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Nonlinear Analysis of Structures by Total Potential Optimization Using Metaheuristic Algorithms (TPO/MA)

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Abstract: Structural analysis problems can be formulized as either root finding problems, or optimization problems. The general practice is to choose the first option directly or to convert the second option again to a root finding problem by taking relevant derivatives and equating them to zero. The second alternative is used very randomly as it is and only for some simple demonstrative problems, most probably due to difficulty in solving optimization problems by classical methods. The method called TPO/MA (Total Potential Optimization using Metaheuristic Algorithms) described in this study successfully enables to handle structural problems with optimization formulation. Using metaheuristic algorithms provides additional advantages in dealing with all kinds of constraints.

Key words: TPO/MA, structural analysis, nonlinear, truss, metaheuristics, algorithms.

1. Introduction

Structural analysis is one of the main tasks of structural engineers. It consists of determining the behavior of a given structural system under the effect of a given set of external effects like loads, temperature differences, mounting errors, forced displacements, etc. Classical approach in solving these problems starts by writing down equilibrium equations. In linear problems these equations form a linear set, i.e., a matrix equation of the form $Ax = b$, where x is an unknown vector with deflections as members, A is a square matrix describing the geometric and material properties of the structure, and b is the known load vector. Such a system can of course be solved using any convenient technique which does not lack in number and quality. In nonlinear problems it is often impossible to write down explicit equations in matrix form. In such cases various iterative techniques are used to reach a solution for problems at hand.

The method described here, the TPO/MA (Total Potential Optimization using Metaheuristic Algorithms) based on the consideration that structural analysis can well be formulated as optimization problems instead of root-finding problems, and the task of optimization that becomes primordial in the procedure can be performed with a meta-heuristic technique. The first part of this consideration is of course not new: a well-known principle in mechanics states that the total potential energy of a body in stable equilibrium is at minimum. Although this principle is a very fundamental one and gives way to many other techniques for structural analysis, it is conventionally used by itself only for demonstrative purposes for solving problems with a very few number of unknowns, say less than four, probably due to the difficulties encountered in reaching at a practical formulation and also in solving optimization problems with a large number of unknowns. Now with the advances in computer hardware and software, especially with the emergence of meta-heuristic algorithms, such a formulation became feasible and enabled the introduction of the TPO/MA technique. A third advantage of this energy formulation is seen at the point that with this way of

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formulation, the difference between linear and non-linear materials completely disappears, linear materials become treated as a simple special case of a general class of non-linear materials, thus enabling non-linear analysis conductible as a black-box operation.

In the paper, the next section is devoted to the short-comings of classical methods. This section will be followed by a description of TPO/MA and then by numerical examples. The last section is on conclusions and hints on future research.

2. Short-Comings of Classical Methods

Linear problems actually do form a very small percentage of real-life structural problems. Classical methods are very effective in solving these kinds of problems; they can be solved with black-box operations with no interaction of the user. Existing commercial and academic software packages are very advanced in forming the matrix equations mentioned above and solving them once the data of the problem are fed into the computer.

This is not the case for non-linear structural analysis problems like

- structures made of general non-linear materials (plastic, elastic-plastic, strain hardening, strain softening, tensionless, compression less materials; those with non-symmetrical stress-strain characteristics like buckling materials, bimodular materials),
- tensegric structures,
- multi-solution cases, before and after bifurcation,
- compliant structures,
- structures with one-sided constraints including those on tensionless foundations,
- under-constrained structures,
- structures with missing or failed elements, and those undergoing progressive failure.

For these cases and for their combinations there do not exist well-established procedures and each problem in this category is solved as a special problem with a near-academic approach. The techniques used for analyzing such structures are mainly based on Newton-Raphson type iterations or incremental loadings of the system [1-9]. The procedures thus developed are valid only for the special problem considered without being applicable to other ones.

3. TPO/MA for Solving Structural Analysis Problems

The formulation of structural analysis as an optimization problem to be solved by meta-heuristic algorithms is given elsewhere [10-13]. In what follows these presentations are summarized.

The deformed shape of a truss is characterized by the displacement vector:

$$\xi^T = [\xi_{11}\ \xi_{12}\ \xi_{13}\ \xi_{21}\ \xi_{22}\ \xi_{23}\ \ldots\ \xi_{Nj,1}\ \xi_{Nj,2}\ \xi_{Nj,3}]^T \qquad (1)$$

where, ξ_{ij}, i = *1*, …, N_j, j = 1, 2, 3 represents the j'th component of the displacement of joint i, N_j being the number of joints. This means that the original coordinates:

$$x_{ij},\ i = 1, \ldots, N_j,\ j = 1, 2, 3 \qquad (2)$$

Of joint i, it become after the deformations.

$$x_{ij}' = x_{ij} + \xi_{ij},\ i = 1, \ldots, N_j,\ j = 1, 2, 3 \qquad (3)$$

According to these definitions, the new length L'_{pq} of the member between joints p and q can be calculated as:

$$L'_{pq} = \sqrt{\sum_{j=1}^{3}(x'_{pj} - x'_{qj})^2} = \sqrt{\sum_{j=1}^{3}\left[(x_{pj} + \xi_{pj}) - (x_{qj} + \xi_{qj})\right]^2} \qquad (4)$$

The uniform strain in the member, dropping the subscripts for simplicity, can then be calculated as:

$$\varepsilon = \frac{L' - L}{L} \qquad (5)$$

The stress in the member can then be computed from the constitutive equation:

$$\sigma = \sigma(\varepsilon) \qquad (6)$$

which we assume to be continuous, single valued and integrable. The strain energy density stored in the member is then given by:

$$s = \int_0^{\varepsilon} \sigma(\varepsilon)\varepsilon d\varepsilon \qquad (7)$$

If the cross-sectional area of the member is A, then the strain energy stored in that member becomes the strain energy density s multiplied by the volume, i.e.,

$$S = ALs \tag{8}$$

In general, stress-strain relation Eq. (6) can be visualized as in Fig. 1, with piecewise continuous linear functions defined by pairs of stress-strain points. Typical examples are:

(1) (-10, -2,000,000), (0,0), (-10, -2,000,000): linear elastic behavior, with $|\varepsilon_{max}| = 10$, i.e. practically infinite, and modulus of elasticity E = 200,000 N/mm^2;

(2) (-0.08, -16,000), (0, 0), (0.08, 16,000): linear elastic behavior, with rupture at $|\varepsilon_{max}| = 0.08$, and modulus of elasticity E = 200,000 N/mm^2;

(3) (-10, -400), (-0.002, -400), (0, 0), (0.002, 400), (10, 400), elastic-plastic behavior, elastic behavior for $|\varepsilon_{max}| < 0.002$ with E = 200,000 N/mm^2;

(4) (-10, -400), (0, -400), (0, 0), (0, 400), (10, 400), plastic behavior, yield stress = 400 N/mm^2;

(5) (-10, -500,400), (-0.04, -600), (-0.02, -400), (0, 0), (0.02, 400), (0.04, 600), (10, 500,400), strain hardening behavior, E starts from the original value of 20,000 N/mm^2 and diminishes gradually;

(6) (-10, -100), (-0.006, -100), (-0.004, -150), (-0.003, -200), (-0.002, -400), (0,0), (0.002, 400), (10, 400), asymmetrical stress strain behavior in tension and compression: elastic-plastic under tension, buckling under compression;

(7) (-10, -200,0000), (0,0), (10, 0), tensionless behavior;

(8) (-10, 0), (0, 0), (10, 200,0000), compression less behavior (cable);

(9) (-10, -100,000), (0, 0), (10, 200,000), bimodular material, with moduli of elasticity E = 100,000 N/mm^2 in compression and E = 200,000 N/mm^2 in tension, etc.

For example, on tension side, these behaviors can be generalized as the r+1 pairs of type.

$$(\varepsilon_i, \sigma_i),\ \varepsilon_{i+1} \geq \varepsilon_i,\ i = 0, 1, \ldots, r \tag{9}$$

Then it can be shown that for a given strain ε, the stress σ and the strain energy density s, can be calculated from Eq. (12).

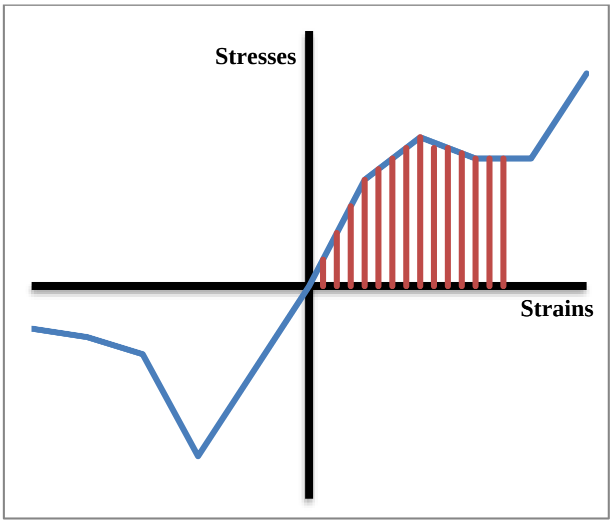


Fig. 1 Stress-strain diagram for nonlinear materials.

Case 1:

For

$$\varepsilon < \varepsilon_r^- \text{ and } \varepsilon > \varepsilon_r^+ \text{ then } \sigma = 0,\ s = 0 \tag{10a}$$

Case 2:

For

$$\varepsilon_p \leq \varepsilon \leq \varepsilon_{p+1}$$

then

$$\sigma = \sigma_p + \frac{\sigma_{p+1} - \sigma_p}{\varepsilon_{p+1} - \varepsilon_p}(\varepsilon - \varepsilon_p),$$

$$s = \sum_{i=0}^{p} \frac{\sigma_i + \sigma_{i+1}}{2}(\varepsilon_{i+1} - \varepsilon_i) + \frac{\sigma + \sigma_p}{2}(\varepsilon - \varepsilon_p) \tag{10b}$$

It can be seen from Eq. (10a) that Case 1 corresponds to the rupture of the material with ε_r^- and ε_r^+ being the rupture values on compression and tension sides, respectively. The strain energy density, s, as calculated from Eq. (10b) is nothing but the shaded area shown in Fig. 1.

This approach obviously indicates that the calculations for a linear elastic material and for a very complicated unsymmetrical behaving material are practically of the same order of difficulty. Linear materials do form only a special case of nonlinear materials. No material necessitates any kind of special treatment; there is no need to make different formulations for any kind of materials.

The only exceptions to these representations are continuous functions. They can be treated in this approach even with less difficulty if the stress-strain function given as in Eq. (6) is integrable, as one of the assumptions states [10].

If all the members of the truss are considered, then it will be more convenient to write Eq. (8) in the form:

$$S = \sum_{i=1}^{N_m} A_i L_i s_i \tag{11}$$

where N_m is the number of members. Adding to this term algebraically, the work done by the external forces, one obtains the final total potential of the system:

$$\Pi = \sum_{i=1}^{N_m} A_i L_i s_i - \sum_{k=1}^{N_P} P_k d_k \tag{12}$$

where P_k's are the external load components applied to the structure, d_k's are the displacements corresponding to these components, and N_P is the number of load components. It is evident that d_k's correspond to the displacements ξ_{ij} where non-zero load component exists.

The symbol Π represents the scalar function to be minimized, which is a function of the variables ξ_{ij} corresponding to the components of the nodal displacements. The number of these components will be $2N_j$ for plane systems, and $3N_j$ for space trusses, respectively, if there are no constraints at all. In fact, some of them, corresponding to the supports, are zero. There may be some other constraints on ξ_{ij}'s of the form.

$$\xi_{ij} < a,\ \xi_{ij} \le a,\ a \le \xi_{ij} < b,\ \xi_{ij} = a\xi_{ik} \tag{13}$$

Then the final formulation of the structural analysis problem becomes: Determining ξ as defined in Eq. (1), satisfying the constraints of the problem and minimizing $\Pi(\xi)$ as given in Eq. (12).

After finding the displacements following this optimization process, the member forces and the relevant support reactions are determined, as in the classical methods, to complete the structural analysis.

The problem, formulated as above can be attacked by any of the appropriate optimization methods. In TPO/MA, the preferred techniques are meta-heuristic algorithms. Applications of various algorithms of this kind have already given very satisfactory results. It seems that the only restriction on the choice of these algorithms is their suitability in dealing with problems of continuous variables.

In meta-heuristic algorithms, the candidate set of vectors to be improved may consist of one vector only, or may form a population with a number of vectors. In the first group one can count Local Search and Simulated Annealing methods, both exploited in TPO/MA. The methods tried in the second group are Genetic Algorithms, Harmony Search, Ant Colony Optimization and Particle Swarm Optimization. Whatever the special algorithm chosen is, the general procedure applied in this study can be shown as in the flow chart demonstrated in Fig. 2 [12]. The only algorithm, as far as the authors are aware, which does not exactly fit in this flow chart is the Ant Colony optimization among the ones applied.

Application procedures of different algorithms do not show significant differences in many steps in this flow chart. The steps that are algorithm-dependent are the ones related to creation of base and candidate configurations, and the one related to redefinition of base configuration. To give some examples, in Local Search and Simulated Annealing, the set of base and candidate vectors consists of one single vector only, while in the others the same set may contain tons of configurations. In Local Search the next base vector is taken to be the best of the old base vector and the candidate vector, while in Simulated Annealing there are instances where the worse of the two vectors can be chosen to be the next candidate vector. Application of Genetic Algorithms necessitates use of many special operators like selection, mating, cross-over, and mutation in the phase of creation of the next candidate vectors. Harmony Search makes use of its memory in this phase. Thus, the flow chart given has to be considered only as a general guide to TPO/MA, it will have many special steps when attached to a particular algorithm.

4. Applications

The TPO/MA method described above is applied to an important number of trusses under different loading and support conditions to test the success of the method as to the accuracy and robustness. In the following paragraphs results of three applications are presented. In these analyses, we have applied metaheuristic algorithms like random search, simulated annealing, harmony search, genetic algorithms, ant colony optimization, all with success.

4.1 2-Bar Truss (Von Misestruss)

The truss shown in Fig. 3a is analyzed for an important range of loads and the results are compared with already existing ones from literature [10, 14]. This simple system which has two degrees of freedom, i.e., the vertical and horizontal displacements of the hinged point at the middle, enabled the drawing of the total potential surface shown in Fig. 3b. The figure clearly shows the two stable—one global and one local, and one unstable solutions of the truss. The stable solutions correspond to the pits on the surface, and the unstable solution is on the ridge between the pits.

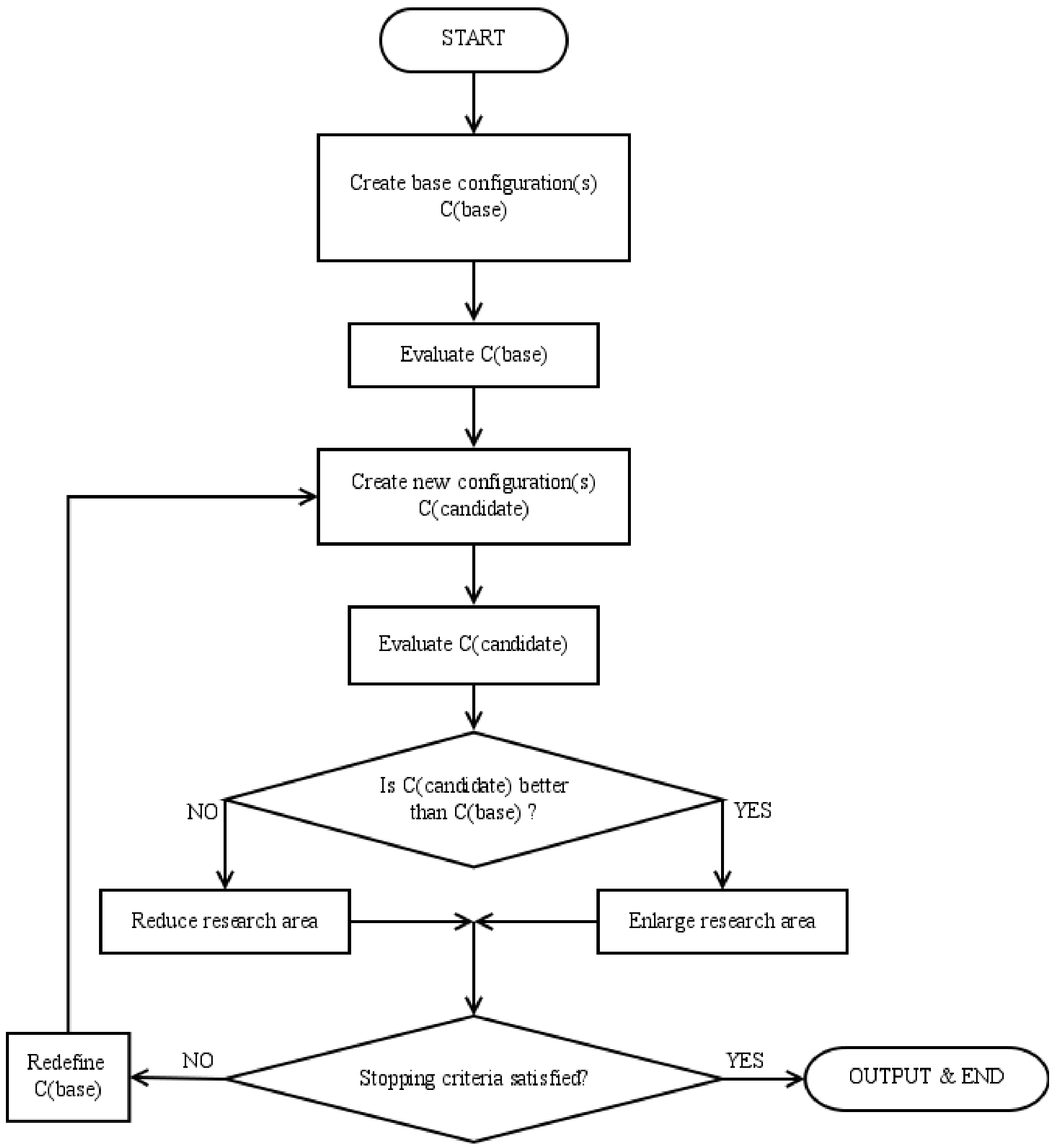
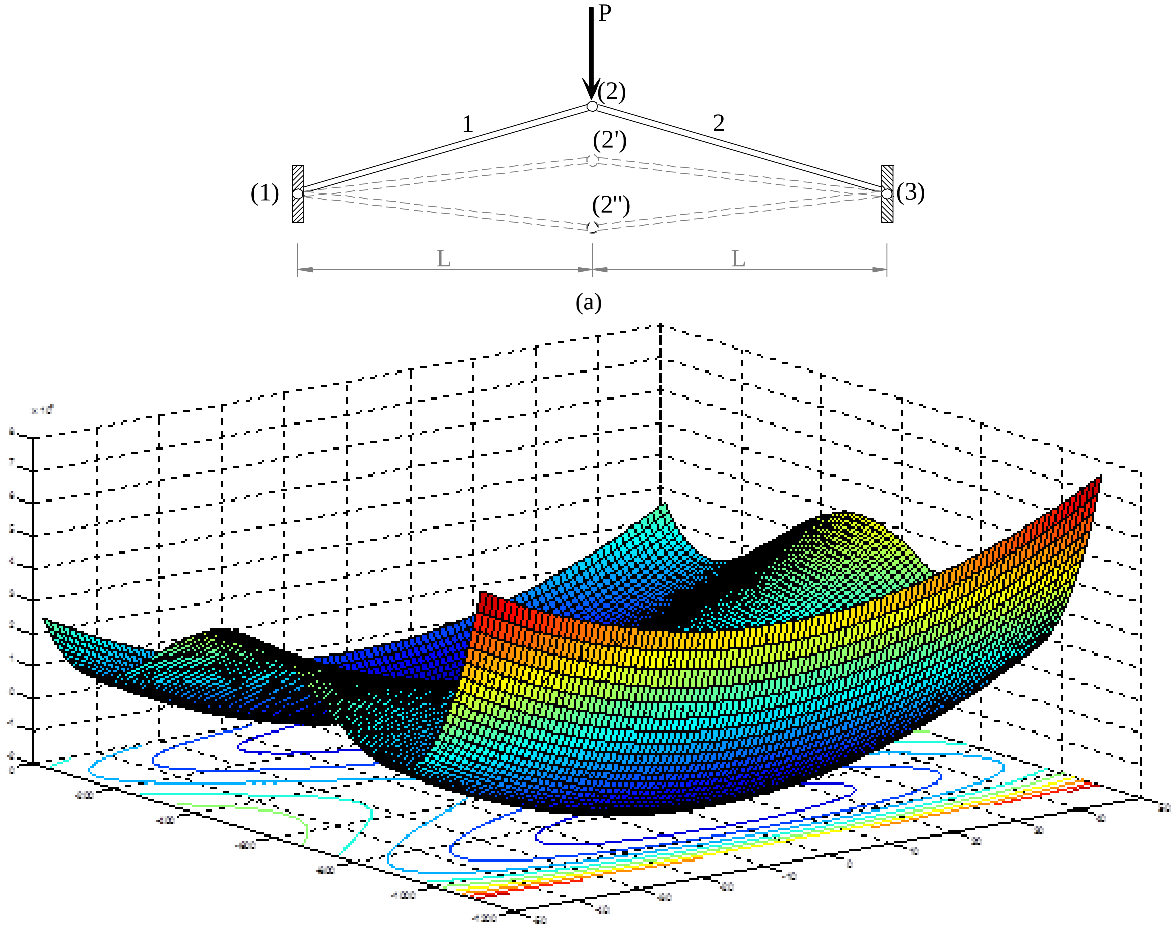


Fig. 2 Flow chart of TPO/MA [12].



(a)

(b)

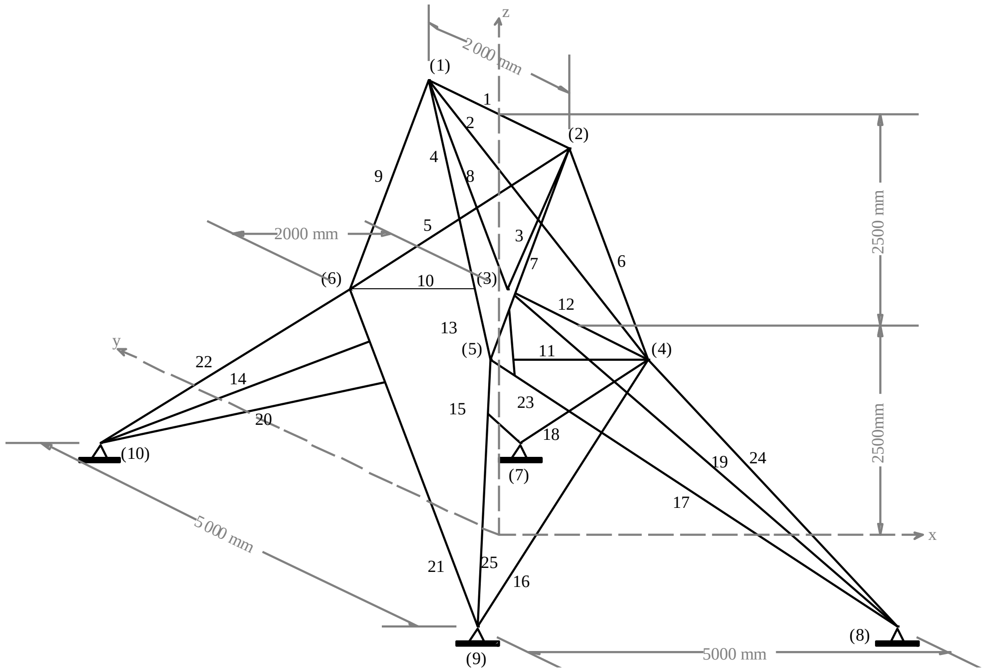
Fig. 3 (a) Probable stable equilibrium configurations of the 2-bar truss; (b) Total potential energy surface.

4.2 25-Bar Space Truss

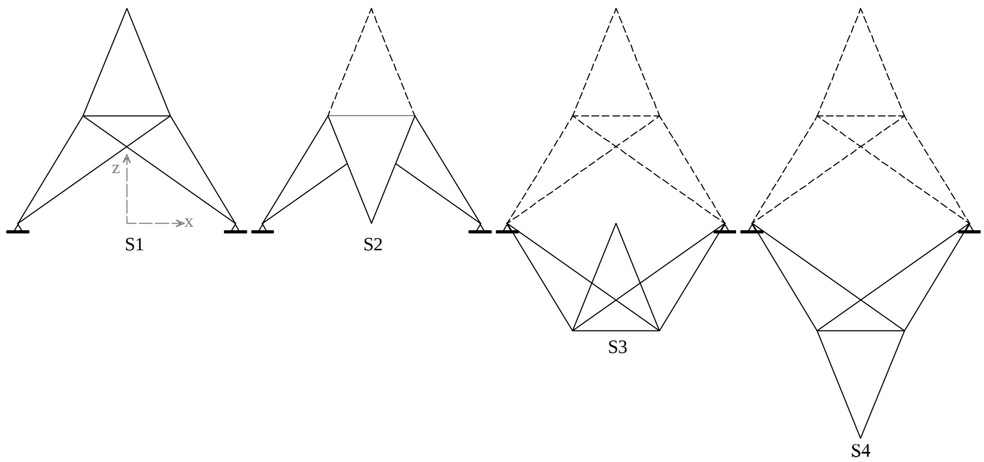
The truss shown in Fig. 4a is solved for a series of loads [14]. The deformed shape S1 in Fig. 4b corresponds to a configuration with loads causing small deformations. The shapes S2, S3, and S4 correspond to configurations that are obtained for loads exceeding a certain level. It is obvious that this situation corresponds to the case of multiple solutions and it is practically impossible to obtain these shapes with normal applications of classical methods including those based on finite element method. Application of the technique TPO/MA, on the other hand, does not necessitate any special arrangement for finding these configurations except some small variations which are nothing more complicated than attacking the problem more than once with random starting values and keeping the step sizes narrower than normal.

4.3 26-Bar Plane Truss

The third example presented in this study is the truss with the original shape shown in Fig. 5 with linear elastic members. The deformed shape corresponding to small loads is shown in Fig. 5a, which can easily be obtained by any classical method also. The other solutions which correspond to the cases of very large deformations, under-constrained truss and missing or failed members are obtained by applying TPO/MA to the problem with no special arrangement at all. It is to be noted that for all these cases the loading on the structure is the same.



(a)

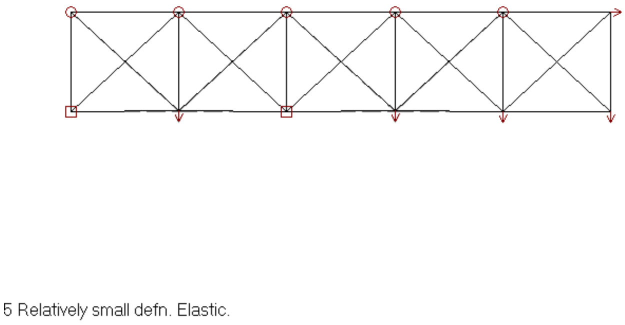


(b)

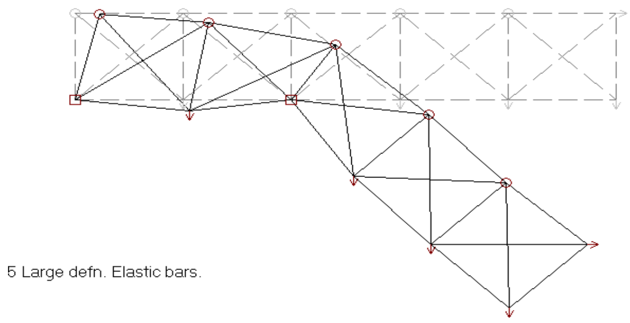
Fig. 4 (a) 25-bar space truss, (b) different configurations after loading.

5. Conclusions

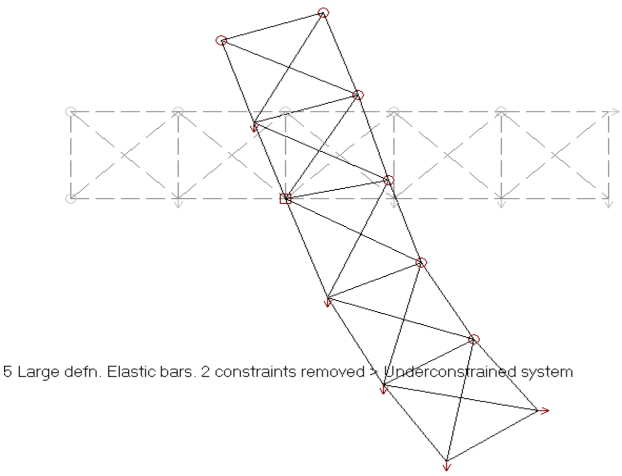
Although the method TPO/MA presented can be applied to analysis of all structural systems, the first applications are made on trusses and truss-like structures for checking its speed, accuracy and robustness. The results have shown that as far as accuracy and robustness are concerned, the method is above all expectations. As to the speed of the method, the satisfaction is not at a perfect level, but this can be understandable since the problems attacked cannot be solved by any other method in a general way. The problems solved include structures like 2 and 3 dimensional normal trusses, cable structures, tensegric structures, etc. with all types of non-linear materials including plastic, elastic-plastic, tensionless materials with or without rupture values and buckling properties. One-sided constraints, multi-solution problems, behavior of structures after bifurcation are also examined. It has been observed that all these problems can be solved by TPO/MA as black box operations.



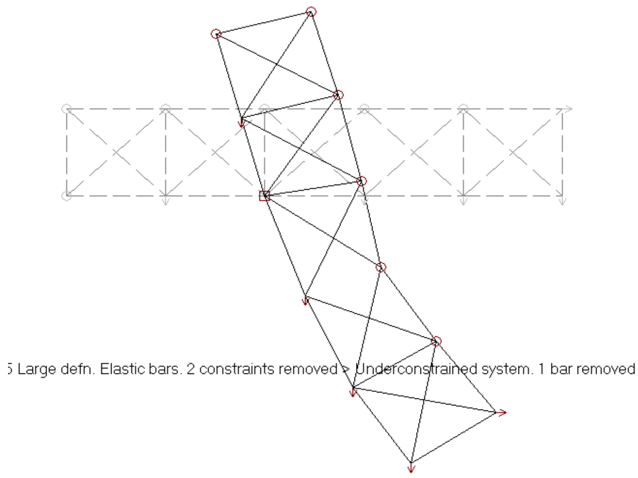
(a) Small deformations



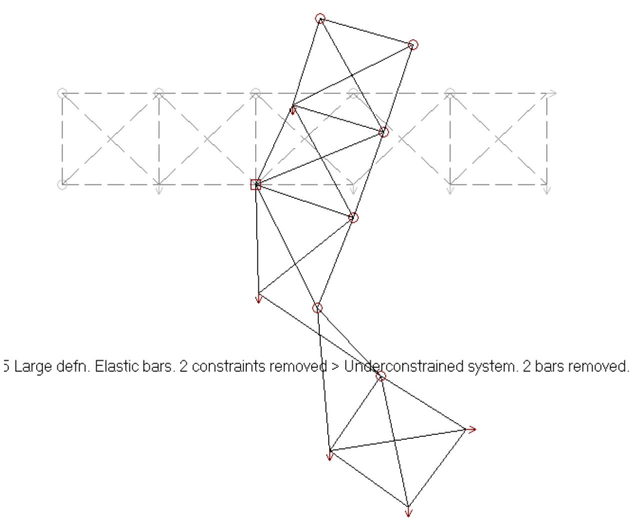
(b) Large deformations



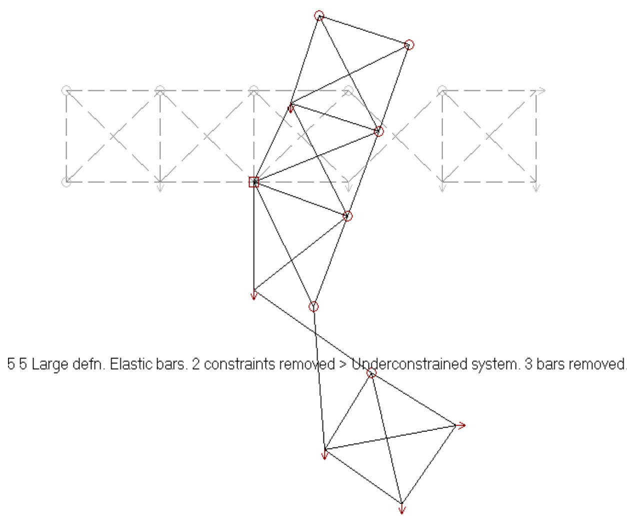
(c) Large deformations, under-constrained structure



(d) Large deformations, under-constrained structure, 1 member missing



(e) Large deformations, under-constrained structure, 2 members missing



(f) Large deformations, under-constrained structure, 3 members missing

Fig. 5 Screen-shots from TPO/MA analysis results for 26-bar truss.

Genetic algorithms, simulated annealing, ant colony optimization, random search and harmony search are optimization techniques used until now in MA part of TPO/MA in our studies, all with great satisfaction. This has shown that other meta-heuristic techniques can also be used in optimization part of the process.

The results presented in the paper demonstrate the level of research on TPO/MA. Future research subjects are also treated.

A literature survey conducted on the applications of meta-heuristic algorithms on structural systems shows that these studies are almost exclusively concentrated on design of structures, like minimization of their costs and finding the most convenient type and topology of such systems. TPO/MA extends this study area to analysis of structures in a very effective way.

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