Management Studies, Nov.-Dec. 2022, Vol. 10, No. 6, 346-362
doi: 10.17265/2328-2185/2022.06.002

A Few Notes on Auctions: The Boom in the Asymmetric Art Auction Market

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The art market, following the example of financial markets, is divided into a primary market, where works are traded directly from artists, and a secondary market that is mainly the auction market. COVID-19 and galloping inflation have influenced the creation of a bull market in artwork. The high incomes of some buyers and inflation have influenced the emergence of the so-called "glittery" art market. Works by neglected artists have "very much taken on a life of their own": fantasy art, works by young poster artists, casting "nightmares"—have been selling well for more than a year. People terrified of inflation are putting their money "in works of art". The auction market dominates the primary market because anonymity makes it easier to enter the art market. The gallery market does not guarantee this anonymity. Very often, the auction market for works of art is used for money laundering. The purpose of this paper is to show that auction mechanisms are a good tool for the efficient allocation of goods and money in an era of galloping inflation, including non-standard objects such as works of art. These mechanisms, due to the information asymmetry, often lead to the generation of all kinds of pathologies and the increasing incidence of the phenomenon known as the winner's curse.

Keywords: bidding, auction mechanism, art auction market, asymmetric auction

Introduction and Literature Review

Works of art were created with the development of civilisation. For art is the ability to produce objects. In later periods, the term "fine arts" was introduced to include painting, music, dance, and theatre. Art is also a field of artistic creation, which is a lasting cultural achievement. Many artistic works are sold at auction. Also, some auctions can be considered art, as they are a mix of theatre and business. The art auction market is currently celebrating triumphs. Works of art in the era of high inflation have become quasi-financial instruments (there is a term for this market as an alternative to the financial market). This is evidenced, *inter alia*, by the specialised art price indexes compiled by the well-known auction houses Sotheby's and Christie's. The data taken for calculating the index come from sales of artworks and antiques from the aforementioned auction houses and include American paintings (from 1750-1950), impressionist and contemporary paintings, and works by the "Old Masters" from 1200-1875. Works of art have become an investment as profitable as financial instruments. Some investment funds focus on the arts rather than the traditional financial market. These include: Fermwood Art Investments or the London-Based Fine Art Fund (Dziuba, 2008). Specialised

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financial services based on art are known as art banking. Several banks around the world are participating in this. Among them are the Societe Generale Art Advisory, UniCredit for Art, ING Art Advisory, etc. A major role in the arena of the global art trade is played by cities such as: Paris, Shanghai, New York, Beijing, London, Milan, Berlin, Tokyo, Hong Kong, and Geneva. The art market, following the example of financial markets, is divided into a primary market, where works are traded directly from artists, and a secondary or auction market.

The auction is one of the oldest forms of exchange of one or more indivisible objects among many competing buyers. It is a commonly used form of sales, especially when it comes to allocating certain non-standard goods. Auctions trade, *inter alia*: works of art, antiques, horses, land, property, mineral resources, cell phone frequencies, etc. Grain, fish, flowers, and many other goods are sold at auctions. Sellers include private individuals, companies, and even countries. The purpose of the auction is to establish a balance between demand and supply. Year on year, hundreds of trillions of dollars are traded at auction. Of course, when such large sums of money are involved, there is a lot of interest from analysts and researchers. Over the past few decades, many papers have been written about auctions. Some of the earliest were the works of William Vickrey: from 1961—*Counterspeculation, auctions, and competitive sealed tenders* and from 1962—*Auctions and bidding games*. In 1996, William Vickrey, along with James A. Mirrlees, was awarded the Nobel Prize in Economics for the works mentioned, among other things. The official announcement read: "The Nobel Prize for fundamental contributions to the economic theory of incentives under asymmetric information".

Many studies define an auction as a mechanism for allocating a single indivisible object among many competing buyers (e.g., in the iconic work of 2020 Nobel Prize winners Milgrom and Weber (Milgrom & Weber, 1982), which they received for the model presented in this work). Other works treat the auction as a typical non-cooperative game between buyers, called players (for example, Wilson in chapter 8 of the iconic *Handbook of Game Theory* (Wilson, 1992)). The auction, like any game, is conducted according to set rules. It is assumed that players follow these rules. They are reliable, in the sense that none of them bid above the amount they are able to pay for the item. Speaking of games, in addition to players and the predetermined rules of the game, payouts are also important. Most often, payouts are defined as follows:

(a) the winner of the auction receives a payout equal to the difference between the valuation, that is, the value that the item being sold represents to the buyer, and the bid made by them,

(b) the payouts of the other players are zero.

In basic auction models, buyers' valuations of the auctioned item are assumed to be independent. This is not a natural assumption, although it is very helpful in developing simple mathematical models. Reality can sometimes be much more complex than what can be described by equations and inequalities. For example, at an auction of paintings, a buyer's valuation is shaped differently when his competitors are private individuals, differently when they are museums, and differently again when they are museums from other countries. Of course, in the latter situation, the valuation is highest because it is influenced by serious competitors, which is unfortunately difficult to factor into a mathematical model. As a result, one can speak of information asymmetry. It is also assumed that buyers have varying degrees of risk aversion, which affects their behaviour and thus the valuation of the object being auctioned. The purpose of this paper is to show that most art auctions are asymmetric; very often its mechanisms are an alternative to those used in the financial markets. For this reason, auctions, not only of works of art, are used for money laundering, while in recent years, in an era of galloping inflation, there is often a phenomenon known as the winner's curse.

The paper is organised as follows. Part one will discuss the basic types of auctions. Due to the specificity of the work regarding art auctions, the next section will be devoted to English auctions. The auction model presented by Milgrom and Weber in 1982 will be discussed in the next section. The formula for the optimal bid for the first-price auction will be presented in the following section. The auction trade in artwork is the next point of this paper. The possibility of using multiple asset auctions and double auctions is discussed in the next two sections. In turn, the practical use and properties of the different types of auctions are discussed in the next section. Due to the fact that symmetry is increasingly being abandoned when modelling economic phenomena, the next section will focus on the problems associated with asymmetric auctions.

Research Design

Research Subject

The purpose of this paper is to show that most art auctions are asymmetric; very often its mechanisms are an alternative to those used in the financial markets. For this reason, auctions, not only of works of art, are used for money laundering, while in recent years, in an era of galloping inflation, there is often a phenomenon known as the winner's curse.

Theory and Hypotheses

The asymmetry affects large fluctuations in the auction market, and also serves to create pathologies such as money laundering. Finally, results from art auctions that have taken place in recent years, including the so-called "glittery" auctions, will be presented. It is also worth noting that nowadays investments in works of art "of all kinds" have become an alternative to money management in financial markets.

Auctions and Their Types

Due to the different rules of sale, the following types of auctions are considered:

1. the first price auction,
2. the Dutch auction,
3. the second price auction (also known as the Vickrey auction),
4. the English auction,
5. the double auction.

Each of the aforementioned auctions has many variations that differ in allocation mechanisms. However, the general rules for the sale of an object have many common features. The first-price auction is a static auction. Buyers submit their bids simultaneously and independently, for example, in envelopes. The buyer with the highest bid wins and pays the seller the exact amount of the bid. The second-price auction can also be static.

In a second-price auction (*nondiscriminating*), the auctioned work is sold to the highest bidder, but the buyer pays not as much as they themselves bid, but an amount equal to the value of the second-highest bid. In other words, the auction item is sold at the highest of the prices contained in the remaining bids, and not, as in the case of a first-price auction, at the highest price of all the bids. The second-price auction is also called the *Vickrey auction* in the literature.

The Dutch auction is a dynamic auction (*descending*, *discriminating*, *open*, *oral auction*) has been used for a very long time in the Netherlands, *inter alia*, for the sale of flowers for export. At present, this form of auction is used primarily by commissaries and when selling perishable goods. The mechanism is that the seller

puts up the "goods" with a very high price that far exceeds the valuations of all the bidders. The price is then gradually reduced until a willing buyer is found. The bidder in this type of auction, if they win, pays as much as they bid. The price for the auctioned item is determined by the highest bid. It turns out that the first-price auction and the Dutch auction are strategically equivalent. This is because the sets of strategies are the same. Equilibrium strategies are also analogous and involve bidding below one's own valuations. In a second-price auction, the winning bid is, as in a first-price auction, the highest bid. The price the buyer pays for the item being auctioned is the second highest price, and this treatment is strategically equivalent to the mechanisms used in an English auction, which will be discussed in the next section.

English Auction

An auction in which bidders outbid each other until there is one that no one will outbid again is called an English auction (*ascending*, *discriminating*, *open*, *oral auction*), or simply *auction*. The English auction is categorised as a *dynamic auction*, where bidders observe opponents' submissions and can make adjustments to their bids as a result.

Bidding is a very natural form of finding a buyer, known and used as early as in ancient times. The seller offers the good and accepts the highest bid submitted above the starting price, as in static auctions. The dynamic feature is that bidders can repeatedly increase their bids. The English auction is thus, in a sense, a dynamic version of the second-price auction. The strategy of "make a bid with price p" at the second-price auction corresponds to the strategy of "outbid as high as p" at the auction, whereby one always "outbids" as low as possible and, as a result, pays a second-price below one's own valuation. It is intuitively clear that it is profitable for the bidder to increase their bid until they exceed their valuation, so in a Nash equilibrium the good being auctioned will be bought by the person who is willing to give the most, but at a price a "penny" higher than the highest of his opponents' valuations. If during the bidding the valuation is exceeded then the buyer can be said to have succumbed to the winner's curse.

There are different variants of the English auction. Another variant is the auction proposed by Cassady and used in Japan, where prices are transmitted and displayed electronically on a screen. It is assumed that in this variant of the auction the price is increased continuously, while the bidders to whom this price corresponds press the appropriate button. The auctioneer then knows which bids are accepted by the bidders. The second-price auction and the English auction are assumed to be equivalent in a narrower sense than the strategic equivalence of the Dutch auction and the first-price auction. While the sets of strategies are not identical, equilibrium strategies also involve bidding below one's own valuation.

The object of the research is usually to check the so-called "efficiency" for each type of auction, to look for the optimal way to distribute goods, and to determine the best bids for each auction participant. The latter consideration corresponds to the search for optimal strategies in a game theoretical sense. Most commonly, the term "optimal auction" corresponds to the problem of maximising the seller's income, assuming that the players, i.e. buyers in the first four types of auctions, have identical access to information, have no risk aversion, and their valuations are independent. This is a situation in which the allocation rules do not affect the seller's income. This means that the first four types of auctions mentioned above are characterised by identical seller income. If the participants' valuations are interdependent then the English auction has the highest income, followed by the second price auction, and finally the first price auction, and the Dutch auction. The higher the income of the seller, the lower the payout of the player-winner. Assuming that auction participants are risk

averse, the seller's income will be higher when the item is sold in a static auction. With the seller's risk aversion, the lowest yield is provided by the English auction.

From the point of view of strategic auction analysis, an important problem is the dissemination of information by the auction house or seller. Also important is the resource and use of private information by each player. The seller can distribute information about the auctioned object in several ways:

1. *honesty—always report all information completely*,
2. *censoring—report only the most favourable information*,
3. *summarising—report only a rough summary statistic*,
4. *randomising—add noise to the data before reporting*,
5. *concealment—never report any information*.

It turns out that the best salesman strategy is honesty (*honesty is the best policy*).

There are many models and studies of auctions. The most well-known and the Nobel Prize winner in 2020 is Milgrom and Weber's 1982 model (Milgrom & Weber, 1982).

General Game Theory Auction Model According to Milgrom and Weber

In 1982, Milgrom and Weber (1982) developed a general econometric model of auctions, which was later cited by authors of many papers and was awarded a Nobel Prize in 2020. In this section, general assumptions relating to the model will be presented. Milgrom and Weber assumed that a single object is allocated. The auction involves *n* buyers and they are the players. The seller is either the auctioneer or the auction house. Each participant has access to differing information about the item being auctioned. This is, so called, the buyers' private information that affects their valuations. The amount (and also the "quality") of private information is a random variable described by a random vector $\mathbf{X} = \left[X^1,\ldots,X^n\right]$. In addition to private information, the valuations of individual players are affected by other factors, which are described by the coordinates of the $\mathbf{S} = \left[S^1,\ldots,S^m\right]$ vector. The $\mathbf{S}$ vector consists of random variables specifying, *inter alia*, information about the auctioned object made available by either the seller or the auction house, the opinions of professionals, and the possible judgments of connoisseurs not participating in the auction. The valuation of *i*-th (*i = 1*, ..., *n*) player is a random variable that depends on the coordinates of vector $\mathbf{X}$ and vector $\mathbf{S}$, that is $V^i = \mathrm{v}^i(S,X)$, where $\mathrm{v}:\Re^{n+m} \to \Re$. The payout of player *i* is determined as follows:

$$g^i = \begin{cases} \mathrm{v}^i - b^i \text{ when player i buy the object} \\ 0 \text{ another case} \end{cases}$$

where:

b^i is the offer of the player *i*.

Let $f(s,x)$ be the density of the probability distribution of the player's valuation. Under the assumption that buyers' valuations are independent, the vector $\mathbf{S}$ ($m = 0$) is omitted. In this situation, the valuation of the *i*-th player depends solely on the *i*-th coordinate of the $\mathbf{X}$ vector, that is, $V^i = X^i$. Also, the probability density function of the distribution of valuations depends solely on the players' information being private. Milgrom and Weber, in constructing the auction model, made the following assumptions:

A1. There is a function *v* defined at $\Re^{n+m}$ such that for each *i* the condition $\mathrm{v}^i(S,X) = \mathrm{v}(S, X^i, \{X^j\}_{i\neq j})$ is satisfied, which says that the *i*-th player's valuation is a symmetric function due to the private information of the other auction participants.

A2. The valuation *v* is a non-negative, continuous, and non-decreasing function due to all variables.

A3. $\forall\, i, \quad E\left[V^i\right] < \infty.$

A4. The probability distribution density function of valuation $f(s,x)$ is symmetric due to *n*-last variables.

A5. Variables $S^1,\ldots,S^m,X^1,\ldots,X^n$ meet the *affiliation condition* given below in Condition (1).

Let *z* and *z'* be points in $\Re^{n+m}$space. $z \vee z'$ means the larger of the elements *z* and *z'*; by $z \wedge z'$ the smaller of the two. The variables in the model are assumed to satisfy the affiliation condition if for each *z*, *z'* occurs:

$$f(z \vee z')f(z \wedge z') \geq f(z)f(z') \tag{1}$$

Condition (1) imposes significant restrictions on the probability density function of the valuation distribution: f. In a nutshell, it can be assumed that Condition (1) serves to "smooth the variables" on which the said density depends. Empirical studies show that the variables affecting individual participants' valuations are positively correlated. An increase (decrease) in some variables can cause an increase (decrease) in the values of others, which in turn, can be the reason for strong fluctuations in the density function *f*. To prevent this, a form of the probability density function for the distribution of valuations is sought that satisfies Condition (1) for different values of *x* and *s*. A good example of a function satisfying Condition (1) is:

$$f(s,x) = h(s)g(x^1|s)\ldots g(x^n|s) \tag{2}$$

where:

$g\left(x^i|s\right)$ is the conditional density of the *i*-th coordinate of the vector $\mathbf{X}$,

h(*s*) is the total edge density of the vector $\mathbf{S}$.

Function (2) is a joint probability density, used, for example, to describe the distribution of valuations of buyers participating in a mineral auction. Condition (1) can be written in the form of a sharp inequality, such as when the model applies to art auctions. The intuitive reasoning is this: a buyer who has discovered a certain beautiful painting expects that the other participants in the auction will also appreciate its value. In turn, Condition (1) can be written as an equality when buyers' valuations are assumed to be independent and *m* = 0. Then the total density function simplifies to the form: $f(\mathbf{x}) = f(x^1)\ldots f(x^n)$.

In this situation, it is possible to make such a permutation of the elements of the vector $\mathbf{X} = \left[X^1,\ldots,X^n\right]$that the valuation of player 1: X^1 is the highest. This permutation is allowed due to the assumption of symmetry. Let Y^1 denote the largest value among the estimators of the random variables $X^2,\ldots,X^n$; Y^2 the second largest and so on up to Y^{n-1}. Therefore, the valuation of player 1 can be written as:

$$V^1 = \mathrm{v}(S^1,\ldots,S^m,X^1,Y^1,\ldots,Y^{n-1}) \tag{3}$$

In this situation, the joint probability density function for the variables $S^1,\ldots,S^m,X^1,\ldots,X^n$ is of the form:

$$f(x,s) = (n-1)!\, f(s^1,\ldots,s^m,x^1,y^1,\ldots,y^{n-1})1_{\{y_1\geq..\geq y_{n-1}\}} \tag{4}$$

where:

$1_{\{\bullet\}}$ is an indicator of the event $\{\cdot\}$.

Milgrom and Weber showed that the probability density function of the distribution of valuations expressed by Equation (4) satisfies conditions A.1-A.5. The Milgrom and Weber model, in a very simple way, can be used to determine equilibrium strategies for first-price auctions.

First-Price Auction—Formal Model

It is assumed that a single indivisible object is put up for sale. The auction involves *n* players and their valuations are independent. The valuation of player *i* is a random variable depending only on the *i*-th coordinate of the vector $\mathbf{X}$, that is: $V^i = X^i$. In the following, the estimator of the *i*-th player's valuation is denoted as v^i. It is further assumed that players do not know the valuations of their opponents. However, the probability distribution of valuations is known, and it is the same for each player. By $F(\cdot)$ means the cumulative distribution function (CDF) of this distribution. Players are assumed to be risk-neutral and bid independently of each other. The optimal bid of a player $i \quad (i = 1,\ldots,n)$ is expressed by the following formula (Milgrom & Weber, 1982, p. 955):

$$b^i = b(\mathrm{v}^i, n, p^0, F) = \mathrm{v}^i - \frac{1}{F((\mathrm{v}^i))^{n-1}} \int_{p^0}^{\mathrm{v}^i} F((x))^{n-1} dx \tag{5}$$

If $\mathrm{v}^i \geq p^0$, whereby the quantity p^0 is called the reservation price or the asking price (R. Wilson, *Strategic Analysis of Auction*, Chapter 8 (Wilson, 1992)). If $\mathrm{v}^i < p^0$ then the bid of player *i*: b^i can take any value, strictly less than the asking price of p^0. In a first-price auction situation, the equilibrium strategy in the Nash sense is to bid below your own valuations, that is, each player $i \quad (i = 1,\ldots,n)$ bids:

$$\begin{cases} b^i \leq \mathrm{v}^i \text{ when } \mathrm{v}^i \geq p^0 \\ b^i = p^0 \text{ when } \mathrm{v}^i = p^0 \end{cases} \tag{6}$$

Let b^w denote the winning bid: $b^w = \max_i b^i$. If $b^w < p^0$ then the displayed object is not sold.

Milgrom and Weber (1982) showed that the highest revenue can be achieved when all available information is made public. In the situation of their partial concealment, buyers could feel uneasy, and this fact could cause potential customers to lose confidence in the auction house. The models discussed are often used to determine strategies (bids) in equilibrium in real auctions. In recent years, due to galloping inflation, art auctions have become particularly important.

The English Auction as the Most Common Mechanism for Selling Artworks

A typical example of an English auction is art auctions, which have a long tradition in the world. Currently, the world art market is breaking new price records. Indeed, it is a viable alternative to the capital market, and the turnover of the world's art auctions is growing year on year. We can talk about the five largest markets in the world: China, the United States, France, the United Kingdom, Germany. The first two countries are now two of the biggest auction powers that will dominate the global art market for years to come. Chinese auction houses are diversifying their branches around the world. China's Poly Auction House dominates all of its competitors, surpassing even the renowned Sotheby's and Christie's[1] in terms of transactions. Economic performance depends not only on access to the exclusive market, but also on the number of transactions involving the sale of works from the lower price ranges. There is also a focus on online sales.

The global art market can be considered segmented. For example, the art market in "Old Masters" represents a segment immune to fashion and investment speculation. The revenue it generates is growing rapidly. The art of the 19th century is equally strong in its stability. Contemporary art is the most vulnerable to price fluctuations. However, regardless of trends and the country in which the auction is held, the most popular and therefore "most expensive" artists are: Pablo Picasso, Claude Monet, Vincent van Gogh, Leonardo da Vinci, Andy Warhol and most recently the legendary street painter Banksy.

One of the main elements in the development of the art market is the "museum industry". In the near term, the art market is likely to reach the threshold of one billion dollars for a single work of art. A large share of the aforementioned market is accounted for by graphic works, sketches, manuscripts, photographs. It should be noted that sculpture or photographs are traded mainly in the Chinese market, which has become dominant in a very short period. In 2009, the Chinese government created and introduced a project to promote procedures for operating in the art market, in principle, on a par with the capital market. The main buyers of the shares were collectors, cultural institutions, and ordinary investors. Thanks to these measures, China has overtaken the United States, the UK, and France, which boasts a history of owning works by "the greatest artists" and an impressive collection of other items. Until 1950, it was France that led the way; today it is fourth. After a number of setbacks and lost lawsuits, such as with British auction house Sotheby's, the French have specialised in certain niches such as street art, photography, and comics.

In Poland, the art trade appeared before the Second World War and continued to operate until the outbreak of the war. Before the war, among the most famous auction houses was the Warsaw House of Art, which began operating in 1921. Auctions were also held at the Palace of Arts, which operated from 1925 to 1944. After the war, the art auction trade virtually ceased. However, the art trade was mediated by DESA[2], which did not deal with auctions. In 1989, with the economic and political changes, art auctions were reactivated, which, as is known, are typical institutions that function in accordance with the principles of the free market and are also its hallmark.

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[1] Sotheby's and Christie's auction houses are the oldest and most reputable auction houses in the world. Sotheby's was founded in 1744, while Christie's was established 22 years later in London. Both houses have representation around the world.

[2] DESA stands for Dzieła Sztuki i Antyki [Works of Art and Antiques, trans]. It was established in 1950 and immediately became a leading art dealing "enterprise" in Poland with many branches. In 1991, DESA was reduced to two branches, Warsaw and Cracow. In 1998, the company was privatised and merged with Dom Aukcyjny Unicum, hence the current name DESA Unicum. Over the following years, there were more than a thousand auctions involving many works of art and collectibles. It has several price records to its credit.

After the Business Act went into effect, the first auction houses began to spring up in major cities, which very often could not cope with the competition. In the early 1990s, there were eight "large", by our standards, auction houses in Poland, of which only four survived until the end of the decade: Agra, Rempex, Polswiss Art, and Unicum. Thus, new auction houses were opened in place of the closed ones, such as Polski Dom Aukcyjny Sztuka in Cracow, the Silesian Auction House, and Polonia Art in Katowice.

In 1989 there were eight art auctions in Poland, in 1990—28, the following year—54. Starting in 1992, there was a decline in the number of auctions held. They were held about 20-30 times a year. After 1997, their number gradually increased. The number of artworks sold has also increased.

Art auctions are held primarily in cities such as: Warsaw, Cracow, Łodź, Poznań, Gdańsk, and Katowice. The centre of the auction trade is, without a doubt, in Warsaw. The objects of transactions on the auction market in Poland are mainly paintings and drawings, sculptures, furniture, antique prints, silverware, jewellery, ceramics, metal and glass artwork, clocks, numismatics. However, mostly, paintings are sold at auction, and this is characteristic of the Polish art market. Paintings sold at domestic auctions are primarily works by Polish artists such as: Jacek Malczewski, Wlastimil Hoffman, Leon Wyczółkowski, Stanisław Ignacy Witkiewicz, Jerzy Kossak, Nikifor Krynicki, Wojciech Kossak, Julian Fałat, Teodor Axentowicz. The largest group among the paintings sold is from 1850-1950. Contemporary paintings account for only a few percent (about 3%) of the total number of works sold. The most popular are oil paintings: landscapes, still lifes, historical scenes, portraits, and genre scenes. Noteworthy is Ryszard Winiarski (1936-2006), a graduate in precision mechanics from the Warsaw University of Technology, who was involved in technological art. Winiarski was one of the leading representatives of the so-called indeterminism in the visual arts in Poland. His first works concerned "an attempt to visually represent statistical distributions". He engaged in "programming" of images and created a series of works inspired by probability calculus. Winiarski proposed the translation of sciences such as statistics, game theory, and stochastic processes. He was not interested in the aesthetics of the painting; the process of creating the work was more important to him. For him, the works were a by-product of his adopted system of thought. At exhibitions, he presented boards on which the programme instructions were written, thanks to which the painting was created.

Occasionally, albeit rarely, the domestic offerings are enriched with paintings by Monet, Renoir, and others, mainly artists of the Ecole de Paris. These paintings fetch high prices, however, which are comparable to some paintings by Polish artists from the turn of the 20th century. This proves that the art auction market in Poland is rather crawling. Especially in the early 1990s, art prices in Poland exceeded Western prices. In addition, in the galleries, collectors were able to purchase artworks more cheaply than at auctions. This fact is explained by the lack of orientation of buyers, the emotions we deal with at a typical English auction. In other words, buyers very often fell into the trap of the winner's curse. Not without influence on prices and irrational buying behaviour was the activity, in the early 1990s, of the controversial Art B company, which made large-scale purchases at the time.

In subsequent years, there was a gradual, this time sustained, recovery. The market has become more realistic, and turnover has begun to rise regularly. Nowadays, collectors are showing interest not only in classical works, but also in "young art" and so-called glitter painting, which draws inspiration from the street. In this style of painting, figures are transformed into puppets, statues, marionettes. Buyers especially appreciate creators who have a recognisable style. Contemporary art, often naive and glitter art, is the subject of one of the final sections of this paper.

Professionals are wondering what to do so that, despite the lack of tradition, the importance of the institution of auctions will also increase in Poland. It turns out that the right thing to do is to follow classical management theory, namely to apply the, so-called, "market niche" strategy and rather focus on developing those auctions that bring splendour to Poland. These include the auction of pure-bred Arabian horses (bred in Poland, of course), as well as organising auctions of other livestock and natural agricultural products, if only through commodity exchanges. This is because the "market niche" strategy aims to achieve a leading position, or even dominance, in a particular market or industry. Such an opportunity is provided by horses, as well as animals raised in natural conditions, and organic agricultural products. Most likely, street art can be disposed of through other types of auctions, which will be presented in the next section.

Multiple Object Auctions

In fact, auctions are not at all limited to the sale of a single indivisible good, but are also sometimes used for the sale of entire collections of identical goods occurring mostly in natural form (grain, fish, etc.), and even securities (for example, shares of a company going on the stock market). Such auctions are called multiple-object auctions.

It is assumed that at the auction the seller offers q identical indivisible goods, which they are ready to sell at a certain unit price (starting price). It is important that in such an auction the bidder's offer must include the desired quantity in addition to the proposed price. Accepted bids are selected on the basis of a clearly defined procedure, and the transaction prices are determined from the prices offered by the bidders by means of a specific rule for its determination (it is not related to the information and preferences of the bidders, or even to their number). In a multiple object auction, players can be either sellers or buyers.

- *discriminating pricing*, which involves accepting bids from bidders at different prices, with the buyer paying as much for each of the identical goods as the bidder offered in his bid

or

- *nondiscriminating pricing*, a method of adopting a price, most often such as the lowest price accepted, which applies to all transactions, involving identical goods.

Related to the way price is determined are the concepts of supply and demand functions. A static auction with n-bidders is considered when a seller offers q identical, indivisible goods at an asking price of p_0 per unit or piece. The determination of the supply function is as follows.

DEFINITION 1. The *supply function* is the function that determines the amount of indivisible goods that a seller intends to sell at auction at a price not lower than the asking price p_0 per unit or piece. The supply function s: $R^+ \to N \cup \{0\}$ is determined by the formula:

$$s(p) = q1_{\{p \geq p_0\}} \tag{7}$$

where:

q—the number of indivisible goods offered for sale,

p—the offer price per unit or piece,

p_0—the requested price per unit or piece (asking price),

$1_{\{\}}$—characteristic function equal to 1 if the inequality in parentheses is satisfied, 0 otherwise.

To recap, the seller presents q indivisible goods and says they cannot sell them at a price lower than a monetary units per unit or piece.

Then each bidder $i = 1, ..., n$ presents a demand function $d_i(p)$ or in reverse form $b_i(x)$ indicating the maximum bid price they are willing to pay for the product x.

DEFINITION 2. The *demand function* is the function that determines the amount of goods that the i-th bidder ($i = 1, ..., n$) can purchase at the price p. The demand function for i-th bidder is denoted by $d_i(p)$.

$$d_i(p): \quad R^+ \to N \cup \{0\}$$

Knowing the supply and demand function, it is possible to determine the equilibrium price a, i.e. the price that equalises demand and supply to the maximum extent possible. If the auction adopts the nondiscriminating method of determining the price of a transaction, the price a is common to each of them. Namely, if we assume that x_i are the number of indivisible goods that the i-th bidder can purchase at a price of a per unit or piece, then the following set of conditions is satisfied:

$$d_i(a) = x_i, \; i = 1, \ldots, n \tag{8}$$

$$\sum_{i=1}^{n} d_i(a) \le s(a) \tag{9}$$

With the discriminating method of determining the price, it is assumed that the i-th bidder for the x-th product ($x \le x_i$) pays the amount $b_i(x)$, where x_i again denotes the number of indivisible goods that the i-th bidder will purchase at the auction.

Double Auctions

An important class of multi-goods auctions are double auctions, which involve multiple buyers and multiple sellers, who can be referred to as players. Both sides of the market offer to buy and to sell. They are the basis for setting *transaction prices* that correspond to the state of competitive equilibrium between demand and supply. A double auction is generally a set of simple rules that define the interactions between traders. It has been observed that any market operating according to these rules is one of the most efficient ways to allocate goods and money. Many markets operate on this principle, some areas of business and finance. The double auction is one of the most well-known forms of exchange on securities markets such as the New York Stock Exchange (NYSE), the Chicago Mercantile Exchange and on securities markets trading options and futures. It is assumed to be one of the simplest and most effective forms of selling securities.

Double auctions are divided into *static* and *dynamic*. In the case of a static auction, prices, in general, are set indistinguishably and are best done by the market. The theoretical methods of determining it proposed by researchers and market analysts raise many questions. To do this, various computer simulations are used which do not always coincide with the situations of reality. Attempts to construct models of the behaviour of auction participants when trading by the rules used in double auctions have been made by many researchers. Robert Wilson sought to evaluate the double auction in terms of efficiency, and also investigated the existence of strategies in equilibrium. He gave necessary, but not sufficient, conditions for the existence of an equilibrium strategy for the double auction treated as a kind of dynamic game with incomplete information. The idea of striking a balance proposed by Wilson was, *inter alia*, that auction participants should not immediately disclose the amount of their valuations. Buyers and sellers are lined up in a certain order because of their valuations. The buyer with the highest bid should be “assigned” to the seller with the lowest unit cost. A similar assignment is made to the other participants-players: the buyer who made the second highest bid is assigned to

the seller with the second lowest unit cost, and so on.... Wilson noted that as the number of traders increases, the efficiency of the auction increases, and prices move toward actual market prices. However, he did not clearly define an equilibrium strategy. He only gave, inductively, a few steps of an algorithm that could be helpful in constructing these strategies (Wilson, 1992).

Other researchers have also considered the double auction as a game with incomplete information repeated multiple times. Players using Bayesian rules, in effect, use strategies used in games with incomplete information. It happens that in the case of the latter, players do not necessarily need to be informed about all the parameters of the game.

In a 1998 paper (Gjestad & Dickhout, 1998), wrote "... the double auction is a persistent puzzle in economic theory" (p. 5). What they were concerned with was not so much the auction model, as they presented the latter as a decentralised economic system, but the way the price of the transaction was determined. They proposed some bidding strategies that they thought were right. However, they drew the general conclusion that in real situations the determination of the competitive equilibrium price and the determination of the rules for the allocation of goods and money among the participants-players, is based mainly on the intuition and experience of market participants. Simple models based on the above assumptions can be illustrated as follows.

Static Double Auctions

During a static double auction, the usual rule is to set the price indistinguishably. This means that if $s_j(p)$ (j = 1, ..., m) is a supply function of the j-th vendor, while $d_i(p)$ (i = 1, ..., n) is the demand function of the i-th bidder, then the equilibrium price a is determined by the condition:

$$\sum_{i=1}^{n} d_i(a) \leq \sum_{j=1}^{m} s_j(a). \tag{10}$$

It is noted that in the best case the exchange for money can be g bundle of goods, where g is the maximum k such that the k-th highest bid to buy b_k exceeds the k-th lowest bid to sell s_k ($g = \max_k \{b_k > s_k\}$). As a rule, however, the trade is g - 1 bundle of goods, as the single least valuable exchange is lost. The transaction occurs between the g - 1 buyers offering the highest prices (they pay the price a_g) and g - 1 sellers demanding the lowest prices (they sell at the price s_q). This creates a monetary surplus of (g - 1)(a_g - s_g). In the equilibrium at such an auction, the bidders, when placing their bids, disclose their valuations.

Dynamic Double Auctions

In dynamic double auctions, sellers have the opportunity to make multiple bids and have them accepted. Many commodity exchanges mainly operate on this principle, as well as some financial markets, such as the Warsaw Stock Exchange. Dynamic double auctions in particular have been extensively studied experimentally, and experimental results on their effectiveness have been presented by Plott (1982) and Smith (1982). As they noted, transaction prices at such auctions usually converge quickly to equilibrium prices. In addition, the efficiency of trade exchanges is very high. Dynamic double auctions are used, for example, when trading CO_2 emissions, as well as energy, and minerals such as crude oil, coal (Drabik, 2007).

Practical Applications of Auctions and Their Properties—Concluding Remarks

Auction theory is currently a widely developing field of economic theory. Despite many significant discoveries, it has many gaps. This is due, *inter alia*, to the fact that people's behaviour is often unpredictable,

and any assumptions and precepts imposed on the rules of the game are too strict. Although the assumptions about the rationality of auction participants' actions are now being abandoned, the interdependence of players' valuations is being taken into account, and therefore the assumption of symmetry is being ignored, the models being built are still imperfect.

Each of the aforementioned types of auctions has many varieties that differ in their allocation mechanisms, but the general principles (rules of operation) for each of them have many common features. Thus, under the assumption that the bidders have no risk aversion and their valuations are independent, it has been shown that the *English auction and the second-price auction are strategically equivalent*, and that the *Dutch auction is strategically equivalent to the first-price auction*. The equilibrium strategies in the English auction and the second-price auction are optimal in the Pareto sense, i.e. the bidder with the highest valuation wins. In this case, the strategy of offering a price equal to its own valuation never yields a worse result than any other strategy, and it yields a better result than any other strategy in certain situations. Such a strategy is called weakly dominant in game theory. In the Dutch auction and first-price auction model, the dominant equilibrium strategy, which is optimal in the Pareto sense, is to make an optimal bid no higher than one's own valuation. It is important that for most models of second-price auctions, as well as in the English auction, rationality is required of the bidders. This means that the amount of each bidder's optimal bid does not depend on the bids of opponents. This is because rational players submit bids in equilibrium with prices equal to their own valuations, while the price is determined by the second-highest bid. This is different in the case of the first-price auction and the Dutch auction, where the prices offered by bidders are lower than their valuations and depend largely on the information they have about the valuations of their opponents.

It also turns out that with equal probability distributions of valuations, if the bidders are not risk averse and know their valuations, but do not know the valuations of their opponents, the expected income of the seller in Nash equilibrium is the same at all four types of auctions. Hence, one speaks of "revenue equivalence". Since the seller's income does not depend on the type of auction, what type of auction the seller chooses seems to be irrelevant. However, the results of numerous experiments clearly indicate that second-price auctions are more efficient than first-price auctions. The seller's expected profit at such auctions, it turns out, is at least as high as at first-price auctions. Therefore, second-price auctions are very often used in practice, especially by large companies or government agencies. Thus, the second-price auction is used by the U.S. Treasury when selling securities it issues. English and Dutch auctions, the so-called oral auctions, are used primarily in situations where quick sales are involved, such as works of art or perishable goods. This distinguishes them from static auctions, where it can even take several months from the time bids are placed to the end of the auction.

Asymmetric Auctions—An Overview of the Problems

The problems associated with asymmetry are many. Assumptions about the symmetry of auction models are so restrictive as to be difficult to accept in reality. Therefore, in recent years, researchers have begun to undermine them, thus creating auction models that we can call asymmetric. The first studies on asymmetric auction models were written in the 1960s, but the theory did not develop in earnest until after 2000. There are also studies treating auctions as Bayesian games. There are various "types of asymmetry" in the literature on auction theory. The more important ones are listed below.

1. Asymmetry caused by differing probability distributions of commodity valuations for buyers.
2. Asymmetry related to unequal access to information by individual auction participants.

3. Asymmetry caused by the fact that the distributions of buyers' valuations have a special stochastic relationship with each other.

Following Milgrom and Weber (1982, theorem 8), it can be assumed that disclosure of information by the seller about the goods being auctioned raises the income from the sale. Thus, it can be thought that access to information has a not inconsiderable impact on both the decisions of individual auction participants and the income from the sale. Many authors have tried to develop this problem in the context of asymmetric auctions.

Empirical studies on the impact of information held by bidders have been studied by researchers (Porter, 1995). They analysed data from the sale of leases for oil drilling rights. The research showed that the profit from the purchase of oil fields depended largely on the bids made by companies that owned oil fields in the vicinity of those being sold. However, it did not depend on bids from companies that did not own land adjacent to the auctioned land.

In short, the conclusions that can be drawn from the work on unequal access to information for buyers can be formulated as follows.

Conclusion 1: Public disclosure by the seller raises sales revenue for all types of (standard) auctions.

Conclusion 2: If the buyers' information about the auctioned goods differs, the second-price auction and the English auction are not equivalent. This is because buyers can use their information during the bidding process (English auction), which leads to more "aggressive play" and thus higher bids. So the English auction brings in a higher revenue than the second-price auction.

Conclusion 3: With unequal access to information, a second-price auction yields higher sales revenue than a first-price auction. In addition, with differential access to information, the first-price auction yields higher revenue than the Dutch auction. In the case of the Dutch auction, well-informed buyers can wait to place their bids until the price drops to a level that is appropriate to the information they have.

It is not only access to information that influences the strategic behaviour of bidders, and thus determines the value of the auctioned goods. The number of buyers, the probability distributions of commodity valuations, but also their attitude to risk, as well as the acceptance and amount of the asking price, also affect the amount of bids placed and also the profit of the bidder.

Another way of conceiving of asymmetry in auction theory is the approach that says there can be "strong" and "weak" auction participants. The probability that a "weak" auction participant will price the commodity higher than a "strong" one is very low. An example would be an auction involving a multinational corporation and several small local entities.

Asymmetry can lead to inefficiency, i.e. winning the auction by a participant for whom the auctioned item would not be worth the most. The phenomenon of asymmetry is very often used in crisis situations such as pandemics or inflation, as well as for money laundering.

Art Market Boom due to So-Called "Young Art"

The year 2020 was very successful in the art auction market. The first half of 2021 was very good, and the third quarter of 2021 was on the mark (Table 1).

Turnover grew rapidly, driven by the high inflation (7%-8%), as well as post-covid interest in works of art (in a pandemic, one looks at both residence and utilitarian objects differently). It is assumed that the turnover in the art market for the past four years has increased three times. Noteworthy is the fact that there was great interest in the so-called "glitter" painting, which does not necessarily correspond to the also fashionable

contemporary art. Collectors, and not only them (also those who seek their investments in works of art), show great interest in "young art". The works of some painters have had a five-fold breakthrough in two-three years (Roethel, 2018). Buyers appreciate artists with a recognisable style, such as characters are dolls, statues or marionettes, geometric graphics, children's style, etc. Therefore, one can speak of a bull market. However, it is worth mentioning that when the market was in a slump, the art of the Old Masters was characterised by high interest, regardless of the prevailing trend.

Table 1

Data for the Polish Art Market for the First Half of 2021

	Half-year 2020	Half-year 2021	Change
Turnover (mln PLN)	153.7	264.2	+71.9%
Number of items offered	18,060	26,280	+45.5%
Number of items bid for	8,949	14,768	+65%
Number of auctions	199	294	+47.7%

Source: ArtInfo.pl.

The Polish auction house DESA Unicum has achieved results from art brokerage unprecedented in the entire history of its existence. As a result, this auction house intends to go public on the stock exchange. In addition to the aforementioned glitter art auctions, auctions of fantasy art, or so-called magic realism, pioneered by Zdzisław Beksiński, are fashionable. In December 2021, sales auctions of Polish artists broke all records. The "classics of the avant-garde" after 1945 are also celebrating.

Art auctions are asymmetrical in nature. There is no doubt that asymmetry, especially information asymmetry, is conducive to money laundering, with China leading the way.

Money Laundering Through the Art Market—An Overview of the Problems

Auction theory is currently a thriving field, especially when it comes to modelling the market mechanisms involved (Drabik, 2007).

However, there is no doubt that in addition to the mechanisms and rules known since Babylonian times, the art market is subject to numerous pathologies and many transactions are speculative. Money laundering does not bypass the art market. Speculation consists, in general, of a customer buying a particular work of art, which he resells very quickly at a profit. Most vulnerable to speculation is contemporary art, which is so ambiguous in nature that it can easily be evaluated in a variety of ways. Operations are very often conducted by organised crime groups. Particularly prevalent in this practice is the[3] Solntsevskaya Bratva [Russian mafia group, trans.].

In Italy, a new organisation called ARCA (Association for Research into Crimes Against Art) has been created to monitor money laundering in the art market. The organisation estimates that the art market is the third largest area of criminal activity in the world, after drug and weapons trafficking. The biggest "tycoon" in this field is China. Experts estimate that this is a market where auctions are rigged and money laundering is the order of the day. It happens very often that the seller (accepting the bribe) and the buyer (giving the bribe) are

[3] The Solntsevskaya Mafia is a criminal group named after a south-western suburb of Moscow. According to the FBI, it is considered the largest European criminal organisation in the world. It is also one of the most important segments of the Russian mafia structure in terms of its control and the extent of its financial power.

in collusion with each other involving the auctioning of a work of art of relatively small value. It is purchased for a predetermined and not inconsiderable amount. Russian-language criminal groups, including the aforementioned Solntsevskaya mafia, specialise in this type of practice. They are interested in the works of Italian Renaissance artists and Flemish painting. Also of interest to the criminal world are interesting properties, often historic ones. The black art market also serves as a "cover" in money laundering. Combating this practice is very difficult, as most transactions are made via the Internet. Local municipalities are also often bribed, allowing very valuable buildings or other objects to be "removed" from the inventory. This practice is called white-collar crime (Pływaczewski, 2015). The secret services of some countries monitor financial operations in the art market. However, it happens that they themselves participate in the described procedure in order to obtain undeclared and thus illegal funds for their own activities (e.g., the so-called FOZZ affair). The art market is also an attractive area for investment by organisations raising funds to finance terrorism. According to U.S. intelligence services, some of the attacks that took place in the U.S. have been financed by the sale of artwork (artefacts) obtained illegally in Afghanistan. The abolition of controls between the countries of the European Union (Schengen Agreement) has increased "freedom" in the circulation of works of art. The great migration of people from conflict-ridden Syria and Iraq also encourages the illegal transportation of artworks, including ancient ones. Also problematic are so-called "chandelier auctions", during which prices are manipulated, artificially inflating them. However, due to the fact that it is a favourable practice for the auction house exhibiting certain artworks (commission, unauthorised advertising, etc.) the practice of overpricing is difficult to eliminate. There is also an illegal auction market called the "dark auction" on the Internet, where other "forbidden" goods are sold in addition to so-called "contraband" artworks.

The art forgery business is also a problem. A network of illegal counterfeiting manufactories has emerged, especially in China. Employees of these manufactories are young painters and often undervalued artists who create various "fakes" wholesale. Often lavishly paid "experts" are engaged to identify and authorise the supposed works of masters. They give, very often, assurance opinions that cannot discount a particular work. Often the "fakes" are so good that they deserve to be called works of art themselves. The forgery procedure is facilitated by the rapid development of digital techniques. Through this, the price of the work being auctioned can be bumped up, encouraging people to take part in the auction.

In the art market, preventing money laundering is much more difficult than in traditional banking. For this purpose, a cell was established to monitor the art market as part of the activities of the General Inspector of Financial Information (GIIF). The European Union has a special group under the aforementioned Financial Action Task Force (FATF). Attempts are being made to create tools for detecting illegal activity, including building related predictive models. In addition, a whole range of indicators used to "estimate" the situation in money laundering prevention models are being introduced (Chodnicka, 2015). The most important of these are:

- Penalty stringency ratio—the quotient of the maximum financial sanctions that can be imposed on financial institutions for non-compliance with money laundering regulations to GDP;
- The ratio of the scale of suprathreshold transactions—the quotient of the value of suprathreshold transactions to GDP;
- current cost ratio—the ratio of suprathreshold expenditures in each country in anti-money laundering to total operating costs for the banking sector.

It is also advisable to conduct public campaigns to provide information about the pathologies and unethical behaviour of art market participants. In this way, illegal buyers can be eliminated.

Conclusion

Auctions, known for centuries, are proving to be the most efficient method of allocating goods and money. They displace the market mechanisms used in financial markets, as the selected works have the lowest investment risk. The auction art market has become a serious alternative to the financial market. China is leading the way, and right after it the United States, the United Kingdom, and France, and investing capital in works of art has become an alternative to investing in real estate, commodities, or securities. However, with the change of "leaders" in the art market, many pathologies have been "generated". The most important of these is "money laundering". Banks and investment funds have had a big impact on this state of affairs. The development of this market is facilitated by the Internet and the multitude of new auction rules used by Internet auction participants. Investments in the arts, regardless of the risks, including pathologies, have proven to be the least risky, with high rates of return. Paradoxically, it has turned out that in recent years (an obvious boom) the highest rate of return was achieved by contemporary art. However, this is the genre most subject to speculation. Prices of works by Old Masters rise in proportion to the artist's fame. It is proving profitable to support artists, regulate ownership and tax rights, educate auctioneers, and create conditions that can facilitate contacts with the foreign market.

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