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Enhancing Student's Ability to Solve Mathematical Word Problems

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Research has shown that American students often need to improve in solving word problems than their international peers. Several factors contribute to this trend, including differences in curriculum, teaching methods, and cultural attitudes toward mathematics education (Mullis, Martin, Foy, & Hooper, 2016; OECD, 2019). "Traditional mathematics instruction has always focused on skills and shot-cut problem-solving," often assessing students' ability to solve word problems solely based on the correctness of their final answers. This results-oriented evaluation fails to effectively diagnose the root causes of students' difficulties in solving word problems. If students can be guided through problem-solving processes and strategies, it would be beneficial in enhancing their problem-solving abilities.

Keywords: mathematical word problems, problem-solving, teaching problem-solving

The Importance of Problem-Solving Teaching

In the ever-evolving landscape of education, problem-solving is a cornerstone of mathematics instruction. Both the National Council of Teachers of Mathematics (NCTM, 2000) and the Common Core State Standards (CCSS) emphasize the importance of developing strong problem-solving skills among students. This focus aims to nurture students' independent thinking and ability to tackle word problems, which are essential for applying mathematical knowledge to real-world scenarios. Word problems mirror common quantitative relationships and practical issues in everyday life, making mathematics a valuable tool for solving real-world problems. This perspective is increasingly recognized by mathematics educators globally.

The emphasis on problem-solving by NCTM and CCSS underscores its vital role in mathematics education. By developing strong problem-solving skills, students gain the ability to understand and apply mathematical concepts, providing them with essential tools for addressing challenges in various aspects of their lives. As educators continue to implement these standards and principles, they help cultivate student's adept at critical thinking, effective problem-solving, and meaningful application of their knowledge.

In traditional teaching, students often struggle to abstract real-world problems into mathematical problems or to apply their mathematical knowledge practically. They may mechanically imitate solutions to common mathematical problems but feel helpless when faced with new challenges. To effectively discover and solve problems, students need an understanding of scientific thinking methods such as observation, analysis, induction,

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analogy, abstraction, generalization, and conjecture. Therefore, it is crucial to strengthen the teaching of word problems to cultivate students' practical problem-solving abilities, ensuring long-term benefits for their overall problem-solving skills.

Teaching word problems helps develop students' logical thinking skills, fosters flexibility and creativity, and equips them with essential skills for becoming well-educated citizens in modern society. The problem-solving experience gained from well-defined and targeted problems enables students to apply mathematical methods to daily life challenges. This practical relevance highlights the urgency for educators and parents to guide students in developing strong problem-solving skills, which are indispensable in real-life situations.

By focusing on these strategies, educators can ensure that students are prepared to meet academic challenges and capable of applying their mathematical knowledge in practical and meaningful ways, enhancing their overall educational experience and preparing them for future success.

Common Types of Errors in Children's Problem-Solving

Research by Russell (2000), Siegler (2003), Booth and Koedinger (2012), and Ashlock (2006) has identified several common errors in elementary school students' problem-solving. These errors include:

(1) Inability to understand the problem: Students often struggle to grasp the problem's context and requirements.

(2) Seeking keywords as the first step: Many students rely on keywords to determine the operation method, leading to incorrect problem-solving approaches.

(3) Weak unit concepts: Misunderstanding or misusing units is a frequent issue.

(4) Inability to form equations: Students often struggle translating word problems into mathematical equations.

(5) Inability to monitor their problem-solving process: Self-monitoring during problem solving is often lacking, leading to errors.

(6) Errors in four operations concepts: Mistakes in addition, subtraction, multiplication, and division are common.

(7) Lack of time and length concepts: Students frequently lack a general understanding of time and length measurements.

(8) Insufficient practical experience with money concepts: Despite theoretical knowledge, practical experience with money is often lacking.

(9) Incomplete understanding of vertical operations concepts: Vertical alignment and columnar calculations are frequently misunderstood.

Currently, when schools assess students' ability to solve word problems, they usually focus solely on the correctness of the final answer. A comprehensive assessment of these skills is necessary to effectively diagnose students' difficulties in solving word problems. Educators and parents must guide students through the problem-solving process and strategies, empowering them to improve their abilities. In addition to addressing these errors, educators can implement various strategies to enhance students' problem-solving skills:

(1) Explicit instruction: Teaching problem-solving strategies explicitly can help students recognize and apply appropriate methods.

(2) Practice with feedback: Providing students with ample practice opportunities and constructive feedback can solidify their understanding.

(3) Encouraging metacognition: Teaching students to think about their thinking and self-monitor their problem-solving process can reduce errors.

(4) Contextual learning: Relating problems to real-life contexts can make learning more meaningful and engaging.

(5) Collaborative learning: Group problem-solving activities can promote discussion, shared strategies, and peer learning.

By focusing on these areas, educators can help students build a stronger foundation in problem-solving, leading to improved performance and confidence in mathematics.

Problem-Solving Process Models

"Problem-solving" is a significant research topic in cognitive psychology, and solving mathematical word problems is a crucial aspect of this broader concept. Cognitive psychologists' research on problem-solving, particularly the models proposed by Polya (1945) and Mayer (1987; 1992), can be directly applied to assist students in solving mathematical word problems. Polya's model offers a structured approach to navigating the problem-solving process, while Mayer's model provides insights into the cognitive aspects of problem-solving. These models make the topic more engaging and relevant to educators, parents, and students by providing practical applications and frameworks for teaching and learning.

Polya's Problem-Solving Process Model

Polya (1945) proposed a four-stage problem-solving process:

(1) Understanding the problem: This involves knowing what the problem is asking and identifying the known and unknown conditions. Students should carefully read the problem and determine what information is given and what needs to be found.

(2) Devising a plan: The next step is formulating a method, strategy, and steps to solve the problem. Students should consider various strategies, such as drawing diagrams, creating equations, or breaking the problem into smaller parts.

(3) Carrying out the plan: Implementing the devised plan involves executing the steps formulated in the previous stage. Students should follow their plan methodically and make necessary calculations or constructions.

(4) Reviewing the solution: Checking the reasonableness of the answer, considering alternative methods to solve the problem, or applying the method to different problems. This stage helps ensure the solution is correct and encourages reflective thinking.

Mayer's Problem-Solving Process Model

From a cognitive psychology perspective, Mayer (1987; 1992) divided the problem-solving process into problem representation and problem-solving. Problem representation is further divided into problem translation and problem integration, while problem-solving is divided into planning, monitoring, and execution. Below are explanations of problem translation, problem integration, problem-solving planning and monitoring, and problem-solving execution:

(1) Problem translation: This stage involves translating each statement into an internal representation, understanding the relationships between sentences, and interpreting each statement using "language knowledge" and "factual knowledge". Problem translation requires a comprehensive understanding of the problem's language

and context. Many students focus solely on the numbers in the problem without grasping their meaning, leading to random and often incorrect calculations.

(2) Problem integration: This involves recognizing the problem type, identifying relevant and irrelevant information, determining the necessary information to solve the problem, and representing the problem using diagrams or symbols, which requires “schema knowledge”. Problem integration synthesizes important information from the problem, discerning essential from non-essential details, and categorizes the problem based on known types. This stage helps form a coherent understanding of the problem.

(3) Problem-solving planning and monitoring: In this phase, students find the direction for solving the problem using various strategies. They represent the problem with “numeric sentences”, “equations”, or “necessary operations”, establish sub-goals, and continuously monitor the problem-solving process. Effective planning and monitoring involve setting up a clear strategy, anticipating potential difficulties, and being prepared to make corrections if errors are found. This requires both strategic thinking and metacognitive skills to evaluate the effectiveness of the chosen approach.

(4) Problem-solving execution: This stage involves using algorithms for simple or continuous calculations and accurately and effectively executing the problem-solving plan, which requires “procedural knowledge”. Problem-solving execution tests students’ computational skills and ability to turn the planned direction into actual calculations. This stage is critical for ensuring the problem is solved correctly and efficiently.

Practical Applications and Implications

(1) Enhanced teaching methods: Educators can use Mayer’s model to structure their lessons, focusing on each stage of the problem-solving process. This can involve teaching students how to translate and integrate problems, plan their approach, monitor their progress, and execute solutions effectively.

(2) Assessment beyond final answers: Instead of focusing solely on the correctness of the final answer, assessments can evaluate students’ understanding and application of each stage in the problem-solving process. This comprehensive assessment can provide deeper insights into students’ strengths and areas needing improvement.

(3) Development of metacognitive skills: Encouraging students to think about their thinking (metacognition) can help them become more aware of their problem-solving processes. Teaching students to plan, monitor, and evaluate their strategies can lead to more effective and reflective learning.

(4) Use of real-world problems: Integrating real-world problems that require students to apply their knowledge in practical contexts can make learning more relevant and engaging. This approach can also help students see the value of the problem-solving skills they are developing.

(5) Collaborative problem-solving: Group activities where students solve problems together can promote sharing strategies and solutions, enhancing their understanding through peer learning. Collaborative problem-solving can also develop communication and teamwork skills.

(6) Use of technology: Incorporating technology, such as interactive software and online tools, can support each stage of the problem-solving process. Technology can provide immediate feedback, alternative strategies, and additional practice opportunities.

By applying Mayer’s cognitive model, educators can create a more structured and effective learning environment that improves students’ mathematical problem-solving skills and prepares them for various complex tasks in their academic and professional lives.

Key Processes in Guiding Children's Problem-Solving

Based on Polya's and Mayer's problem-solving process models, the author divides the psychological processes of problem-solving into five parts to analyze the key processes parents or teachers should pay attention to when assisting children in solving mathematical word problems.

Understanding the Problem

This includes (1) carefully reading the Problem from start to finish, (2) asking oneself if the Problem is understood, (3) restating the Problem in one's own words, (4) identifying and highlighting keywords, and (5) confirming the necessary conditions, known conditions, unknown conditions, and problem goals. The first task in solving a mathematical problem is fully understanding its meaning, which involves comprehending the problem's context. Only then can the solution strategy be found. In teaching this process, parents or teachers can lead children to read the problem, help them understand the Problem through questions, and ensure they truly understand the Problem by asking them to restate it. Parents and teachers should avoid teaching children to search for "keywords" in the problem, such as using subtraction if "difference" appears or addition if "total" appears. Such teaching methods focus only on a few keywords and neglect understanding the overall problem context, leading to incorrect solutions.

Problem Representation

In representing the problem, the solver needs to analyze the problem's context and determine the problem type. Students should identify and translate the appropriate schema from the Problem's context into a computable representation. Parents or teachers should teach students to grasp the relationships between important information in the problem and use auxiliary tools like diagrams, tables, manipulative aids, multiple attempts, and pattern recognition to help represent the Problem. If representation is challenging, students should be guided to reread the Problem to understand the relationships between variables and recall similar past problem-solving experiences.

Devising a Plan

Devising a plan involves finding the problem-solving direction based on the Problem's understanding. Students should identify the necessary and unnecessary conditions for solving the Problem, understand the steps and sub-goals, and plan the solution. Parents or teachers should encourage students to recall relevant concepts or formulas. If similar problem-solving experiences are lacking, the Problem should be simplified. Teachers should also teach problem-solving strategies and ensure students understand when to use them to devise a plan.

Carrying Out the Plan

This stage involves actual numerical calculations. During calculations, procedural knowledge is essential. Parents or teachers should ensure students have proficient procedural knowledge and can calculate accurately and effectively, such as mastering the four operations of addition, subtraction, multiplication, and division.

Evaluating the Solution

After finding the solution, parents or teachers should encourage students to verify and review their answers. Verification ensures accuracy since students may need clarification on whether their answers are correct. After verification, the entire problem-solving process should be reviewed and reflected upon to identify areas for improvement.

Enhancing Students' Mathematical Problem-Solving Beliefs

Schoenfeld (1985) highlighted that students' beliefs and attitudes toward mathematics significantly influence their learning outcomes alongside their mathematical knowledge. Developing a positive mindset and confidence in solving mathematical problems is crucial for students' success. Several strategies can be employed to enhance students' mathematical problem-solving beliefs:

(1) Using appropriate teaching materials and strategies: Select teaching materials that are age-appropriate, engaging, and challenging. Employ strategies that cater to different learning styles, ensuring all students can grasp complex concepts.

(2) Positive role models: Parents and teachers should model positive attitudes towards mathematics. Demonstrating enthusiasm and perseverance when tackling mathematical problems can inspire students to adopt similar attitudes.

(3) Timely encouragement: Provide consistent and timely encouragement. Acknowledge students' efforts and progress rather than just their correct answers. This reinforcement can build their confidence and motivate them to continue improving.

(4) Remedial teaching: Offer additional support and remedial teaching for students who struggle with certain concepts. Tailored interventions can help these students catch up and build a stronger foundation in mathematics.

(5) Engaging and diverse content: Incorporate various content and teaching methods to keep students engaged. Use games, puzzles, and interactive activities to make learning mathematics enjoyable and stimulating.

(6) Connecting learning content with real-life applications: Show students how mathematical concepts apply to real-world situations. This can make learning more relevant and interesting and help students understand the importance and utility of mathematics.

(7) Emphasizing Problem-Solving as a Process: Teach students that problem-solving involves understanding, planning, executing, and evaluating. Please encourage them to view mistakes as learning opportunities and emphasize the value of the problem-solving journey over merely obtaining the correct answer.

(8) Developing a growth mindset: Foster a growth mindset in students by encouraging them to embrace challenges and persist through difficulties. Teach them that intelligence and abilities can be developed through dedication and hard work.

(9) Collaborative learning: Encourage group work and collaborative problem-solving activities. Working with peers can expose students to different perspectives and strategies, enhancing their understanding and problem-solving skills.

(10) Use of technology: Incorporate technology, such as educational software and online resources, to provide interactive and adaptive learning experiences. Technology can offer immediate feedback and personalized learning pathways, aiding students' mathematical development.

(11) Regular reflection and self-assessment: Encourage students to reflect on their learning and problem-solving processes regularly. Self-assessment can help them identify areas for improvement and better understand their strengths and weaknesses.

Implementing these strategies can help educators and parents create a supportive and motivating learning environment. This environment will help students build positive beliefs about their mathematical abilities, leading to greater confidence and success in solving mathematical problems.

Strategies for Problem-Solving Instruction

Traditional mathematics teaching often emphasizes results rather than processes, which can lead to inflexible knowledge application. To address this, the following strategies for problem-solving instruction are suggested for teachers:

(1) Encourage students to read problems aloud: Have students read problems aloud and restate them in their own words. This helps internalize the problem and ensures they fully understand it. For example, if a problem involves calculating the area of a rectangle, students should restate it by saying, “I need to find the area by multiplying the length and width.”

(2) Promote problem-solving records: Encourage students to keep detailed records of their problem-solving processes. This could include noting down each step they take, their strategies, and any difficulties they encounter. Teachers can guide students in reflecting on these records to identify patterns and areas for improvement.

(3) Think-aloud strategies: Promote think-aloud strategies, where students verbalize their thought processes, strategies, and difficulties as they work through problems. This helps make their thinking visible and allows teachers to provide immediate feedback. For example, a student might say, “I’m dividing 36 by 4 because I need to find out how many groups of 4 are in 36.”

(4) Vocabulary understanding: Ensure students understand the vocabulary related to calculation methods. For instance, terms like “sum”, “difference”, “product”, and “quotient” should be clearly explained and used regularly in context.

(5) Introduce non-routine mathematics materials: Use non-routine problems that require students to develop and apply independent problem-solving strategies. For example, presenting puzzles or real-world scenarios that do not have straightforward solutions can stimulate creative and critical thinking.

(6) Problem formulation exercises: Provide exercises where students create their own problems. This deepens their understanding of problem structures and helps them see mathematics as a dynamic and creative subject. For example, ask students to write a word problem involving fractions that their classmates can solve.

(7) Understand fundamental operations: Guide students to understand the true meanings of addition, subtraction, multiplication, and division. Use concrete examples and manipulatives to illustrate these operations. For instance, show how multiplication can be seen as repeated addition using counters or blocks.

(8) Dynamic assessment: Implement dynamic assessment techniques, providing clues and hints during problem-solving. This formative assessment helps identify students’ needs and guide them through their thinking processes. For example, if a student is stuck, ask guiding questions like, “What happens if you divide both sides by 2?”

Conclusion

Problem-solving activities have become the central focus of mathematics education, intertwining instructional content with real-life applications to deepen mathematical understanding through processes grounded in students’ prior knowledge and experiences. Emphasizing a problem-solving-oriented approach transforms the learning experience from rote memorization to active engagement. Educators can create a dynamic and effective learning environment by integrating real-life applications, building on students’ prior knowledge, and fostering a deep conceptual understanding. This approach not only helps students master mathematical concepts but also equips them to tackle complex problems in their academic and professional futures. As problem-solving takes center stage in mathematics education, students will develop the skills and confidence necessary to navigate the challenges of the 21st century.

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