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Students' Mathematics Performance via *APOS* Instruction and Learners' Cognitive Ability on Mathematical Problem Solving: An Analysis

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The study analyzed students' performance in mathematics through APOS instruction on learners' cognitive ability and mathematical problem solving performance of Senior Secondary School students in Sokoto State, Nigeria. The objective of the study was to find out whether APOS instruction has effects on learners' performance in mathematics when compared with teacher-centered traditional method of teaching, and whether there is any difference in learners' cognitive ability in mathematical problem solving performance when taught with APOS instructional method (AIM). Three (3) research questions were answered and three (3) null hypotheses were tested and analyzed at 0.05 level of significance. Four hundred and twenty two (422) students from SS 2 served as sample. Quasi-experimental research design was used, involving a pre-test—treatment—post-test, non-randomized approach. Two instruments were used for data collection: Mathematical Problem Solving Performance Test (MPSPT) and Learner Cognitive Ability Test (LCAT). Data were analyzed using independent *t*-test, ANOVA, as well as Scheffe's test. The study found out that experimental group outperformed their control group counterpart in the post-test when taught with AIM than traditional method. In terms of cognitive abilities, it was found that experimental group outperformed their control group counterpart in all the domains that were examined i.e. knowledge, intelligence, and creativity. However, Scheffe's test further revealed that experimental group did better in intelligence domain than the two other domains. Thus, the study recommended that students at Senior Secondary Schools in Sokoto state should be taught using AIM in addition to the usual traditional, teacher-led teaching method that has been in use for long.

Keywords: APOS-theory, instruction, cognitive-ability, mathematics and performance

Introduction

Mathematics performance denotes individuals' ability to address mathematical problems. Such problems could be found in all facets of Mathematics: Algebra, Calculus, Analysis, Modelling, Statistics, Fluids, etc., and in utilitarian application or personal experience—formal or informal, depending on the need for its contextual application. Recently, the quest for producing mathematically sound individuals amongst people—teachers, learners, administrators, artisans, engineers, pharmacists, doctors, etc., is overwhelming and has been emphasized by all stakeholders in education industry (Kumar, 2023). This enables such individuals to function well in the society in which they live and interact without much problem. In view of this, Makar, Fry, and English (2023),

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and Westheimer (2015) argued that to be a functional and effective member of the society, the need for developing mathematical skills is instrumental. Therefore, since society today is overwhelmed by myriad of needs, enormous problems, and ample opportunities, students should be trained to become familiar with the best ways these needs could be attained through training—formally or informally, such problems resolved and opportunities explored, harnessed, and used judiciously. Thus, students have to be taught to leverage on their mathematical ability which is explained through their performance in the subject, thereby harnessing their innate mathematical abilities in carrying out these necessary human interactions without many problems. To achieve this, the full potentials of learners' cognitive abilities which include but are not limited to knowledge, intelligence, memory, creativity, spatial ability, visualization have to be properly developed in order to make that ultimate education goal achievable.

Furthermore, learner cognitive abilities require students to learn mathematical problems, internalize the information learnt, and interpret it as an object-like entity (Gravemeijer, 2024; Ellis, Paoletti, & Lockwood, 2024), which could be in a narrative form, expository in nature, and graphic text to reflect real life happenings when effectively guided by a teacher (Brown & Palinscar, 2018). Thus, learner cognitive abilities help students in extracting central ideas from a given mathematical text (problem) and organize the extracted text into meaningful array of information so that it could be calculated, predicted, explained, described, manipulated, and shared in their own way and according to each student's learning phase and/or style (Kellman & Massey, 2013).

Statement of the Problem

The ability to address mathematical problems an individual is confronted with depends, to a large extent on how mathematically-literate that person is. Quite a number of daily engagements in the society today have a lot to do with Mathematics ranging from buying and selling, education, agriculture, aviation, health care delivery, marketing, and a host of other pressing issues. All these phenomena, as they happen in the society, depend to a greater extent on Mathematics as to their occurrence and interpretation. For instance, they rely on an aspect of Mathematics known as statistics—which collects, organizes, presents, analyses, and interprets data for the purpose of carrying out those engagements effortlessly. Peoples' interaction therefore is complementary. Therefore, in order to make the interaction easier, they have to be mathematically sound and numerically coherent. In other words, intellectual numeracy is required to ensure the sustainability of the interaction, hitch-free. This is perhaps what informed the need for Mathematics literacy to be taught across the continuum of students especially at the formative level of its learning. By and large, this mathematical literacy enhances students' capacity to perform better in Mathematics and allows for addressing mathematical problems through the instrumentality learners' cognitive abilities. These cognitive abilities range from knowledge, intelligence, memory, and creativity. Others are verbal ability, reading ability, spatial ability as well as quantitative reasoning.

Academics are of different views regarding what could be termed as students' mathematical problem solving performance which, some argue, depends on teachers' pedagogical competence while others alluded to the inherent students' cognitive abilities as to their mathematical competence. Thus, in line with this assertion, cognitively-inclined theories have to be used by teachers to address mathematical problem solving performance. Such theories as APOS, variation, SOLO Taxonomy, reification and commognition to mention but a few, are suitable for addressing Mathematics problem solving performance in relation to students' cognitive abilities. As a consequence, some countries around the world incorporate Mathematics problem solving in their curriculum because of its importance to everyday life and bearing to national development in terms of their planning

processes (budget, price indices, financial inclusion, annuities, exchange rates, etc.). To achieve this, students' mathematical problem solving performance has to be given special consideration from the outset when they are in secondary school before they eventually venture into various life careers having to do with those planning areas mentioned earlier. Yet, in spite of this need for Mathematics, in recent years, students' performance in Mathematics, based on some examination results indicators, has been steadily declining. Therefore, this study was embarked upon to examine the extent to which this problem has been affecting Mathematics learning in Nigeria, and specifically within Sokoto. Table 1 indicates the downward trend of students' performance in West African Examination Council (WAEC) Mathematics results spanning three years (2019-2021), and this serves as a justification for embarking on this study in order to address the problem.

Table 1

Summary of WAEC Results Statistics (2019-2021) for Sokoto State

S/N	State	Year	Enrollment		Total	% Pass at five credits including in Mathematics
			Male	Female		
	Sokoto	2019	13,229	7,404	20,633	39.03
	Sokoto	2020	8,952	6,484	15,436	37.88
	Sokoto	2021	1,087	592	1,679	32.77

Source: National Bureau of Statistics (2022).

Objectives of the Study

The objectives of the study were to find out:

(1) the difference in the mathematical problem solving performance of students taught using APOS instruction and those taught using traditional method of teaching in Sokoto state;

(2) gender difference in the mathematical problem solving performance of students taught using APOS instruction in Sokoto state;

(3) the difference in learners' cognitive abilities on mathematical problem solving performance taught using APOS instruction in Sokoto state.

Research Questions

The following research questions were answered in the study:

(1) Is there any difference in the mathematical problem solving performance of students taught using APOS instruction and those taught using traditional method of teaching in Sokoto state?

(2) Is there any gender difference in the mathematical problem solving performance of students taught using APOS instruction in Sokoto state?

(3) Is there any difference in learners' cognitive abilities on mathematical problem solving performance taught using APOS instruction in Sokoto state?

Null Hypotheses

The following null hypotheses were tested and analyzed at 0.05 level of significance.

(1) There is no significant mean difference in the mathematical problem solving performance of students taught using APOS instruction and those taught using traditional method of teaching in Sokoto state.

(2) There is no significant difference in the mathematical problem solving performance of male and female

students taught using APOS instruction in Sokoto state.

(3) There is no significant mean difference amongst the learners' cognitive abilities in mathematical problem solving performance taught using APOS instruction in Sokoto state

Review of Related Literature

Learner cognitive abilities are some aspects of students' mental settings responsible for cognition, such as memorizing and remembering information that had been learnt; exhibiting and focusing attention on what is to be learnt; speed of information processing; and spatial and causal reasoning amongst others (Uus, Seitlinger, & Ley, 2020). Individual differences amongst students are measured by comparing scores on tests of these mental abilities in order to establish which one of them has far-reaching effect on learning. For instance, tests of general intelligence as an aspect of learner cognitive ability, such as the Wechsler Adult Intelligence Test, are based on a broad sample of these mental ability tests, and measures of aptitudes for learning in specific instructional domains, such as Mathematics or language learning (Vista & Alahmadi, 2022; Li, 2016; Robinson & Lubienski, 2011). Therefore, within the context of this research, the learner cognitive abilities that were examined in their relation to students' mathematical problem solving performance are: knowledge, intelligence, and creativity.

Knowledge is taken to mean becoming aware of, or understanding of someone or something and being conscious of them, such as factual statements—a definitive knowledge (Small, 2020), skills formation—procedural knowledge, or objectification—symbolic knowledge (Rata, 2019). In other words, it is familiarization with a given set of information required to be learnt. Therefore, to acquire knowledge, some of these sources and schemata have to be observed with individual learners i.e. ability to perceive information, to reason, to develop retentive memory, to testify to the acquisition of the said knowledge, scientific inquiry skills development, and in the end, education and practice of the acquired knowledge (Choo, 2016).

Intelligence, as an aspect of learners' cognitive ability, has been defined differently: firstly, the ability of learner to cultivate skills of abstraction, logic, understanding, self-awareness, self-worth, learning, knowledge conception, reasoning, creativity, critical thinking, and problem-solving skills (A. G. Renatovna & A. S. Renatovna, 2021) amongst others. More succinctly, it can be described as the ability and/or capacity to perceive or infer information, and to retain it as an acquired knowledge to be applied towards solving behavioural problems within an environment or a given context of learning (Wang, 2009). Intelligence is most often studied in humans but has also been observed in both non-human animals and in plants despite controversy as to whether some of these forms of life exhibit intelligence. Thus, in this research, the concern is that of human intelligence (learning), which involves learners' cognitive ability and how they use it in learning Mathematics and solving mathematical problems. Within the context of this research, comprehension ability of students in mathematics was studied.

Creativity is a phenomenon whereby something new and valuable is formed; such that the innovated idea may be readily used or applied such as a novel idea, a new scientific theory, an outstanding musical composition, or a masterstroke joke or even a physical object such as an invention, a printed literary work, or a painting (Bhatnagar & Badke-Schaub, 2017). Creativity as an area of interest to the scholars is found in a number of disciplines, which primarily include but are not limited to psychology, business studies, and cognitive science and mathematics (Hernandez-Torrano & Ibrayeva, 2020). These disciplines cover areas relating to creativity, intelligence, personality, mental health, or artificial intelligence; the potential for fostering creativity through education and training; the fostering of creativity for national economic benefit; and the application of creative resources to improve the effectiveness of teaching and learning. In a summary of scientific research into creativity,

Mumford suggested: “Over the course of the last decade, however, we seem to have reached a general agreement that creativity involves the production of novel, useful products” (Mumford, 2003, p. 110).

Furthermore, students’ performance in Mathematics had been very disturbing (Garba, Ismail, Osman, & Rameli, 2020; Suleiman & Hammed, 2019). According to Suleiman and Hammed (2019), this was occasioned by a number of factors which include students’ biological readiness, instructional strategy employed, peer pressure, curriculum inflexibility, frequent transfer of Mathematics motivation, poor socio-economic background of students amongst others. Much literature suggests that the problem is universal—that virtually every country has a problem of students’ poor performance in Mathematics (Hanushek, Peterson, & Woessmann, 2010). Within the country, statistics had shown how some regions fared relative to others in Mathematics performance. The variation also is state-wide, in which some states were found to be better than others. Thus, this study is an attempt to address some of these problems affecting learners in Sokoto, Kebbi, and Zamfara after having looked at the problem from global perspective, scaling it to the regions and subsequently states.

Methodology

Quasi-experimental design was used for the conduct of the study. Thus, a pre-test—treatment—posttest, non-randomized approach was used to ascertain the impact of APOS instruction of teaching on learners’ cognitive abilities and mathematical problem solving performance. Experimental and control groups were used in the study. Experimental group received instruction using APOS instruction while control group received instruction using traditional method of teaching.

The subjects in the study were Senior Secondary School II (SS 2) students in Sokoto state. Their age ranged from 16-19 years. The total number of this category of students for the 2021-2022 academic year when the study was conducted was put at 23,730 students distributed across the six education zones of the state. The sample selected for the research was 422 (285 experimental and 137 control) SS 2 students. The procedure used in the selection of the samples was a multi-stage sample selection. In the first stage, stratified sampling was used in the selection of 12 schools cutting across the six education zones of the state. The second stage involved the use of simple random sampling approach in the selection of six schools, one from each of the education zones in the state and one intact class from the selected schools that participated in the study was used. In the third stage, simple random sampling approach was used to assign experimental and control groups to the intact classes selected. Two types of instruments were used in the study. The two instruments were developed by the researcher using the contents of SSS Mathematics curriculum developed by NERDC. The test items were constructed drawing from the provisions of the syllabus in the theme “Algebraic Processes” aspect of the curriculum. The instruments were:

(1) Mathematics Problem Solving Performance Test (MPSPT) which was designed to measure students’ mathematical knowledge.

(2) Learner Cognitive Ability Test (LCAT) which was designed to measure students’ mathematical intelligence and mathematical creativity.

The validity of the instruments (MPSPT and LCAT) was determined by experts in the field of Mathematics Education, Test and Measurement, and Curriculum and Instruction in the Faculty of Education and Extension Services, Sokoto. The experts made constructive contribution and independent corrections. All the observations were harmonized and incorporated to ensure high validity of the two instruments. The instruments were administered to 100 SS 2 students each from the two secondary schools in Sokoto state for pilot testing. The

schools used for pilot testing fall within the population of the study but were not part of the sample for the study. Using test re-test reliability measurement, the two results were compared and analysed using PPMCC, and their indices obtained. The reliability index of MPSPT was 0.70 while the reliability index for LCAT was 0.85. The indices for the two instruments were substantially within the range of reliability level in the test and measurement and were therefore considered adequately consistent and reliable.

The procedure for the administration and collection of data was in three phases. Phase one featured the administration of pre-test to both experimental and control groups which revealed homogeneity of the groups to the intended treatment. The second phase was four weeks of instruction on APOS instruction in mathematical problem solving performance to the experimental group. The control group was also taught for four weeks using traditional method of teaching in mathematical problem solving performance. Both experimental and control groups received instruction from the researcher in order to control the effect of teacher variable. The third phase featured the administration of post-test to both groups. The data collected were subjected to statistical analysis. The statistical tools employed were mean, standard deviation, and *t*-test.

Results of the Study

The findings of the study were presented in two different categories; i.e. descriptive and inferential levels. At descriptive level, mean, standard deviations were used, while at inferential level, *t*-test and ANOVA were used to report the differences observed between the set of treatments administered; i.e. between experimental and control groups as well as amongst the three cognitive abilities.

Testing Null Hypotheses

Null Hypothesis One

There is no significant mean difference in the mathematical problem solving performance of students taught using APOS instruction and those taught using traditional method of teaching in Sokoto state.

Table 1

Summary of T-Test Analysis of Students' Performance Between APOS Instruction and Traditional Method in Sokoto State

Method of teaching	N	Mean	SD	DF	*t*-cal.	*p*-value	Decision
APOS instruction	285	41.48	6.33				
				420	9.64	0.007	H_0 rejected
Traditional method	137	34.77	7.42				

Source: Research Fieldwork (2023), $\alpha = 0.05$.

Table 1 showed the summary of *t*-test analysis for the difference in the performance of students taught Mathematics using APOS instruction and those taught using traditional method in Sokoto state. The result of the analysis showed that *p*-value = 0.007, which is less than $\alpha = 0.05$. Thus, when $p \leq 0.05$, the null hypothesis is rejected. This implies that the null hypothesis which says that there is no significant mean difference between the performance of students taught Mathematics using APOS instruction and those taught using traditional method was rejected. Hence, the result revealed that there was significant difference in the performance of students when taught with APOS instruction and traditional method in Sokoto state.

Null Hypothesis Two

There is no significant difference in the mathematical problem solving performance of male and female students taught using APOS instruction in Zamfara state.

Table 2

Summary of T-Test Analysis Between Male & Female Students' Performance in Sokoto State Taught Using APOS Instruction

Gender	*N*	Mean	SD	DF	*t*-cal.	*p*-value	Decision
Male	200	43.07	5.03				
				350	10.62	0.00	H_0 rejected
Female	152	35.80	7.79				

Source: Research Fieldwork (2023), $\alpha = 0.05$.

Table 2 showed the summary of *t*-test analysis between the performance male and female students taught using APOS instruction in Sokoto state. The result of the analysis showed that *p*-value = 0.00, which is less than $\alpha = 0.05$. Thus, when $p \leq 0.05$, the null hypothesis is rejected. This implies that the null hypothesis which says that there is no significant mean difference between the performance of male and female students taught Mathematics using APOS instruction was rejected. Hence, the result revealed that there was significant difference between the performances of students when taught with APOS instruction. Therefore, it is concluded that there is significant gender difference in the mathematical problem solving performance of students.

Null Hypothesis Three

There is no significant mean difference amongst the learners' cognitive abilities in mathematical problem solving performance taught using APOS instruction in Sokoto state.

To test this null hypothesis, three cognitive abilities were examined. They are: knowledge, intelligence, and creativity. Each was considered as a unique domain of students' cognitive ability that requires to be tested independently.

Table 3

Summary of T-Test Analysis Between Experimental and Control Groups on Knowledge Domain in Sokoto State

Variables	*N*	Mean	SD	DF	*t*-cal.	*p*-value	Decision
Experimental group	285	41.91	5.08				
				420	9.70	0.00	H_0 rejected
Control group	137	36.26	6.57				

Source: Research Fieldwork (2023), $\alpha = 0.05$.

Table 3 showed the summary of *t*-test analysis between the performance of experimental and control groups on knowledge domain in Sokoto state. The result of the analysis showed that *p*-value = 0.00, which is less than $\alpha = 0.05$. Thus, when $p \leq 0.05$, the null hypothesis is rejected. This implies that the null hypothesis which says that there is no significant mean difference between students' cognitive ability in mathematical problem solving performance in Sokoto state was rejected. Hence, the result revealed that there was significant difference between the performances of experimental group who were taught using APOS instruction and their control group counterpart who were taught using traditional method of teaching. Therefore, it is concluded that there is significant difference in the mathematical problem solving performance of students in favour of experimental

group. Hence, the experimental group who were taught using APOS instruction performed significantly higher in mathematical knowledge as a domain in cognitive ability.

Table 4

Summary of T-Test Analysis Between Experimental and Control Groups on Intelligence Domain in Sokoto State

Variables	*N*	Mean	SD	DF	*t*-cal.	*p*-value	Decision
Experimental group	285	46.92	6.91				
				420	12.14	0.00	H_0 rejected
Control group	137	38.43	7.05				

Source: Research Fieldwork (2023), $\alpha = 0.05$.

Table 4 showed the summary of *t*-test analysis between the performance of experimental and control groups on intelligence domain in Zamfara state. The result of the analysis showed that *p*-value = 0.00 is less than $\alpha = 0.05$. Thus, when $p \leq 0.05$, the null hypothesis is rejected. This implies that the null hypothesis which says that there is no significant mean difference between students' cognitive ability in mathematical problem solving performance in Sokoto state was rejected. Hence, the result revealed that there was significant difference between the performances of experimental group who were taught using APOS instruction and their control group counterpart who were taught using traditional method of teaching. Therefore, it is concluded that there is significant difference in the mathematical problem solving performance of students in favour of experimental group. Hence, the experimental group who were taught using APOS instruction performed significantly higher in mathematical intelligence as a domain in cognitive ability.

Table 5

Summary of T-Test Analysis Between Experimental and Control Groups on Creativity Domain in Sokoto State

Variables	*N*	Mean	SD	DF	*t*-cal.	*p*-value	Decision
Experimental group	285	36.86	4.52				
				420	3.26	0.00	H_0 rejected
Control group	137	35.23	5.28				

Source: Research Fieldwork (2023), $\alpha = 0.05$.

Table 5 is the summary of *t*-test analysis between the performance of experimental and control groups on creativity domain in Sokoto state. The result of the analysis showed that *p*-value = 0.00 is less than $\alpha = 0.05$. Thus, when $p \leq 0.05$, the null hypothesis is rejected. This implies that the null hypothesis which says that there is no significant mean difference between students' cognitive ability in mathematical problem solving performance in Sokoto state was rejected. Hence, the result revealed that there was significant difference between the performances of experimental group who were taught using APOS instruction and their control group counterpart who were taught using traditional method of teaching. Therefore, it is concluded that there is significant difference in the mathematical problem solving performance of students in favour of experimental group. Hence, the experimental group who were taught using APOS instruction performed significantly higher in mathematical creativity as a domain in cognitive ability.

Table 6

Summary of ANOVA Between Experimental and Control Groups Amongst the Cognitive Abilities in Sokoto State

Variables	Sum of squares	DF	Mean square	*F*	*p*-value	Decision
Between groups	22,972.275	5	4594.455	133.229	.000	Sig.
Within groups	43,451.495	1,260	34.485			
Total	66,423.769	1,265				

Source: Research Fieldwork (2023), $\alpha = 0.05$.

Table 6 showed the summary of ANOVA result of students' mean performance on cognitive abilities between experimental and control groups in Sokoto state. From Table 6, the *p*-value of 0.00 is less than the alpha value of 0.05 ($p < 0.05$), with *F*-ratio value of 133.229. This implies that there is probably significant mean difference in cognitive abilities between experimental groups who were taught mathematical problem solving using APOS instruction over control groups who were taught using traditional method. Therefore, to find out which group performed significantly better in the cognitive abilities with the use of APOS as the instruction tool, Table 7 presents the Scheffe's Post-Hoc analysis of the result.

Table 7

Summary of Scheffe's Test Between Experimental and Control Groups Amongst the Cognitive Abilities in Sokoto State

Variables	*N*	Subset for alpha = 0.05			
		1	2	3	4
Control creativity domain	137	35.2336			
Control knowledge domain	137	36.2555	36.2555		
Experimental creativity domain	285	36.8596	36.8596		
Control intelligence domain	137		38.1533		
Experimental knowledge domain	285			41.9053	
Experimental intelligence domain	285				46.9228
Sig.		0.215	0.086	1.000	1.000

Source: Research Fieldwork (2023), $\alpha = 0.05$.

Table 7 showed the Post-Hoc analysis of the null hypothesis which stated that there is no significant mean difference amongst the learners' cognitive abilities in mathematical problem solving performance taught using APOS instruction in Sokoto state. The result revealed that experimental group performed better in intelligence than their counterparts in other cognitive abilities (i.e. creativity and knowledge). This is because the value of Scheffe's test in respect of experimental group is 46.92 at $\alpha = 0.05$ level of significance which is greater than other groups in remaining cognitive abilities.

Findings of the Study

Below are the findings from the study:

(1) Hypothesis one revealed that the difference between students taught Mathematics using APOS instruction and those taught using traditional method in Sokoto state is significant. This is because the hypothesis was rejected, indicating that students using APOS instruction outperformed their counterpart who were taught using traditional method of teaching.

(2) Hypothesis two revealed that the difference between male and female students taught using APOS

instruction in Sokoto state is significant, indicating that male students outperformed their female counterpart. Hence, there is gender difference in the performance of students.

(3) The findings of the null hypothesis three are in respect of the three cognitive abilities examined in the study. They are: knowledge, intelligence, and creativity. Hence, the findings are:

(i) In terms of knowledge domain, the study revealed that the experimental group who were taught using APOS instruction outperformed their control group counterpart who were taught using traditional method. Therefore, experimental group showed high level of mathematical knowledge than the control group in Sokoto state.

(ii) In terms of intelligence domain, the study revealed that the experimental group who were taught using APOS instruction outperformed their control group counterpart who were taught using traditional method. Therefore, experimental group showed high level of mathematical intelligence than the control group in Sokoto state.

(iii) In terms of creativity domain, the study revealed that the experimental group who were taught using APOS instruction outperformed their control group counterpart who were taught using traditional method. Therefore, experimental group showed high level of mathematical creativity than the control group in Sokoto state.

(4) Lastly, through post-hoc analysis, the study revealed that the experimental group in Sokoto did better in intelligence domain than the two other domains of knowledge and creativity as well as their control group counterpart in all the domains.

Discussion of Findings

The study examined the impact of APOS instruction on learners' cognitive abilities and mathematical problem solving performance of senior secondary school (SSII) students in Sokoto state, Nigeria. Findings from the study indicated that experimental group that received instruction through APOS instructional framework performed better in their posttest mean score in mathematical problem solving performance. This finding is in agreement with the study of Arnawa, Sumarno, Kartasasmita, and Baskoro (2007), on applying APOS to improve students' ability to prove in elementary abstract algebra, which indicated that experimental group performed significantly higher than the control group.

Another finding of this study is that, there was no significant gender gap in mean score differences of students' performance in Zamfara state. In agreement with this was a study by Arnawa and Nita (2020) on improving students' achievement in elementary linear algebra through APOS theory approach. The study was a pre-experimental one-shot case study design in which 32 students participated. The students were taught Chemistry Mathematics II in the academic year 2018/2019 at Andalas University. The study found that APOS theory can improve students' understanding in linear algebra and there was no significant gender gap in students' achievement in elementary linear algebra.

A study by Baye, Ayele, and Wondimuneh (2021) on implementing GeoGebra integrated with multi-teaching approaches guided by the APOS theory to enhance students' conceptual understanding of limit in Ethiopian Universities is in agreement with one of the findings of this study because the study indicated that the results of the quantitative (posttest) data analysis proved that students' mean score performance (i.e. knowledge of Mathematics) on conceptual understanding of limit in the experimental group was significantly better than those in the control group.

Conclusion

Findings from this study suggest that the strategy proved potent at enhancing mathematical problem solving performance of secondary schools in Sokoto state. The conclusion drawn from the study was that APOS instruction has the potential of improving students' mathematics performance. When effectively utilized by teachers, the strategy has the potential to improve students' mathematical intelligence. Knowledge and creativity can also be improved in learners, even though the strategy proved more potent on intelligence.

Recommendations

In line with the findings of the study, the following recommendations were offered:

(1) APOS instruction should be adopted as instructional tool to complement traditional method in teaching Mathematics in Nigerian secondary schools. This will help greatly in improving students' understanding of abstract concepts of most themes in the revised senior secondary Mathematics curriculum that have been deemed difficult to teach and learn by both teachers and students, thereby improving their academic performance.

(2) APOS instruction was found to be gender sensitive and has proven to discriminate between male and female students' performance in Sokoto state. It is therefore imperative for teachers of Mathematics to incorporate this method in order to enhance the academic performance of female students in secondary schools across the state.

(3) Teachers should give special attention to female students when teaching so as to develop their cognitive abilities like their male counterparts because in all the domains measured male students outperformed their female counterparts.

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