

# Decision Models for Cargo Container Loading with an Intermediate Hub

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**Abstract:** In this research, optimization models for air freight forwarders that considers renting air containers and assigning cargos to containers are proposed with the objective to minimize total operating cost. The transportation network consists of an intermediate hub which is used for unloading and sorting cargos before the delivery to destinations. The models provide operational decision at both regions and hub. The proposed models are formulated based on the cargo demand information provided by the forwarders' customers. The models are applicable for air freight forwarders to plan their bookings with airlines with either regular shipment or irregular demand of cargos during peak season. The proposed models take into account constraints related to weight and volume limits of containers and also on constraints related to available containers. The computational results show that the proposed models can be used for practical air cargo planning. A two-phase method is also proposed in order to generate solutions for large instance data sets. The obtained results from the paper also suggest new research directions in this field.

**Key words:** Air cargo loading, air cargo planning with intermediate hub, air cargo decision support system.

## 1. Introduction

One of the most important transportation channels in global distribution network is shipping based on air cargo. Air transport is the best choice for delivering high value products or short life products since it provides reliable service with short lead-time. In air cargo distribution network, goods are moved from origins to destinations through several parties such as a shipper, a forwarder, a trucker, an airline, and a consignee [1]. The process of air shipment requires involvement from all parties, beginning from shipper point until reaching consignee at the destination point [2]. Within this process, air freight forwarders, who act as intermediary between shippers and airlines, make profit by buying cargo space from airlines and re-selling it to shippers. The cargo space can be

bought under a contracted basis or a request-reply basis [3]. Although space can be reserved in advance before the shipment date, the freight forwarders need to be careful in planning a booking with airlines since renting or cancelling air containers urgently will cause them to pay a high penalty cost.

This paper addresses the decision of air freight forwarders on renting air containers from airlines for loading their cargos to minimize the total cost. Cargos are shipped from regions to destinations via a hub. The research considers real situation experienced by forwarders based on the accuracy of cargo information received from shippers. A deterministic model is presented for the case when certain cargo demand is known. Also, a two-phase method for solving model with large instance of data sets is proposed.

## 2. Literature Review

The research begins with the consolidation problem of air freight logistics forwarders. Suggestion of

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transforming consolidation problem into a set covering problem was found in studies [4] and [5], using Lagrangian relaxation and heuristic algorithm in which results were obtained within reasonable time. Another work on consolidation issue addressed the idea of dividing shipments into multiple jobs operated by many processing units. The problem was solved using branch-and-bound and heuristic method [6]. Later on, the studied in [6] was extended by adding constraints on shipment target cost, capacity limit, and delivery time [7].

The studies on cargo container loading problems were conducted by many research works. Containers are large boxes with standardized dimensions and used for holding goods to transport from one destination to another [8]. The early work on air cargo loading decision was found in the case study of air freight forwarder in Hong Kong [9]. They presented a mixed integer programming model to minimize total cost of renting containers without violation on constraints related to container volume and weight limits. Also, the work of [10] considered an extension by adding constraint on cargo quantity besides other constraints found in [9]. The problem of pallet selection and cargo loading was proposed in [11]. They introduced a two-phase intelligent decision support system for minimizing total operating cost. The application of large-scale neighborhood search heuristic approach for planning of cargo loading in real time was introduced by [12]. In [13] a mixed integer programming and a piece-wise cost function were discussed for the problem of transporting goods from multiple origins to multiple destinations. The constraints were subjected to volume and weight limit, flight departure/arrival time, shipment ready date, capacity limit, and over-declaration constraint. The scheduling of cargos into the aircraft based on aircraft capacity to maximize profit and customer satisfaction was presented in [14]. Other works related to loading plan of cargos into the aircraft were found in [15] and [16].

The cargo loading problem becomes more complicated when uncertainty is involved. In 2008, Yan and his colleagues introduced a nonlinear mixed integer programming for cargo container loading problem of air express carriers. They considered stochastic demand with shipment flow from origins to destinations via a hub under cost minimization [17]. Later on, this work was further studied by [18], using a scenario decomposition technique combining with genetic algorithm to obtain results in a reasonable time. In [19] a dual-response forwarding approach was presented by the use of two-stage stochastic model to deal with uncertainty and the case of no delayed shipment. The extension of this work was addressed in [20] under the consideration of delayed cargos. [21] also discussed air cargo forwarding problem with demand uncertainty. A hub was presented as a warehouse for receiving cargos transported from regions before shipping them to destinations. A Two-stage stochastic model was presented in this study to optimize total cost. More studies and review of works related to air cargo operations can be found in [22].

### 3. Model

In this part, a model for container booking and cargo loading decision is described. The shipment is transported from regions to a hub for unloading and sorting before delivering to destinations. Note that containers for renting at the hub come from two sources, previously used containers from regions and new containers renting at the hub. The model is a deterministic model, applicable for regular shipment when certain cargo quantity is known. The model distinguishes well from the previous study in [21] by taking consideration on constraints related to container weight and volume limit, and constraints on quantity of booked containers which cannot exceed the available quantity of containers provided by airlines.

Parameters:

$I$ : Set of container types

$J$  : Set of cargo types

$R$  : Set of regions

$D$  : Set of destinations

$K_i$  : Set of breaking-point for container type  $i$

$L_{ri}$  : Set of available quantity of container  $i$  in region  $r$

$L_i^h$  : Set of available quantity of container  $i$  at the hub

$a_{ik}$  : Upper Weight in breaking-point  $k$  of container  $i$

$p_{ik}$  : Unit charge rate in range  $[a_{i(k-1)}, a_{ik}]$  of container  $i$

$f_i$  : Fixed cost of renting container  $i$

$b_i$  : Unit repacking cost of container  $i$  at the hub

$\theta$  : Discount rate on fixed cost when using previous containers from regions at the hub

$q_{rdj}$  : Quantity of cargo  $j$  transported to destination  $d$  from region  $r$

$Sub_{rj}$  : Supply of cargo  $j$  in region  $r$

$Dem_{dj}$  : Demand of cargo  $j$  at destination  $d$

$v_j$  : Volume of cargo  $j$

$w_j$  : Weight of cargo  $j$

$V_i$  : Volume limit of container  $i$

$W_i$  : Weight limit of container  $i$

Variables:

$X_{ril}$  : Binary variable which is equal to 1 if the  $l^{th}$  container of type  $i$  is used in region  $r$ ; otherwise, it is equal to 0

$Y_{rilj}$  : Integer variable indicating the quantity of cargo  $j$  loaded into the  $l^{th}$  container of type  $i$  in region  $r$

$G_{rilk}$  : Continuous variable indicating the cargo

weight distributed in range  $[a_{i(k-1)}, a_{ik}]$  inside the  $l^{th}$  container of type  $i$  in region  $r$

$Z_{rilk}$  : Binary variable which is equal to 1 if  $G_{rilk}$  is within range  $[a_{i(k-1)}, a_{ik}]$ ; otherwise, equal to 0

$X_{dil}^h$  : Binary variable which is equal to 1 if the  $l^{th}$  container of type  $i$  is used to destination  $d$  at the hub; otherwise, it is equal to 0

$Y_{dilj}^h$  : Integer variable referring to the quantity of cargo  $j$  loaded into the  $l^{th}$  container of type  $i$  to destination  $d$  at the hub

$G_{dil}^h$  : Continuous variable indicating the cargo weight distributed in range  $[a_{i(k-1)}, a_{ik}]$  inside the  $l^{th}$  container of type  $i$  at the hub

$Z_{dil}^h$  : Binary variable which is equal to 1 if  $G_{dil}^h$  is within range  $[a_{i(k-1)}, a_{ik}]$ ; otherwise, equal to 0

$X_{dril}^{hp}$  : Binary variable which is equal to 1 if the  $l^{th}$  previous container of type  $i$  from region  $r$  is used to destination  $d$  at the hub; otherwise, it is equal to 0

$Y_{drilj}^{hp}$  : Integer variable presenting the quantity of cargo  $j$  loaded into the  $l^{th}$  previous container of type  $i$  from region  $r$  to destination  $d$  at the hub

$G_{dril}^{hp}$  : Continuous variable presenting the cargo weight distributed in range  $[a_{i(k-1)}, a_{ik}]$  inside the  $l^{th}$  container of type  $i$  to destination  $d$  at the hub

$Z_{dril}^{hp}$  : Binary variable which is equal to 1 if  $G_{dril}^{hp}$  is within range  $[a_{i(k-1)}, a_{ik}]$ ; otherwise, equal to 0

Objective function: Minimize

$$\begin{aligned}
 & \left( \sum_{r=1}^R \sum_{i=1}^I \sum_{l=1}^{L_{ir}} f_i \cdot X_{ril} + \sum_{r=1}^R \sum_{i=1}^I \sum_{l=1}^{L_{ir}} \sum_{k=1}^{K_i} p_{ik} \cdot G_{rilk} \right) + \\
 & \left( \sum_{r=1}^R \sum_{i=1}^I \sum_{l=1}^{L_{ir}} b_i \cdot X_{ril} \right) + \\
 & \left( \sum_{d=1}^D \sum_{i=1}^I \sum_{l=1}^{L_i^h} f_i \cdot X_{dil}^h + \sum_{d=1}^D \sum_{i=1}^I \sum_{l=1}^{L_i^h} \sum_{k=1}^{K_i} p_{ik} \cdot G_{dil}^h \right) + \\
 & \left( \sum_{d=1}^D \sum_{r=1}^R \sum_{i=1}^I \sum_{l=1}^{L_{ir}} (1-\theta) \cdot f_i \cdot X_{dril}^{hp} + \sum_{d=1}^D \sum_{r=1}^R \sum_{i=1}^I \sum_{l=1}^{L_{ir}} \sum_{k=1}^{K_i} p_{ik} \cdot G_{dril}^{hp} \right)
 \end{aligned} \tag{1}$$

The first part of objective function in (1) is the fixed and variable cost of renting containers in regions. The second part is the container repacking cost at the hub. The third part is the fixed and variable costs of renting new containers at the hub. The fourth part is the fixed and variable costs of using previously used containers from the regions at the hub. The objective function is to minimize the total cost.

Constraints:

Cargo supply/demand constraints:

$$Sub_{rj} = \sum_{i=1}^I \sum_{l=1}^{L_{ri}} Y_{rilj} \quad \forall r \in R, j \in J \quad (2)$$

Demand  $d_j =$

$$\sum_{i=1}^I \sum_{l=1}^{L_i^h} Y_{dilj}^h + \sum_{r=1}^R \sum_{i=1}^I \sum_{l=1}^{L_{ri}} Y_{drilj}^{hp} \quad \forall d \in D, j \in J \quad (3)$$

Container volume constraints:

$$V_i \cdot X_{ril} \geq \sum_{j=1}^J v_j \cdot Y_{rilj} \quad \forall r \in R, i \in I, l \in L_{ri} \quad (4)$$

$$V_i \cdot X_{dil}^h \geq \sum_{j=1}^J v_j \cdot Y_{dilj}^h \quad \forall d \in D, i \in I, l \in L_i^h \quad (5)$$

$$V_i \cdot X_{dril}^{hp} \geq \sum_{j=1}^J v_j \cdot Y_{drilj}^{hp} \quad \forall d \in D, r \in R, i \in I, l \in L_{ri} \quad (6)$$

Container weight constraints:

$$W_i \cdot X_{ril} \geq \sum_{j=1}^J w_j \cdot Y_{rilj} \quad \forall r \in R, i \in I, l \in L_{ri} \quad (7)$$

$$W_i \cdot X_{dil}^h \geq \sum_{j=1}^J w_j \cdot Y_{dilj}^h \quad \forall d \in D, i \in I, l \in L_i^h \quad (8)$$

$$W_i \cdot X_{dril}^{hp} \geq \sum_{j=1}^J w_j \cdot Y_{drilj}^{hp} \quad \forall d \in D, r \in R, i \in I, l \in L_{ri} \quad (9)$$

Weight distribution constraints:

$$\sum_{k=1}^{K_i} G_{rilk} = \sum_{j=1}^J w_j \cdot Y_{rilj} \quad \forall r \in R, i \in I, l \in L_{ri} \quad (10)$$

$$\sum_{k=1}^{K_i} G_{dil}^h = \sum_{j=1}^J w_j \cdot Y_{dilj}^h \quad \forall d \in D, i \in I, l \in L_i^h \quad (11)$$

$$\sum_{k=1}^{K_i} G_{drilk}^{hp} = \sum_{j=1}^J w_j \cdot Y_{drilj}^{hp} \quad \forall d \in D, r \in R, i \in I, l \in L_{ri} \quad (12)$$

Weight break-point constraints:

$$G_{rilk} \leq Z_{rilk} \cdot (a_{ik} - a_{i(k-1)}) \quad \forall r \in R, i \in I, l \in L_{ri}, k \in K_i \quad (13)$$

$$G_{ril(k-1)} \geq Z_{rilk} \cdot (a_{i(k-1)} - a_{i(k-2)}) \quad \forall r \in R, i \in I, l \in L_{ri}, k \in K_i \cap k \geq 2 \quad (14)$$

$$G_{dil}^h \leq Z_{dil}^h \cdot (a_{ik} - a_{i(k-1)}) \quad \forall d \in D, i \in I, l \in L_i^h, k \in K_i \quad (15)$$

$$G_{dil(k-1)}^h \geq Z_{dil}^h \cdot (a_{i(k-1)} - a_{i(k-2)}) \quad \forall d \in D, i \in I, l \in L_i^h, k \in K_i \cap k \geq 2 \quad (16)$$

$$G_{drilk}^{hp} \leq Z_{drilk}^{hp} \cdot (a_{ik} - a_{i(k-1)}) \quad \forall d \in D, r \in R, i \in I, l \in L_{ri}, k \in K_i \quad (17)$$

$$G_{dril(k-1)}^{hp} \geq Z_{drilk}^{hp} \cdot (a_{i(k-1)} - a_{i(k-2)}) \quad \forall d \in D, r \in R, i \in I, l \in L_{ri}, k \in K_i \cap k \geq 2 \quad (18)$$

Previous container constraints:

$$X_{ril} \geq \sum_{d=1}^D X_{dril}^{hp} \quad \forall r \in R, i \in I, l \in L_{ri} \quad (19)$$

Constraint (2) refers to the supply quantity of cargo in each region. Constraint (3) is the cargo demand quantity for each destination. Constraints (4)-(6) are the container volume constraints which ensure that the total volume of cargoes loaded inside a container cannot be higher than the limited volume of the container. Constraints (7)-(9) represent container weight constraints which limit the total weight of cargoes loaded inside a container. Constraints (10)-(12) guarantee that the total cargo weight distributed over all weight ranges of a container equals the total weight of all cargoes loaded inside the container.

Constraints (13)-(18) ensure that the cargo weight in the range  $[a_{i(k-1)}, a_{ik}]$  cannot be positive if the range  $[a_{i(k-2)}, a_{i(k-1)}]$  is not fully used. That is,  $Z_{rilk}, Z_{dil}^h, Z_{drilk}^{hp}$  equal 1 if  $G_{rilk}, G_{dil}^h, G_{drilk}^{hp}$  reach the range  $[a_{i(k-1)}, a_{ik}]$ , and  $G_{rilk}, G_{dil}^h, G_{drilk}^{hp}$  are less-than or-equal-to the difference between  $a_{ik}$  and  $a_{i(k-1)}$ . Constraint (19) makes sure that previous container can be selected at the hub for any destinations only if it was selected from region.

#### 4. Computational Results

This section aims to test the models using CPLEX with several examples of loading cargos into air containers. Computer used for testing has CPU 2.50GHz, RAM 8.00GB, and 64-bit operating system, and is equipped with IBM ILOG CPLEX Optimization Studio version 12.4. Data sets for testing are subsets of a full data from electronic product shipping with 315 cargos. However, it is assumed that cargos are classified into 3 types: large, medium and small, with weight 340; 308; and 213 kilograms and volume 1,278,116; 989,192; and 511,712 cubic centimetres, respectively. Also, it is assumed that airlines provide 5 different types of air containers for forwarders to rent at both regions and hub; each type has one container available. The demand of cargos is picked up randomly from the full data set for testing the model.

Table 1 shows an example of cargo quantity needed for shipping with certain shipment.

For air containers, the information including fixed cost, repacking cost, penalty cost for returning/renting container on the shipping day, volume limit, weight limit, weight breaking-point and unit charge rate is provided as shown in Table 2. However, the information regarding to prices is treated confidentially, so the data related to prices is generated randomly with excel in a given range presented in Table 2.

All the shipments are assumed to transport from two regions (R1, R2) to a hub first. At the hub, cargos are unloaded and consolidated before shipping to two destinations (D1, D2). Note that containers at regions and hub are assumed to have the same characteristics. If previous containers from regions are selected to reuse at the hub, fixed cost is discount 5%.

**Table 1** Cargo quantities with certainty.

Region	Destination	Cargo quantity		
		Large	Medium	Small
R1	D1	1	6	3
	D2	2	3	5
R2	D1	6	2	2
	D2	3	2	5

**Table 2** Air Container Information.

Container type (i)	1	2	3	4	5	
Fixed cost (\$)	600-700	500-600	400-500	300-400	200-300	
Repacking cost (\$)	300-350	250-300	200-250	150-200	100-150	
Unit penalty cost for returning container (\$)	550-650	450-550	350-450	250-350	150-250	
Unit penalty cost for urgent renting container (\$)	800-900	700-800	600-700	500-600	400-500	
Volume limit (cm <sup>3</sup> )	15900000	10800000	7200000	6900000	5000000	
Weight limit (kg)	5035	4624	3176	2450	1588	
Weight break-point (kg)	a <sub>i1</sub>	100	100	100	100	100
	a <sub>i2</sub>	300	300	300	300	300
	a <sub>i3</sub>	500	500	500	500	500
	a <sub>i4</sub>	1000	1000	1000	1000	1000
	a <sub>i5</sub>	3000	3000	3000	3000	3000
	a <sub>i6</sub>	6000	6000	6000	6000	6000
Unit charge rate (\$)	p <sub>i1</sub>	0	0	0	0	0
	p <sub>i2</sub>	3.5 - 5	3.5 - 5	3.5 - 5	3.5 - 5	3.5 - 5
	p <sub>i3</sub>	0	0	0	0	0
	p <sub>i4</sub>	2.5 - 3.4	2.5 - 3.4	2.5 - 3.4	2.5 - 3.4	2.5 - 3.4
	p <sub>i5</sub>	0	0	0	0	0
	p <sub>i6</sub>	1 - 2.4	1 - 2.4	1 - 2.4	1 - 2.4	1 - 2.4

The test on deterministic model took around seven seconds (00:00:06:46) with an optimal solution cost (USD 24401.05). This model contains 593 constraints and 671 variables. The results of container booking and cargo loading plan at regions and hub are summarized in Table 3 and Table 4. Note that “L, M, S” are used to represent large, medium and small types of cargos to be shipped to destination 1. “l, m, s” represent large, medium and small types of cargos to be transported to destination 2.

To test the performance and runtime of the proposed model, different data sets with different sizes were generated randomly with excel and run with different configurations using CPLEX. Searching time is set at 7200 seconds (2 hours). The results after solving other data sets are shown in Table 5.

In Table 5, columns 1 and 2 show the description of the test data, number of cargos and containers. Column 3 gives the solution time from CPLEX. Note that the runtime is limited to 2 hours for this study. Columns 4 and 5 represent objective value and the gap from optimality. From the results, the optimal solutions can be obtained from data sets 1 to 16 only. The solutions for data sets 17 to 30 cannot be obtained due to the size of the data sets. In order to handle large data instances, a two-phase method is introduced in the next section.

**Table 3 Container rental and cargo loading plan at regions.**

Rented container type	Loading plan	
	R1	R2
1	6M, 1S, 2l, 1m	6L, 2l, 2m
4	1L, 2S, 2m, 5s	2M, 2S, 1l, 5s

**Table 4 Container rental and cargo loading plan at the hub.**

Rented container type		Loading plan	
		D1	D2
1	From hub	6L, 1M, 3S	
	From R1	1L, 7M, 2S	
	From R2		5l, 2m, 3s
4	From R1		3m, 7s

**Table 5 Test results with different data sets.**

Data Set#	#Cargos :#Containers	Time (second)	Objective Cost	Gap
1	40:5	3.80	23882.71	0.00%
2	40:5	6.16	25140.41	0.00%
3	40:5	3.63	21021.85	0.00%
4	40:5	3.80	23345.00	0.00%
5	40:5	5.67	24086.00	0.00%
6	60:5	12.06	36695.60	0.00%
7	60:5	6.33	34549.07	0.00%
8	60:5	7.00	37335.61	0.00%
9	60:5	11.03	38157.30	0.00%
10	60:5	5.45	37182.60	0.00%
11	80:5	75.19	45157.75	0.00%
12	80:5	20.27	50006.00	0.00%
13	80:5	55.72	48752.33	0.00%
14	80:5	24.13	46005.37	0.00%
15	80:5	64.58	44825.95	0.00%
16	120:5	3259.33	67802.41	0.00%
17	120:5	Time limit exceeded	67144.81	0.44%
18	120:5	Time limit exceeded	74364.88	0.58%
19	120:5	Time limit exceeded	70128.31	1.50%
20	120:5	Time limit exceeded	63039.02	0.94%
21	160:5	Time limit exceeded	84964.04	0.37%
22	160:5	Time limit exceeded	86385.13	0.72%
23	160:5	Time limit exceeded	83836.21	0.53%
24	160:5	Time limit exceeded	81348.06	0.71%
25	160:5	Time limit exceeded	82921.24	0.36%
26	200:5	Time limit exceeded	105054.58	0.21%
27	200:5	Time limit exceeded	108917.84	0.59%
28	200:5	Time limit exceeded	107829.92	0.30%
29	200:5	Time limit exceeded	104313.21	0.24%
30	200:5	Time limit exceeded	106632.78	0.46%

### 5. Enhanced Solution Methodology

In this section, a two-phase method is proposed for selecting air container and making cargo loading plan. There are two main parts to be considered in the two-phase method. The first part is the decision to be

made from regions to hub. The second part is the decision from hub to destinations, with the use of results obtained from the first part. The details of the two-phase model are described as follows:

### 5.1 Phase 1: Model for making decision from regions to hub

Variables:

$X_{ril}$  : Binary variable which is equal to 1 if the container of type  $i$  is selected

$Y_{rilj}$  : Integer variable indicates the quantity of cargo  $j$  loaded into the  $l^{\text{th}}$  container of type  $i$  in region  $r$ .

$G_{rilk}$ : Continuous variable indicating the cargo weight distributed in range  $[a_{i(k-1)}, a_{ik}]$  inside the  $l^{\text{th}}$  container of type  $i$  in region  $r$

$Z_{rilk}$ : Binary variable which is equal to 1 if  $G_{rilk}$  is within range  $[a_{i(k-1)}, a_{ik}]$ ; otherwise, it is equal to 0

Objective function:

Minimize

$$\left( \sum_{r=1}^R \sum_{i=1}^I \sum_{l=1}^{L_{ir}} f_i \cdot X_{ril} + \sum_{r=1}^R \sum_{i=1}^I \sum_{l=1}^{L_{ir}} \sum_{k=1}^{K_i} p_{ik} \cdot G_{rilk} \right) + \left( \sum_{r=1}^R \sum_{i=1}^I \sum_{l=1}^{L_{ir}} b_i \cdot X_{ril} \right) \quad (20)$$

The objective function in (20) is to minimize total cost including fixed cost, variable cost, and repacking cost of containers from all regions. The constraints consist of constraints from (2), (4), (7), (10), (13) and (14).

### 5.2 Phase 2: Decision from hub to destination

Note that the output data of selected containers in regions from phase 1 are used as the input data of phase 2 model. The variables, objective function and constraints used in phase 2 are described as follows:

$X_{dil}^h$  : Binary variable which is equal to 1 if the  $l^{\text{th}}$  container of type  $i$  is selected for destination  $d$  at the hub

$Y_{dilj}^h$  : Integer variable indicates the quantity of cargo  $j$  loaded into the  $l^{\text{th}}$  container of type  $i$  to destination  $d$  at the hub

$G_{dil}^h$ : Continuous variable indicating the cargo weight distributed in range  $[a_{i(k-1)}, a_{ik}]$  inside the  $l^{\text{th}}$  container of type  $i$  to destination  $d$  at the hub

$Z_{dil}^h$  : Binary variable which is equal to 1 if  $G_{dil}^h$  is within range  $[a_{i(k-1)}, a_{ik}]$ ; otherwise, it is equal to 0

$X_{dril}^{hp}$ : Binary variable which is equal to 1 if the  $l^{\text{th}}$  previously used container of type  $i$  from region  $r$  is selected for destination  $d$  at the hub; 0 otherwise

$Y_{drilj}^{hp}$  : Integer variable indicates the quantity of cargo  $j$  loaded into the  $l^{\text{th}}$  previously used container of type  $i$  from region  $r$  to destination  $d$  at the hub

$G_{drilk}^{hp}$ : Continuous variable indicating the cargo weight distributed in range  $[a_{i(k-1)}, a_{ik}]$  inside the  $l^{\text{th}}$  previously used container of type  $i$  from region  $r$  to destination  $d$  at the hub

$Z_{drilk}^{hp}$  : Binary variable which is equal to 1 if  $G_{drilk}^{hp}$  is within range  $[a_{i(k-1)}, a_{ik}]$ ; otherwise, it is equal to 0

Objective function:

Minimize

$$\left( \sum_{d=1}^D \sum_{i=1}^I \sum_{l=1}^{L_i^h} f_i \cdot X_{dil}^h + \sum_{d=1}^D \sum_{i=1}^I \sum_{l=1}^{L_i^h} \sum_{k=1}^{K_i} p_{ik} \cdot G_{dil}^h \right) + \left( \sum_{d=1}^D \sum_{r=1}^R \sum_{i=1}^I \sum_{l=1}^{L_{ir}} (1-\theta) \cdot f_i \cdot X_{dril}^{hp} + \left( \sum_{d=1}^D \sum_{r=1}^R \sum_{i=1}^I \sum_{l=1}^{L_{ir}} \sum_{k=1}^{K_i} p_{ik} \cdot G_{drilk}^{hp} \right) \right) \quad (21)$$

The objective is to minimize the total cost containing fixed cost and variable cost of renting new containers and previously used containers from regions at the hub.

The constraints consist of constraints from (3), (5)-(6), (8)-(9), (11)-(12), (15)-(18), and the following constraints:

$$X_{ril} \geq \sum_{d=1}^D X_{dril}^{hp} \quad \forall r \in R, i \in I, l \in L_{ri} : X_{ril} = 1 \quad (22)$$

$$\sum_{d=1}^D X_{dril}^{hp} + \sum_{d=1}^D \sum_{j=1}^J Y_{drilj}^{hp} + \sum_{d=1}^D \sum_{k=1}^K (G_{drilk}^{hp} + Z_{drilk}^{hp}) = 0$$

$$\forall r \in R, i \in I, l \in L_{ri} : X_{ril} = 0 \quad (23)$$

**Table 6** Test results from two-phase method

Data Set#	#Cargos :#Containers	Time (second)	Objective Cost	Gap
1	40:5	2.66	12157.92	0.00%
2	40:5	3.83	13493.14	0.00%
3	40:5	2.08	11247.76	0.00%
4	40:5	3.87	11213.70	0.00%
5	40:5	3.58	13146.80	0.00%
6	60:5	4.25	23882.71	0.00%
7	60:5	4.30	25140.41	0.00%
8	60:5	4.22	21021.85	0.00%
9	60:5	4.28	23345.00	0.00%
10	60:5	4.80	24086.00	0.00%
11	80:5	5.64	36695.60	0.00%
12	80:5	5.24	34549.07	0.00%
13	80:5	4.61	37335.61	0.00%
14	80:5	5.64	38157.30	0.00%
15	80:5	5.80	37182.60	0.00%
16	120:5	14.24	45157.75	0.00%
17	120:5	7.11	50006.00	0.00%
18	120:5	24.78	48752.33	0.00%
19	120:5	14.11	46005.37	0.00%
20	120:5	11.04	44825.95	0.00%
21	160:5	238.63	67802.41	0.00%
22	160:5	143.70	67144.81	0.00%
23	160:5	170.37	74364.88	0.00%
24	160:5	748.76	70128.31	0.00%
25	160:5	223.14	63039.02	0.00%
26	200:5	238.44	84964.04	0.00%
27	200:5	1708.36	86385.13	0.00%
28	200:5	2157.92	83836.21	0.00%
29	200:5	1287.62	81348.06	0.00%
30	200:5	391.44	82921.24	0.00%

Constraint (22) ensures that the previously used containers at the hub can be selected if it was selected in regions. Constraint (23) ensures that unused containers at regions will not be selected at the hub.

By testing with the same data sets used for the original model, the results are obtained and summarized in Table 6.

From the results shown in Table 6, the two-phase method provides optimal solutions in shorter time, comparing to the original model. Also, this model can generate solutions for all the unsolved cases from the original model within reasonable time.

## 6. Conclusions

This paper deals with the forwarder's decisions on renting air containers for loading their cargos which are transported from regions to destinations via a hub. A model was proposed to deal with the real situations normally faced by forwarders relating to the shipping demand of cargos. The model is a deterministic model, applicable for regular shipments when certain demand is known.

The deterministic model presents container booking and cargo loading decision of the air freight forwarders when they receive actual cargo information from their customers. The model can be applied with regular shipment in which demand of cargo is known. More practical constraints are considered in this model, making it different from previous studies, by adding container's volume and weight limit constraints and container quantity constraints. Several data sets were generated with different configurations to test the performance of the model. Computational results from CPLEX show optimal solution for some configurations; however, there were some cases that solutions were not obtained within searching time limit. A two-phase decision method was proposed to deal with unsolved cases. Results show that the two-phase model can generate optimal solution in very short time and can find solution for the unsolved cases from the original model within a reasonable time.

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