debt is offset by the firm's losses caused by increasing risks of bankruptcy. Leland's model includes the firm and a security representing a claim on the firm, which continuously pays for a nonnegative coupon per instant of time when the firm is solvent. Leland uses the GBM equation (Merton, 1974) with Black-Cox's default line (1976) as a boundary condition to describe the firm value V:

$$dV/V = \mu dt + \sigma dW,\ V(DL) = 0 \tag{1}$$

where μ is the expected rate of return, σ^2 is the instantaneous variance of the return per unit of time, dW is a standard Gauss-Wiener process, $DL(t)$ is the default line. The value of claim $F(V, t)$ on the firm that continuously pays a nonnegative coupon, C, follows Merton's general equation (Merton, 1974) for security pricing:

$$F_t + rVF_V + 0.5\sigma^2V^2F_{VV} - rF + C = 0 \tag{2}$$

(r is the expected interest rate) with boundary conditions determined by payments at maturity, and by payments in bankruptcy should it happen prior to maturity. Leland puts a hard restriction on debt: It is the debt of infinite maturity. To use Equation (1), Leland assumes that "any net cash outflows associated with the choice of leverage must be financed by selling additional equity" (Leland, 1994, p. 1217). This equity must be external to the firm because "bond covenants restrict firms from selling their assets" (p. 1217). The author accepts all Merton's assumptions, plus introduces a new one that capital structure decisions, once made, remain static.

Because a closed-form solution of the problem (Equation (2)) is unknown, Leland constructs its stationary version, tending time to infinity:

$$rVF_V + 0.5\sigma^2V^2F_{VV} - rF + C = 0. \tag{3}$$

He develops a closed-form solution for this marginal case writing explicit equations for the firm's debt $D(V, C)$, equity, bankruptcy costs $BC(V, C)$, tax-benefit function $TB(V,C)$, and the firm value $v(V, C)$:

$$\begin{gathered} D(V, C) = C/r + [(1 - \alpha)V_B - C/r](V/V_B)^{-X}, \\ BC(V, C) = \alpha V_B(V/V_B)^{-X}, \\ TB(V, C) = (\tau C/r)[1 - (V/V_B)^{-X}], \end{gathered} \tag{4}$$

$$\begin{gathered} v(V, C) = V + TB(V, C) - BC(V, C), \\ X = 2r/\sigma^2. \end{gathered} \tag{5}$$

The asset value at which bankruptcy is declared is V_B; the fraction of value which is lost to bankruptcy is α ($0 \le \alpha \le 1$), leaving stockholders with nothing and debtholders with a value of (1 - α)V_B.

These equations make a quantitative basis of the trade-off theory, which Leland applies for determining the optimal asset structure for exogenous default. On the other hand, if the firm management can choose the moment of default, they can do it, maximizing firm's equity at that moment (the endogenous default). Leland presents a multilateral analysis of various debt covenants and their influence on debt variables. Equation (5) reveals that someone but the firm pays for the debt; else this equation will include (infinite) expenses for coupon payments. On the other side, when the firm does not pay for its debt but receives tax returns, it becomes a classic arbitraging machine generating arbitrage profits. This circumstance explains the most surprising result that the greater the debt, the greater the total firm value (compare it against the MMP3 discussed above), and that the firm gets full benefits when its debt leverage is 100 percent if there are no bankruptcy costs. The bankruptcy costs make the total value as a function of debt a concave down function, securing the existence of optimal leverage. The arbitrage catastrophe makes void all presented results. Here we do not mention the fact that the problem (Equations (1)-(2)) has no stationary solution, and formulas (Equations (4)-(5)) are false (see Shemetov, 2023 for details).

Leland and Toft (1996) revisit the problem of optimal capital structure with a debt of finite maturity. The authors describe firm's productive asset, V, by continuous-time diffusion process with constant proportional volatility σ:

$$dV/V = [\mu(V, t) - \delta]dt + \sigma dW, \tag{6}$$

where $\mu(V, t)$ is the total expected rate of return on asset value V, δ is the constant fraction of the value paid out to security holders, and dW is a standard Gauss-Wiener process. The process continues until V falls to default-triggering value V_B (the default line). The authors consider a bond issue with maturity t from the present, which has principal $p(t)$ and continuously pays a constant coupon flow $c(t)$. In the event of bankruptcy, the debt of maturity t receives the fraction $\varrho(t)$ of asset value V_B. Introducing the density of the first passage time τ from V to V_B, $f(\tau; V, V_B)$, and its cumulative distribution $F(\tau; V, V_B)$, and using the risk-neutral technique, the authors wright an equation for the debt value of maturity t, $D\,(V; V_B, t)$:

$$D\,(V; V_B, t) = \int e^{-r\tau} c(t)\,[1 - F(\tau; V, V_B)]d\tau + e^{-rt} p(t)\,[1 - F(t; V, V_B)] + \int e^{-r\tau}\rho(t)V_B f(\tau; V, V_B)d\tau \tag{7}$$

(here integral limits are 0 and t).

The authors do not discuss sources and methods of financing coupon flow $c(t)$, although those sources and methods are of great importance for the GBM schemes (see Black & Cox, 1976). However, one can restore the source and method of firm financing, taking account of the author's evidence that the last equation for the debt value of maturity t converts to Equation (4) for the debt value of infinite maturity when $t \to \infty$ and $\rho = 1 - \alpha$. It becomes clear that the method of coupon flow financing remains the same as it is in Leland (1994, p. 1217): "any net cash outflows associated with the choice of leverage must be financed by selling additional equity". At that, this equity must be external to the firm because "bond covenants restrict firms from selling their assets" (p. 1217). This financing method makes the firm an arbitraging machine generating arbitrage profits and, correspondently, makes void all results of Leland and Toft (1996). One should remember that the risk-neutral technique is equivalent to the GBM model considering self-financing firms only. Therefore, all results derived within this framework have very little to do with the market of real firms paying their business securing expenses, e.g. debts and taxes.

The root of failure of Leland (1994) and Leland and Toft (1996) is in using the GBM equation for the description of the firm value, especially for firms with debts. Any model neglecting debt redemption in borrowing is completely inadequate. Unfortunately, most contemporary models studying capital structure dynamics are founded on the GBM equation (contingent claim models or real option models; Merton, 1974). Therefore, all results achieved with those models must be checked-out with better models taking into account all firm's payments including debts and taxes. We present such a model (the extended Merton model, EMM) in Section 2.

The EMM model shows that debt in most cases deteriorates firm's dynamics and decreases the firm value; the greater the debt, the faster the firm's state deteriorates. The function $V(D, t)$ has no maximum; it is a decreasing function of debt D; the optimal leverage and the maximum firm value it allegedly provides are just phantoms. Therefore, all debates about the optimal capital structure in the sense of Leland's model are meaningless. However, in long years since 1973, when Kraus and Litzenberger put forward their hypothesis about existence of the optimal capital structure, this idea has gradually seized minds of both scholars and business managers. Some scholars are so sure in the trade-off theory that observing the median corporate debt-to-capital ratio in the United States over 1965 to 2000 only 31.4%, with 40% of firms having this ratio of less than 20%, while traditional trade-off models produce substantially higher values, they call it "the *low value* puzzle" (Strebulaev, 2007).

Beattie et al. (2006) in a comprehensive survey of the corporate financing decision-making process in UK companies try to enable a comparison between practice and extant theories of capital structure. The author's objective is to understand how companies determine their overall financing strategy, why they pick up a particular mix of financing instruments, and why they choose to limit borrowings or set up spare borrowing capacity. One of the key findings of this study is that firms are heterogeneous in their capital structure policies. About half of the firms (51%) seek to maintain a target debt level consistent with the trade-off theory, while 59% of responding firms argued that they follow the pecking-order theory. At that, 32% of respondent firms claim that they follow both theories, and 22% to follow neither of them. It means that these two theories are not viewed by respondents as either mutually exclusive or exhaustive. The firms following the trade-off theory consider targets ranging from 0% to 300% with a median of 40% and with 80% of companies indicating a target level of debt of 50% or less. Why the firms issue such great debts? It is because they are deceived by the false trade-off theory and have no way of evaluating the state of the firm in five-year or longer time intervals while a destructive effect of debt develops slowly at first steps and gains momentum with each new year revealing its threat in some time. To the best of our knowledge, existing models make their forecast for one year only (e.g. CreditMetrics from JP Morgan, Moody's KMV, CreditRisk+ from Credit Risk Financial Products) that does not reveal a full picture of debt effects. The significant influence of the trade-off theory on financial decisions of business practitioners is due to a long and systematic psychological suggestion that reputable financial theorists exert on corporate management.

On the question "Who/what is influential in setting target capital structure ratios?", the firms answer (in decreasing order): (a) company senior management, (b) existing shareholders, (c) commercial bankers, (d) investment bankers, (e) debt covenants, (f) outside investment analysts, (g) potential shareholders, (h) comparison with ratios of industry competitors, (i) major trade creditors. The findings by Beattie et al. (2006) convincingly demonstrate that most managers believe in existent capital structure theories. Yet the trade-off theory is absolutely false and misleading. We discuss shortcomings of the pecking-order theory in the next section.

Information asymmetry and pecking-order theory. Unfortunately, the pecking-order theory has no strong mathematical basis; it is mostly an empirical theory. Mayers and Majluf (1984) fail to set up an optimization problem in the pecking-order model; there is no such problem setting up to now. That is why Mayers (1984) calls it not a theory but a story.

The cornerstone of pecking-order theory is the idea of information asymmetry between managers and existent and future investors. Theoretically this idea is good and sound, but its practical realization now is extremely difficult, maybe even impossible. All market participants have information about the market state consisting of a part common for all participants and a unique part specific for that particular group (for example, firm's managers have exclusive information about the real state of their firm and its prospectives in the future). But how one can effectively use this information asymmetry for practical purposes remains unclear. To estimate the state of the firm in the near future, both sides (managers and investors) must use the same model of firm development with all necessary parameters and initial data. The sides can have the same model, but managers do know all parameters and initial data of their firms, whereas investors can only estimate them with errors increasing uncertainty for them. It goes without saying that a positive effect of the information excess is possible if both sides use an adequate model correctly describing the firm development. If the sides use inadequate models, this information excess becomes useless and even harmful. We illustrate this thesis by the following example.

Giesecke and Goldberg (2008) consider a problem of market evaluation of firm's credit risks. They use a structural model with a default line to show that information asymmetry can induce an event premium for abrupt changes in security prices that occur at firm default. If public investors are unable to observe the threshold asset value at which firm's management liquidates the firm, then they face an abrupt default risk as they cannot discern the firm's distance to default. The authors think that investors must have premium for this non-diversifiable risk. From the investor point of view, the credit risk problem includes a randomly jumping default line. Technically the authors suggest another kind of a jump-diffusion process (JDP) adding an extra jump risk to a low GBM risk. To apply the martingale technique to problem solving, Giesecke and Goldberg use the GBM model supplemented with jumps in the firm value to a default line whose location is allegedly randomly varying. Random leaps to the default line simulate an unexpected occurrence of the firm's default. However, this default happens unexpectedly not due to random jumps of the default line, but due to the difference between the real high default probability specific for the firms with payments and unknown both to managers and investors and its low GBM estimation used by investors as well as by firm's management. In this case, both sides use the inadequate GBM model for estimation of the firm's state and risks. The information asymmetry on the firm state and its prospects becomes negligible against the background of a wrong model. Therefore, managers and investors are equally unpleasantly surprised by the firm default. The risk premium for investors in those circumstances looks unjust. One can conclude that for information asymmetry to be important, both firm's managers and potential investors must use adequate models; otherwise information dominance of one side will be lost in errors of an inadequate model.

JDP models supplement the GBM model with jumps in the firm value. However, economists believe that leaps have various causes: It could be new information about the firm (Zhou, 2001; Hilberink & Rogers, 2002; Kou, 2002; Chen & Kou, 2009), or informational asymmetry between public investors and firm's management (Giesecke & Goldberg, 2008). Both interpretations are equally far from reality because any firm has compulsory payments, which JDP models neglect. As we know now, the real value distribution is skewed, and this skewness continuously varies. No JDP with fixed statistical jump characteristics can provide for the distribution with varying skewness.

The pecking-order theory sets up the following order of financing: (1) internal finance (retained earnings), (2) if the internal finance is insufficient, it comes to external finance issuing the safest security first; the firm starts with debt, then convertible bonds, and equity as a last resort. There is no well-defined target of the debt-equity mix; an observed debt ratio reflects firm's cumulative requirements for external finance (Myers, 1984). Firm's unwillingness to issue common stock stems from the fact that most managers do not accept the idea of market efficiency. For example, 86% of UK managers felt that the market fairly priced their shares less than three quarters of the time (Beattie et al., 2006). This share of UK managers compares against 52% of US managers with the same views (Pinegar & Wilbricht, 1989).

It is time to mark that Myers and Majluf (1984) trust Modigliani-Miller Propositions and believe that a moderate debt makes a positive effect on firm finance and does not increase firm risks. We show that any debt deteriorates the firm state and increases default risks. That is, issuing debt is not a harmless operation; it has its price in decrease of returns and growth of default risks.

Concerning new equity issuing, we can show that the market price of shares must drop consistently with the observed practice. For successful market investment, investors must have as much information about their target firm as possible. The asset distribution provides complete information about the firm: Using it, investors can compute the mean return on assets, variance, skewness, other statistical moments they need, and also the default probability. This information helps investors optimize their choice of target shares. Unfortunately, investors do not know the asset distribution of the firm; all they observe is a time series of stochastic share prices. The investors can only approximately estimate the mean returns of a particular stock analyzing a time series of its prices over a unit time interval. Their eagerness to have a particular stock in their portfolio affects the share price. Two motives govern this eagerness: investor's striving for wealth-maximizing on the one side, and risk-avoiding on the other. The relative importance of those motives does not remain constant for the investor; one or the other motive dominates at particular time periods depending on a state of the stock and market. This subjective behavior of individual investors introduces additional shocks in the dynamics of market returns. The cumulative effect of all investors working with a particular stock at the exchange determines the market price distribution of this stock.

The share price structure consists of two parts: the change in firm's assets over a time unit and the premium paid by investors for the right of possessing a particular stock. A part of firm's assets is paid out to shareholders as dividends. The change in assets runs relatively slow as corporate assets develop in the process of manufacturing and marketing firm's goods. This process includes multiple payments and infusions of funds and is definitely not a self-financing process, and, therefore, it is always skewed. The asset distribution and asset change over a unit time interval completely determine the first part of the share price. The extended Merton model (EMM) computes the asset distribution, its mean returns and riskiness as a function of time and the firm and market parameters. The premium which the investor is ready to pay for the right of possessing the share is determined by investor's expectations about the growth rate of firm's assets, the time interval of asset growth which depends on the firm riskiness, the market demand for this share, and the mean rate of returns of investor's portfolio. Changes in share prices happen very fast and premium dynamics makes a martingale process.

Now let us consider what happens to the share price when the firm issues a new stock. Suppose, the firm's asset at time t is X, a number of old shares is N_1, a number of new shares is N_2. A part of firm's assets in one share before issuing new equity is X/N_1; after that it falls to $X/(N_1+ N_2)$ making the stock less attractive for potential investors. The first component of the share price, the expected rate of return on a share, decreases. It is possible that the market demand will also get lesser because for lesser rate of return some investors will find it inexpedient to include the stock with that low rate of return into their portfolios. If the expected time interval of the expected growth rate remains the same, the premium value after issuing new equity still decreases, and the share price becomes lesser than it was before equity issuing. As one can see, issuing new equity leads to changes in both parts of the share price, that is, the change in firm's assets which is additive, and the premium which value changes non-linearly with a number of shares. Knowing the premium and firm's assets, one can find the market value of the firm at a fixed moment t in the case of issuing new equity. Finding the firm value at the same time (e.g. the date of debt maturity) in the case of debt financing, and comparing these two market values of the firm, one can choose the best method of firm financing by debt or by new equity.

EMM Description

The extended Merton model (EMM; Shemetov, 2020; 2023) uses the celebrated continuous-time equation describing development of the firm value in a stochastic environment (Merton, 1974):

$$dX = (\alpha_0 X - P)dt + CXdW. \tag{8}$$

Here $X(t)$ is a firm value at time t, constant α_0 is an instantaneous expected return on the firm per unit time, P is total dollar payouts by the firm per unit time, constant $C > 0$ is the instantaneous variance of returns, W is a Wiener process representing a cumulative effect of normal shocks. The firm makes various payments while doing business. Some of them are necessary for manufacturing and marketing of firm's goods (current payments); these payments are considered as continuous-time payments (*CP*). The other payments secure the firm's presence in business. Such expenses include fixed costs (*FC*), taxes (*TAX*), dividends (*DIV*), and debt payments (*DP*); these payments are made by installments in the end of time unit (a year). One can write for business securing expenses (BSEs):

$$\begin{gathered} P = FC + DP + TAX + DIV, \\ CP(t) = P_0\,\pi(t),\ CP(0) = P_0 > 0,\ \pi(0) = 1. \end{gathered} \tag{9}$$

Equation (8) for random variable $x = ln\ (RX / P_0)$ by Ito's Lemma transforms to

$$\begin{gathered} dx = R(1 - \pi(t)e^{-x})dt + CdW, \\ x(0) \equiv x_0 = \ln\left(\frac{RX_0}{P_0}\right),\ R = \alpha_0 - C^2/2. \end{gathered} \tag{10}$$

Writing a Fokker-Plank equation for Equation (10), one comes to an equation for the probability distribution $V(x, t)$, or x-distribution; V_y is a partial derivative over a variable y:

$$V_t + R(1 - \pi(t)e^{-x})V_x - 0.5C^2V_{xx} + R\pi(t)e^{-x}V = 0. \tag{11}$$

The initial condition is

$$\begin{gathered} V(x,0) = V_0(x; H_0, \sigma_0^2), \\ H_0 = \langle x_0 \rangle = \int_{-\infty}^{\infty} xV(x,0)dx,\ \sigma_0^2 = \langle (x_0 - H_0)^2 \rangle = \int_{-\infty}^{\infty} (x - H_0)^2 V(x,0)dx, \end{gathered} \tag{12}$$

where $V_0(x; H_0, \sigma_0^2)$ is a normal function, and a boundary condition implying that a firm comes to default when its value falls to X_D ($0 < X_D < X_0$) is

$$V(DL, t) = 0, DL = \max\left[ln\left(\frac{RX_D}{P_0}\right), 0\right]. \tag{13}$$

If X_D is an outstanding debt as it is in Black and Cox (1976), Equation (13) is an *exogenous* constraint. If the firm has no debt, the line $x = 0$ makes a soft *endogenous* boundary that separates a profitable business from its failure. Below the line $x = 0$, the firm can continue its activities only selling its assets. A solution of the problem (Equations (11), (12), (13)) is the log-value distribution for a firm in financial distress; it is denoted as $\hat{V}(x,t)$. If one knows a solution $V(x, t)$ in the open space, then a solution of the boundary problem (Equations (11), (12), (13)) can be written as

$$\hat{V}(x,t) = V(x,t) - V(2DL - x, t). \tag{14}$$

The intensity of default probability *DPInt*(t) is

$$DPInt(t) = 2\int_{-\infty}^{DP} V(x,t)dx. \tag{15}$$

The model calculates the following statistical moments:

$$\begin{gathered}\widehat{H}(t) = \int_{DL}^{\infty} x\widehat{V}\,(x,t)dx, \quad \widehat{VAR}(t) = \int_{DL}^{\infty}(x-\widehat{H})^2\widehat{V}\,(x,t)dx,\\ \widehat{SK}(t) = \int_{DL}^{\infty}(x-\widehat{H})^3\widehat{V}\,(x,t)dx,\ \widehat{KT}(t) = \int_{DL}^{\infty}(x-\widehat{H})^4\widehat{V}\,(x,t)dx,\\ \widehat{MX}(t) = \frac{P_0}{R}\int_{DL}^{\infty} e^x\widehat{V}\,(x,t)dx,\ Skew(t) = \widehat{SK}(t)\ \widehat{VAR}^{-3/2}(t),\\ Kurt(t) = \widehat{KT}(t)\widehat{VAR}^{-2}(t).\end{gathered} \tag{16}$$

The objective of credit risk analysis is estimating the firm's default probability over a chosen time interval (e.g. debt maturity):

$$DPr(t_s,\tau) = \int_{t_s}^{t_s+\tau} DPInt(t)dt. \tag{17}$$

The solution (Equations (11)-(14)) describes the behavior of firm's assets in the interval $(t_{k-1} \le t < t_k)$ when the firm makes only continuous payments. In the end of the year, $t = t_k$, the firm pays its obligations to debtholders (*DP*) and the corporate income tax (*TAX*). We will not consider dividend payments.

The effect of one-time payment *DP* leads to an instant left shift *LDP* of each point of the distribution $V(x,t_k)$:

$$LDP(x,t_k) = ln\left(\frac{X-DP}{X}\right),\ X > DP. \tag{18}$$

Payment *DP* makes a boundary separating the firms who will survive that debt payment from those who will default. In *x*-space the default boundary is $BP = ln(R\cdot DP/P_0)$. The distribution function for the firms survived the debt payment is

$$\begin{gathered}\tilde{V}(x,t_k) = V(x,t_k) - V(2BP-x,t_k),\\ ln\left(\frac{X-DP}{X}\right) = \ln(1-e^{BP-x}),\end{gathered} \tag{19}$$

and the transformed function after the shift can be written as

$$x^{(DP)} = x + \ln(1-e^{BP-x}), \tag{20}$$

$$\begin{gathered}V_1\left(x^{(DP)},t_k^+\right) = (1-e^{BP-x})\tilde{V}(x,t_k^-),\\ BP \le x < \infty,\ -\infty < x^{(DP)} < \infty.\end{gathered} \tag{21}$$

Here t_k^- and t_k^+ are the moments in the end of t_k-year *before* and *after DP* payment; factor $1-e^{BP-x}$ in Equation (21) provides for the integral identity:

$$V_1\left(x^{(DP)},t_k^+\right)dx^{(DP)} = \tilde{V}(x,t_k^-)dx. \tag{22}$$

This transformation affects differently high $(X \gg DP)$ and low $(X \cong DP)$ values in the distribution: High values shift insignificantly while low values travel large distances to the left forming negative distribution skewness. This skewness increases default probability for all firms, and brings to imminent default the firms with $X \le DP$.

Tax payments follow the rule:

$TAX(X,t_k) = 0$, if taxable firm's year income $INCOME(X,t_k) \le 0$, and

$$TAX(X,t_k) = tax\left(1-\frac{D(t_k)}{X}\right)INCOME(X,t_k), \text{ if } INCOME(X,t_k) > 0, \tag{23}$$

here tax is the tax rate and the term in parenthesis represents the tax shield.

$$\frac{D(t_k)}{X} = \frac{RD(t_k)}{P_0}e^{-x} \equiv b(t_k)e^{-x},\ INCOME(X,t_k) \equiv \frac{P_0}{R}F(x,t_k). \tag{24}$$

So, one has for the tax value in *x*-variable:

$$TAX(x,t_k) = tax[1-b(t_k)e^{-x}]F(x,t_k). \tag{25}$$

The effect of tax payment leads to an instant left shift of each point of the distribution $V_1(x,t_k)$:

$$\begin{gathered} ln\left(\frac{X-TAX}{X}\right) = ln[1-tax(1-be^{-x})F(x,t_k)], \text{ if } F(x,t_k) > 0, \text{ and} \\ ln\left(\frac{X-TAX}{X}\right) = 0, \text{ if } F(x,t_k) \le 0. \end{gathered} \tag{26}$$

For the levered firm paying its debts and taxes, one has for a new variable $x^{(TAX)}$ and the asset distribution after tax payment $V_2\left(x^{(TAX)},t_k^+\right)$:

$$\begin{gathered} x^{(TAX)} = x + ln[1-tax(1-be^{-x})F(x,t_k)], \text{ if } F(x,t_k) > 0, \text{ else } x^{(TAX)} = x. \\ V_2\left(x^{(TAX)},t_k^+\right) = \frac{1-tax(1-be^{-x})F}{1-tax[F+(1-be^{-x})F_x]}V_1(x,t_k^+), \text{ if } F(x,t_k) > 0, \text{ else} \\ V_2\left(x^{(TAX)},t_k^+\right) = V_1(x,t_k^+); \\ F(x,t) = \int_{-\infty}^{x}(1-e^{-(x-x')})g(x,t|\,x',t')V(x',t')dx'. \end{gathered} \tag{27}$$

For the unlevered firm, one must use a function $V(x,t_k)$ instead of $V_2\left(x^{(TAX)},t_k^+\right)$ with the default line *DL* = 0. Omitting in the after-tax distribution index (*TAX*), one can re-wright the after-tax asset distribution as $V_2(x,t_k^+)$. This distribution is fed as an initial condition to the entrance of the problem (Equations (11)-(14)):

$$V(x,t_k) = V_2(x,t_k^+) \tag{28}$$

which then is solved in the next year interval $t_k \le t < t_{k+1}$.

Effect of Debt on Firm Development

To illustrate effects of debt on the firm development in the EMM framework, we consider the following setting. Suppose there is a business promising an expected annual rate of return of μ percent on firm's assets, to enter which, a firm must have assets of no less than 1,000 dollar units. Suppose also that a firm has assets *X* = 1,000 and enters this business immediately (the all-equity or unlevered firm). This firm pays its BSEs in the form of fixed costs and taxes, *PU* = *FC* + *TAX*. We suppose a continuous mode of fixed cost payments and taxes paid in lump sum once a year. Other firm parameters are H_0 = 2.0, VAR_0 = 0.02, *R* = 0.1, MX_0 = 1,000, P_0 = 13.3989, *tax* = 0.3; see parameter descriptions in the previous section.

The dynamics of the unlevered firm distribution is shown in Figures 1-4. A comparison of the initial asset distribution and the asset distribution in the end of first year before tax payment is shown in Figure 1. One can see that earnings drive the asset distribution right. Figure 2 presents the asset distribution in the end of the first year before- and after-tax payment; tax payments increase distribution right skewness.

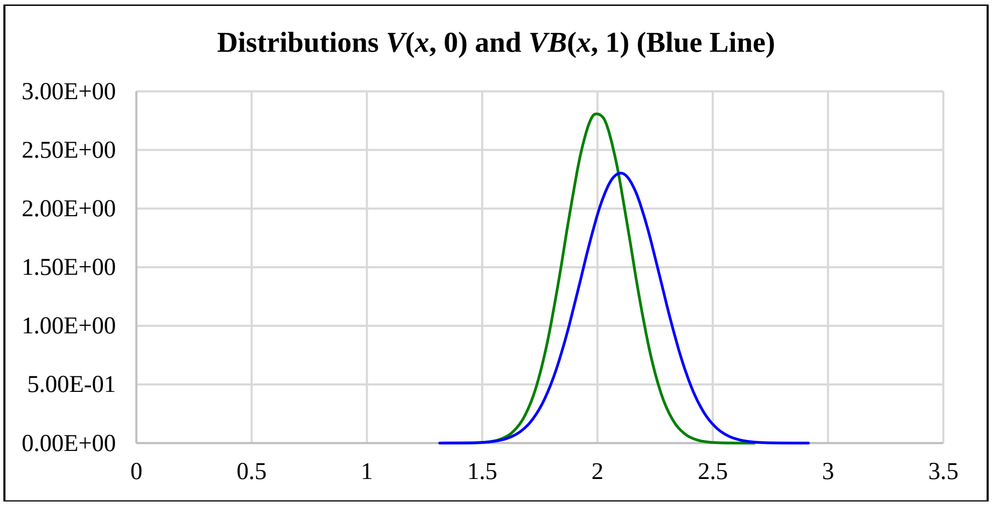

Figure 1. Log-asset distributions of unlevered firm at $t = 0$, $V(x, 0)$, and $t = 1$ before tax payment, $VB(x, 1)$ (blue line).

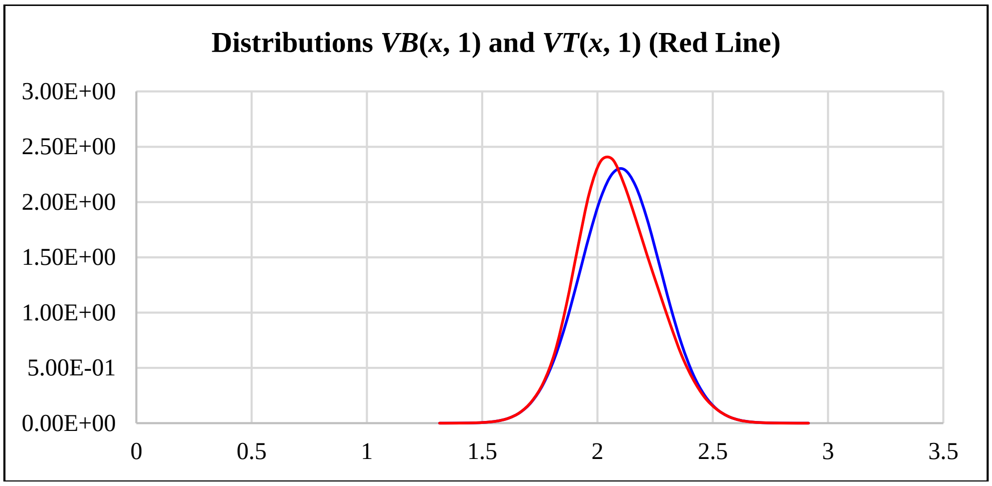

Figure 2. Log-asset distributions of unlevered firm at $t = 1$ before $VB(x, 1)$ and after $VT(x, 1)$ tax payment (red line).

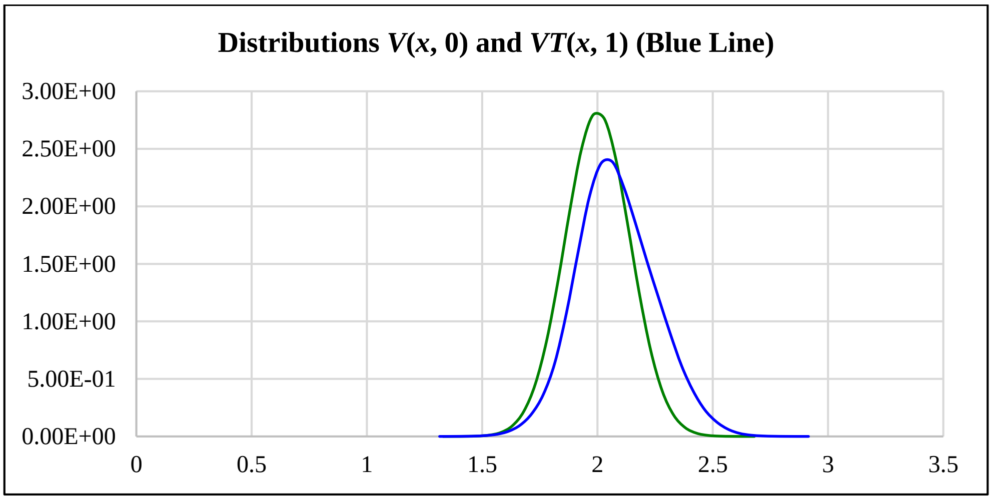

Figure 3. Log-asset distribution of the unlevered firm at $t = 0$, $V(x, 0)$, and $t = 1$ after debt and tax payment, $VT(x, 1)$ (blue line).

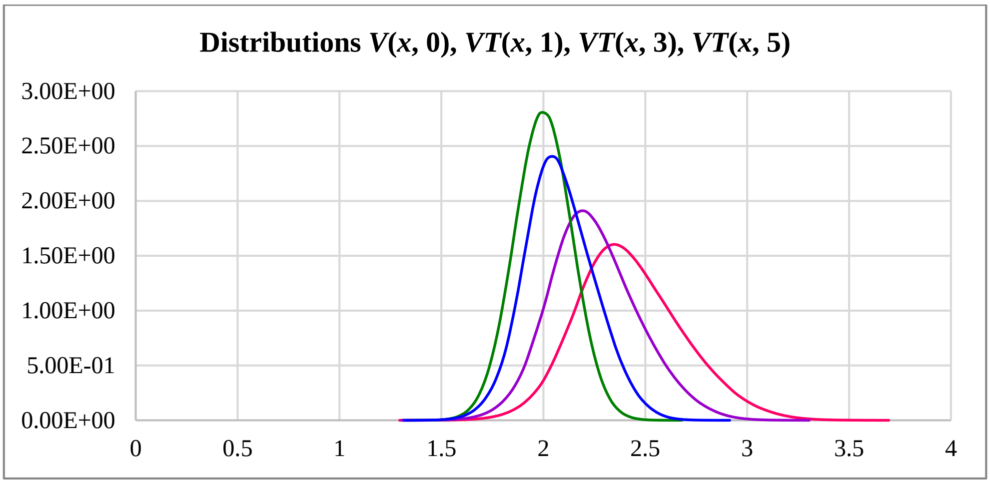

Figure 4. Evolution of log-asset distributions of unlevered firm at $t = 0$ ($V(x, 0)$, green), 1 ($VT(x, 1)$, blue), 3 ($VT(x, 3)$, purple), 5 ($VT(x, 5)$, red) after tax payment.

Table 1

Evolution of Statistical Moments for the Asset Distribution of the Unlevered Firm After Tax Payments

Year	Mean return $H(t) - H_0$	Variance $VAR(t) - VAR_0$	Skewness	Kurtosis
0	0	0	0	0
1	0.081	0.0096	0.184	0.070
2	0.165	0.0191	0.252	0.126
3	0.247	0.0287	0.343	0.179
4	0.336	0.0386	0.352	0.165
5	0.422	0.0485	0.361	0.160

The evolution of asset distribution after tax payments over the time interval of five years is given in Figure 4. It shows that the distribution invariably shifts right and the positive distribution skewness grows over time; the default probability remains very low. One can find also the evolution of firm statistical moments in Table 1. All data prove steady positive development of the unlevered firm.

Now we consider another firm identical to the first one in all respects but the size of assets. Let it has X = 1,000 - A dollar units. To enter the business, the firm takes a bank loan of A units for T_m years at the annual interest rate of r percent (the levered firm). The total debt is $D(0) = Aexp(rT_m)$. Suppose also that the firm pays its debt once a year in equal installments $DP = (A/T_m)exp(rT_m)$. Total BSEs for the levered firm are $PL = FC + DP + TAX$. As before, FC is continuous payment. Other firm parameters are $H_0 = 2.0$, $VAR_0 = 0.02$, $R = 0.1$, MX_0 = 1,000 doll. units, A = 100 doll. units, T_m = 5 yrs, $r = 0.05$, total debt 128.4 doll. units, $P_0 = 13.399$, $tax = 0.3$. We will trace the evolution of firm's assets in the interval of 10 years.

The dynamics of the levered firm's distribution is shown in Figures 5-11. Figure 12 demonstrates a shocking difference between the after-tax distributions of the levered and unlevered firms computed for the same time t = 5 (the date of debt maturity of the levered firm, the lowest point in this firm evolution). Figure 5 demonstrates changes in the asset distribution of the levered firm after accumulating year earnings; the distribution $VB(x, 1)$ shifts right about the initial distribution $V(x, 0)$. After payment of the first debt instalment (Figure 6), the distribution $VD(x, 1)$ fells to the left not only of $VB(x, 1)$-distribution, but also about the initial distribution $V(x, 0)$ indicating that debt payment of this size is detrimental to growth of firm's assets.

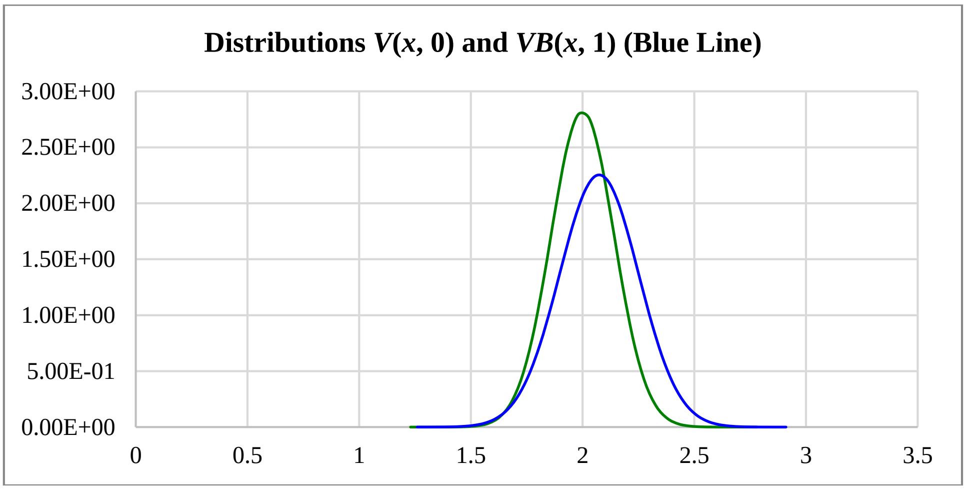

Figure 5. Log-asset distributions of levered firm at $t = 0$, $V(x, 0)$, green, and $t = 1$ before debt payment, $VB(x, 1)$, blue line.

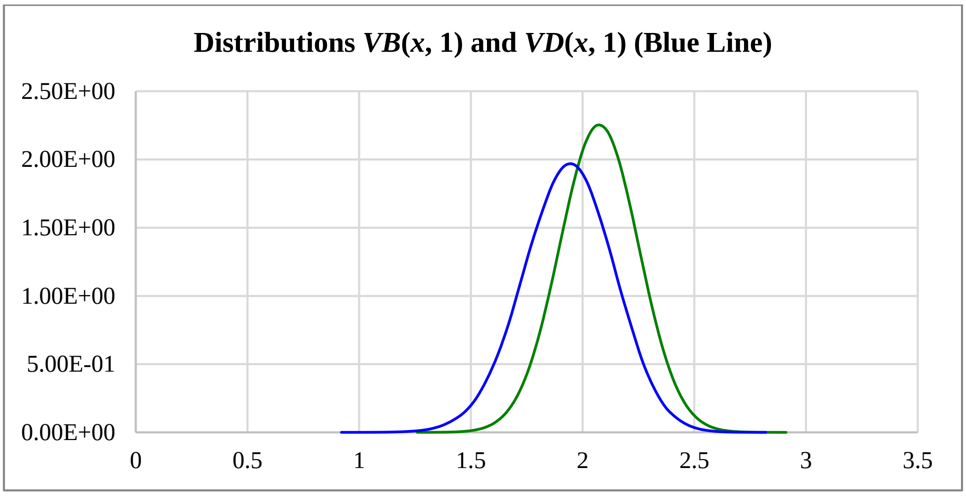

Figure 6. Log-asset distributions of levered firm at $t = 1$ before (green, $VB(x, 1)$) and after debt payment (blue line, $VD(x, 1)$).

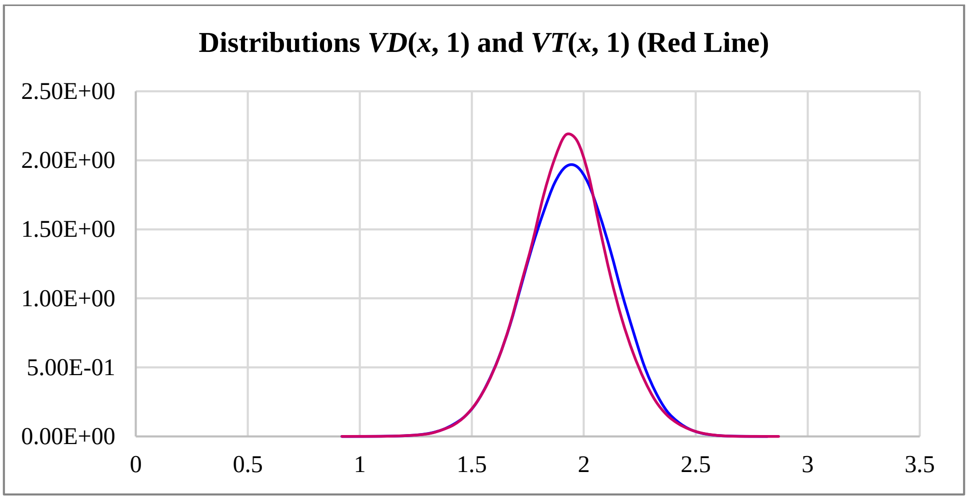

Figure 7. Log-asset distributions of levered firm at $t = 1$ before (blue line, $VD(x, 1)$) and after tax payment (red line, $VT(x, 1)$).

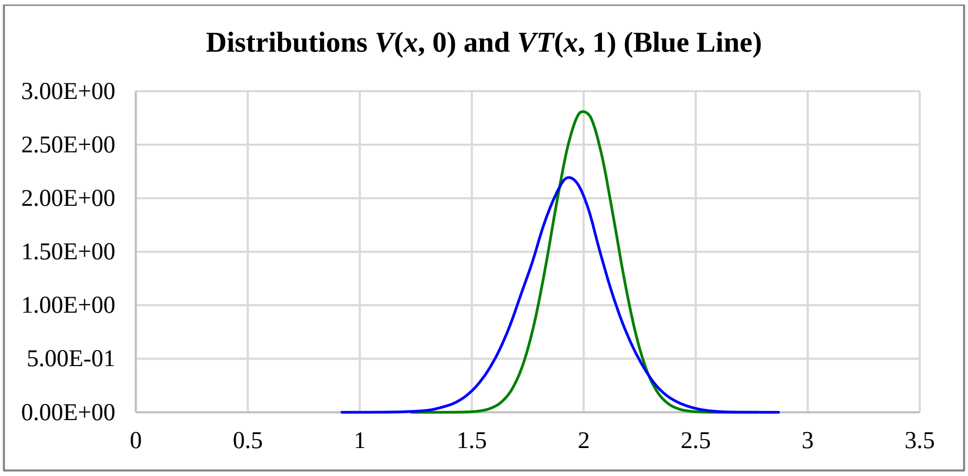

Figure 8. Log-asset distributions of levered firm at $t = 0$, $V(x, 0)$, green, and $t = 1$ after debt and tax payments, $VT(x, 1)$, blue.

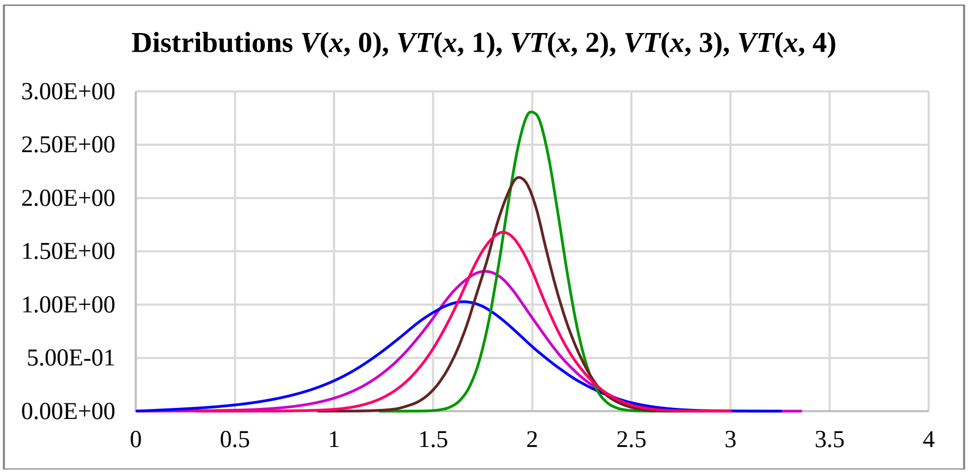

Figure 9. Log-asset distributions of levered firm at $t = 0$, $V(x, 0)$, green, and $VT(x, t)$ at $t = 1$ (brown), 2 (red), 3 (magenta), 4 (blue line).

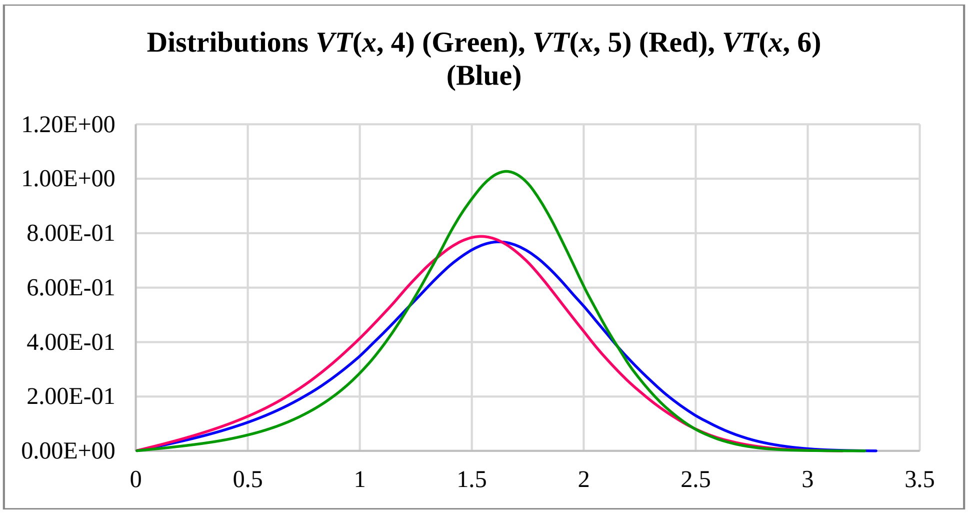

Figure 10. Log-asset distributions $VT(x, t)$ of levered firm in the neighborhood of debt maturity, $t = 4$ (green), 5 (debt maturity, red), 6 (blue).

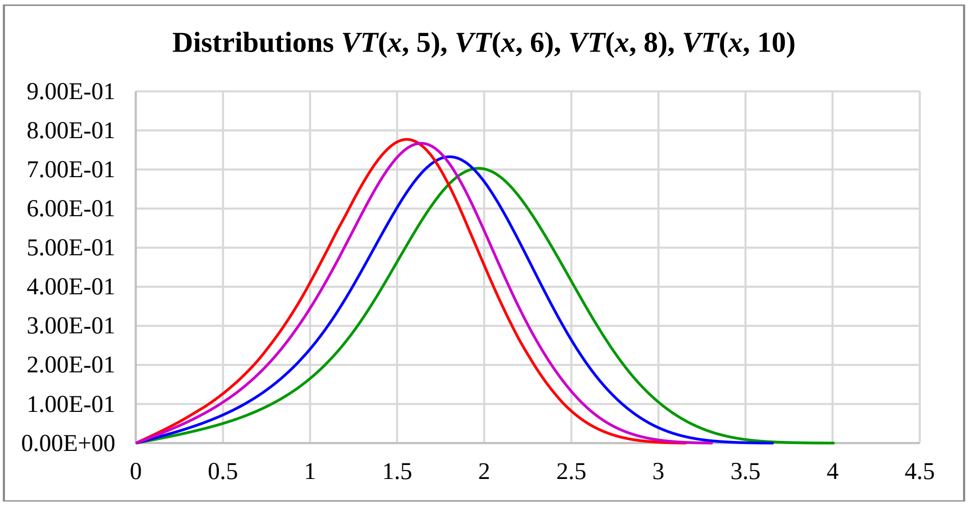

Figure 11. Log-asset distributions $VT(x, t)$ of levered firm in its recovery: $t = 5$ (debt maturity, red), 6 (purple), 8 (blue) and 10 (green).

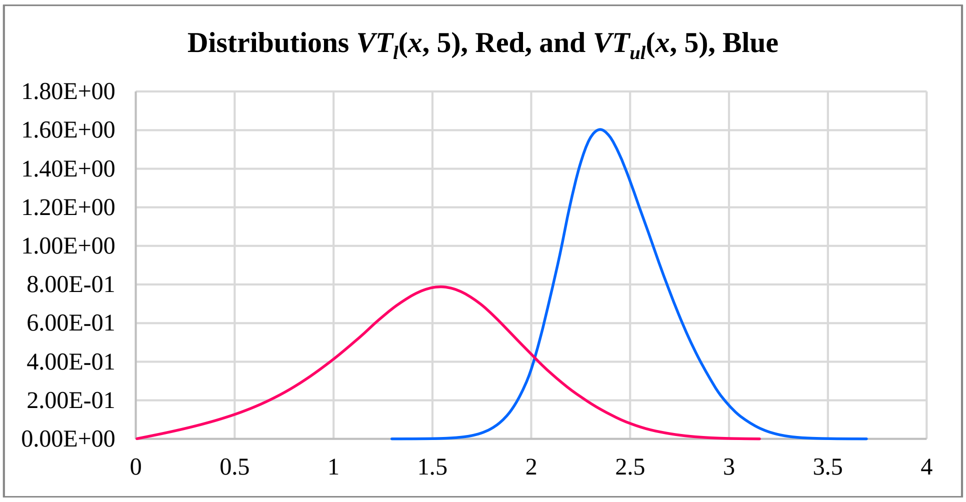

Figure 12. Log-asset distributions $VT_l(x, t)$ of levered firm at $t = 5$ (debt maturity, red) and $VT_{ul}(x, t)$ of unlevered firm, $t = 5$ (blue).

Table 2

Evolution of Statistical Moments for the Asset Distribution of the Levered Firm After Debt and Tax Payments

Year	Mean return $H(t) - H_0$	Variance $VAR(t) - VAR_0$	Skewness	Kurtosis	$DPInt(t)$
0	0	0	0	0	0
1	-0.077	0.018	-3.65e-03	0.278	0
2	-0.164	0.047	-0.144	0.421	3.58e-07
3	-0.276	0.092	-0.338	0.903	3.67e-04
4	-0.425	0.158	-0.880	3.887	1.01e-02
5	-0.632	0.212	-1.462	6.461	6.26e-02
6	-0.539	0.228	-1.389	6.163	5.36e-02
7	-0.447	0.246	-1.318	5.883	4.60e-02
8	-0.356	0.264	-1.248	5.616	3.95e-02
9	-0.264	0.281	-1.180	5.364	3.40e-02
10	-0.172	0.292	-1.119	5.130	2.94e-02

Tax payment (Figure 7) makes the distribution *VT*(x, 1) right skewed compared to *VD*(x, 1)-distribution but makes no significant effect on the location of asset distribution in the x-axis. Figure 8 shows a joint effect of earnings, debt, and tax payments on the asset distribution; one can see that this effect is negative: The resulting *VT*(x, 1)-distribution falls on the left of the initial *V*(x, 0)-distribution. The same behavior of *VT*(x, t_k)-distribution in respect to *VT*(x, t_{k-1})-distribution of the previous year can be traced for all years of debt payments, k = 2, 3, 4, 5 (Figures 9 and 10). Positive asset dynamics begins after debt maturity when the debt is paid completely out and the firm pays just taxes. The most eloquent evidence is Figure 12 comparing asset distributions of the levered and unlevered firms at t = 5 (the year of debt maturity corresponding to the lowest point in the development of levered firm's assets). This picture destroys the myth about positive effect of debt on the firm development. Table 2 also contributes to the exposure of this myth: the difference between the mean returns at t = 5, $H_{ul}(5) - H_l(5)$ = 1.052 in favor of the unlevered firm, the difference between variances, $VAR_l(5) - VAR_{ul}(5)$ = 0.163 again in favor of the unlevered firm, and the intensity of the default probability is negligibly small for the unlevered firm and rather high for the levered firm, *DPint*(5) = 6.26e-2. Table 2 and Figure 11 demonstrate that the firm needs five more years for achieving a partial recovery from the damages inflicted by debt.

It is interesting that managers of the levered firm consider the firm development as quite normal. They watch stochastic outcomes of asset returns which are mostly positive, they can estimate the variance which is rather high, but they explain it with a current market state, and they have no information about the intensity of default probability threatening the firm with default in the near future. The estimates of the firm stability they can get from financial experts founding their judgment on structural models (also known as real option models or contingent claim models) rooting from Merton (1974) undervalue the default probability inspiring false feeling of safety in the managers. Having no opportunity to compare the consequences of various financing scenarios, managers usually choose between the trade-off and pecking order recommended by scientific experts. Remember that about 50% of UK firms follow the trade-off theory with target debt spreading from 0% to 300% with the median debt of 40% (Beattie et al., 2006). Such a high level of the median debt significantly deteriorates the firm development, and 60% of UK managers look for the alternative in pecking order financing which also pushes them to capital borrowing though of a lesser scale. Our paper shows managers a real instrument for estimating long-term consequences of debt financing.

Here we have considered the effect of debt when the firm cannot borrow capital at the market and has to take a bank loan. In the following papers we will consider the case of a public debt which significantly distinguishes from a bank loan. We also plan to transform the pecking-order theory from an empiric qualitative story into a quantitative theory allowing the optimal choice between debt financing and new equity financing.

Conclusion

Analyzing seminal papers of Modigliani and Miller (1958; 1963), Merton (1974), Leland (1994), Leland and Toft (1996) constituting theoretical underpinning of the trade-off theory of capital structure, we have shown that all of them are wrong. Correspondingly, all following studies of capital structure, making a weighty part of financial economics, are unreliable: None of their conclusions except the most trivial is true, and no recommendation can be accepted without doubts. After the MMP3 (1963) and Kraus and Litzenberger (1973) supposing that debt brings absolute tax benefits to the firm, Leland (1994) and Leland and Toft (1996) develop a mathematical basis for the trade-off theory determining the optimal capital structure. We prove that this optimal capital structure is just a phantom. Their error stems from the fact that taking into account tax benefits of debt, they disregard an increase in firm's payments caused by debt compensation. The structural model by Merton (1974) they use for studying the firm development admits no firm's payment at all and leads to a serious distortion of reality.

Using the extended Merton model (EMM; Shemetov, 2020; 2023), we have shown that a cumulative effect of debt and tax negatively affects the firm value, returns, and stability supporting the empirical study by Sarkar and Zapatero (2003) demonstrating that the higher the debt, the lesser the mean returns. All papers teaching how to choose the optimal debt leverage in static or dynamic conditions (Kraus & Litzenberger, 1973; Leland, 1994; Leland & Toft, 1996; Goldstein et al., 2001; Strebulaev, 2007; etc.) are false and can seriously damage the firm if it takes their recommendations for practical guidance. However, empiric studies by Beatty et al. (2006) and Graham and Harvey (2003) show that the trade-off theory of capital structure is still very popular: About 50% of managers follow its recommendations in this or that degree subjecting their firms to extra risks. The agency theory recommends to have a moderate debt in the firm's capital structure just to keep the managers busy (Stulz, 1990).

Our conclusions on cumulative effects of debt and tax on the firm value are:

(1) on the contrary to MMP1 (1958), debt does affect the firm value and its survival,

(2) on the contrary to MMP3 (1963), this influence is negative diminishing the firm value and its chances to survive,

(3) the debt pressure increases as the debt grows, and the greater the debt, the sooner the default comes,

(4) the main factors, depressing the levered firm, are its increased debt payments added to BSEs of the identical unlevered firm and the length of debt maturity,

(5) corporate taxes cause development of positive skewness in the firm's asset distribution, but do not affect the location of this distribution in the asset-axis.

Our results reject the methods and conclusions of the MMP3 (1963), Merton, (1974), Leland, (1994), Leland and Toft (1996) and all following studies of capital structure problems based on the structural model by Merton, (1974). Our analysis of the cumulative effect of debt and tax on the firm value is a quantitative instrument computing consequences of choosing this or that level of debt. This instrument can be helpful for practising financial managers.

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