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Credit Risk Model Taking Account of Inflation and Its Contribution to Macroeconomic Discussion on Effect of Inflation on Output Growth*

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We use Extended Merton model (EMM) for estimating the firm's credit risks in the presence of inflation. We show quantitatively that inflation is an influential factor making either a benign or adverse effect on the firm's survival, supporting at the microeconomic level New Keynesian findings of the nonlinear inflation effect on output growth. Lower inflation increasing the firm's expected rate of return can raise its mean year returns and decrease its default probability. Higher inflation, decreasing the expected rate return, makes the opposite effect. The magnitude of the adverse effect depends on the firm strength: for a steady firm, this effect is small, whereas for a weaker firm, it can be fatal. EMM is the only model taking account of inflation. It can be useful for banks or insurance companies estimating credit risks of commercial borrowers over the debt maturity, and for the firm's management planning long-term business operations.

Keywords: inflation, corporate credit risks, structural model, non-linear inflation effect on output growth, New Keynesian macroeconomics

Introduction and Literature Review

Entities in the financial services industry must estimate credit risks for an individual borrower with the highest possible accuracy and precision, taking account of all factors affecting the risks. There is evidence from macroeconomics that inflation is among such factors. Fisher (1993; 1996) has shown a nonlinear relation between inflation and output growth with a threshold inflation value separating the interval of low inflation, making a "greasing" effect on output growth, from the region of high inflation, "throwing sand" into the machinery of output growth. The New-Keynesian literature argues that these two effects appear systematically when the inflation rate moves from its lower to higher levels. In the long run, the moderate inflation rate assures the level of economic activity, which is higher than the one under complete price stability (Ackerlof et al., 1996). On the negative effect of inflation on output growth (the sand effect), Keynes (1920) says:

> As the inflation proceeds, and the real value of the currency fluctuates wildly from month to month, all permanent relations between debtors and creditors, which form the ultimate foundations of capitalism, become so utterly disordered as to be almost meaningless, and the process of wealth-getting degenerates into a gamble and a lottery. (p. 220)

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Trying to explain the mechanisms providing these effects, macroeconomists use exclusively macroeconomic arguments, considering influences of the nominal wages and prices rigidity, imperfect competition, non-neutrality of money, and persistent aggregate fluctuations (Goodfriend & King, 1997; Rotemberg & Woodford, 1997; Woodford, 2003; Gali, 2002; 2008). (Wages and prices are rigid if a change in demand or costs is not fully transmitted to prices or wages.) In their empirical analysis of output growth, macroeconomists usually include the macroeconomic explaining variables such as the initial level of GDP per capita, trade openness, measured as a ratio of the sum of exports and imports to GDP, government expenditures to GDP, investments, presented as a ratio of gross fixed capital formation to GDP, population growth, and the inflation rate, defined as a growth rate of the consumer price index (CPI) (Levine et al., 2000; Beck et al., 2000; Khan & Senhadji, 2001; Lopez-Vallavichencio & Mignon, 2011; Khan, 2013).

A natural question arises if there is such nonlinear relation between inflation and output and stability of the firm. With this objective in mind, one has to include inflation into the model describing the firm's development. Unfortunately, as it follows from literature (e.g., Bohn, 2006; Canbolat & Gumrah, 2015; Crouhy et al., 2006; Hamerle et al., 2003; Kern & Bernd 2001; Leland, 2006b; Sundareshan, 2013), no contemporary theoretical or practical model considers inflation when estimating credit risks of a commercial borrower. Here we give examples illustrating the general inability of the existing credit risk models to take account of inflation.

CreditMetricsTM (JP Morgan; *CreditMetricsTM*—Technical Document, 2007) applies primarily to bonds and loans and estimates a forward distribution of a credit portfolio value at a time horizon of one year. The changes in the portfolio value occur due to random migrations in a borrower's credit quality up to default. The credit risk estimation extensively uses average credit risk ratings imported from rating agencies (Moody's or Standard & Poor's). The model does not include inflation as a factor influencing the portfolio value.

CreditPortfolioView (McKinsey) estimates credit risks using a discrete-time multi-period model with default probabilities considered as functions of macroeconomic variables. Namely, levels of the long-term interest rate, the growth rate of GDP, the global unemployment rate, the exchange rates, the public spending, and the savings (Derbali & Hallara, 2012). This system uses a too broad approach to the credit risk problem to be sufficiently precise when estimating the credit risks of an individual firm.

Moody's KMV credit risk technique (Bohn, 2006) uses the geometric Brownian model (GBM, Eq. (3) along with an extensive default database to assess the default probability and the loss distribution at a time horizon of one year. The method applies to publicly traded firms for which the market value of equity is known. A GBM deficiency is in its lognormal distribution, producing the default probabilities much lesser than the default frequencies observable in practice. To overcome this deficiency, Moody's KMV calculates in GBM the distance to default for the firm's current state, and then using the default database, determines a share of the firms with that distance to default, who have defaulted within a year. They call this share the Expected Default Frequency (EDF). As one can see in Section 1, EDF is a rough estimate of the intensity of default probability (IPD) computed at a horizon of one year. Moody's KMV does not include inflation as a model parameter.

The vector of studies in the credit risk theory shifts from various modifications of GBM models (e.g., Merton, 1974; Black & Cox, 1976; Leland, 1994; Longstaff & Schwartz, 1995; Leland & Toft, 1996) to the more advanced jump-diffusion processes (JDPs). JDPs use a combination of GBM with Poisson's jumps down in the firm value (Merton, 1979; Hilberink & Rogers, 2002; Kou, 2002; Leland, 2006a; Zhou, 2001), or jumps in both directions like the double-exponential jump process introduced by (Chen & Kou, 2005). Because JDPs consider only publicly traded firms whose market values are known, jumps in the firm value are allegedly caused by new information about the firm. Unfortunately, no procedure of direct measurement of jump parameters (their intensity and length) is specified. To determine jump parameters, Leland (2006b) and other researchers, following the ideology of Moody's KMV, calibrate their models to specific default databases. Downward jumps lead to negative skewness in a log-value distribution, which pure GBM lacks. However, in its current form with a time-independent jump part, no JDP can correctly describe the firm value distribution because of its permanently growing skewness. These models also do not consider the effect of inflation on credit risks because neither their diffusion part nor their jump part includes the inflation rate as a parameter. None of the theoretical or practical credit risk models takes account of inflation while estimating the default probability of an individual firm.

GBM can feel inflation only through the expected rate of return. Inflation produces a benign/adverse effect on the development of the firm if inflation causes an increase/decrease in its expected rate of return. At that, the firm value distribution remains lognormal with a greater/lesser drift rate. If the price increase exactly offsets the decrease in demand, the expected rate of return remains the same, and nothing will change to the firm in the GBM model. Our objective is to show that inflation affects the firm value in this case too.

Here we consider a continuous-time credit risk model estimating the probability that the firm in its development will face a lack of liquidity, leading the firm to default. We trace down the dependence of default probability on factors of the firm's business environment, including inflation. Our model supports the macroeconomic empirical observations and theoretical results of New Keynesian literature that moderate inflation rates are good for the national economy, but high inflation rates make an adverse effect on it. The model proves that the roots of this phenomenon are in the microeconomic properties of the firm and, therefore, they are hardly visible at the macroeconomic level. Concerning credit risks of an individual firm, we conclude that inflation can make a significant effect on the firm default probability and its mean value; thus, credit risk models must take account of it.

In Section 1, we briefly introduce a continuous-time Extended Merton model (EMM) comprehensively presented by Shemetov (2020). We show that the firm's compulsory payments (or business securing expenses—BSEs) including fixed costs, debt payments, taxes, and dividends) lead to a skewed log-value distribution with its variance and skewness growing fast. At that, the log-value mean is always a concave-down function of time whose characteristics depend on the firm's business parameters.

In Section 2, we briefly show that the skewed distribution generated by EMM sheds new light on the no-arbitrage pricing principle. This principle is considered now as unconditionally correct; however, it is unconditionally valid only for the market of GBM-firms which do not take account of BSEs. We show when these payments are included into a model, the no-arbitrage pricing principle becomes a time-dependent characteristic of individual stocks and the firms issuing them, rather than a universal property of the market as a whole. From a practical point of view, it means that the long-term investors such as pension funds, mutual funds, banks, insurance companies, and big firms suffer unnecessary losses misled by the structural models and no-arbitrage pricing principle.

In Section 3, we analyze the inflation effects on the firm's development and show that inflation can make a positive effect as well as a negative effect on the firm's state and survival. The magnitude of this effect depends on the state of the firm and its business environment, and also on the elasticity of demand on the firm's production. Results of this Section demonstrate that the New Keynesian nonlinear dependence of output growth on inflation has its roots in the microeconomic properties of the firm hardly visible from the macroeconomic level.

Model Description

The Extended Merton model (EMM) is described by the equation:

$$dX = (\mu X - P)dt + CXdW,\ X(0) = X_0, \tag{1}$$

$$P = FC + DP + TAX + DIV. \tag{2}$$

Here $X(t)$ is a firm market value at time t, μ is an instantaneous expected return on the firm per unit time, P is the total dollar payouts by the firm for its fixed costs (FC), debt payments (DP), corporate taxes (TAX), and dividend payments (DIV), all per unit time. C is the volatility of the firm value, and W is a Wiener process representing a cumulative effect of normal shocks; μ and C are constants (Shemetov, 2020). *Business securing expenses P* (*BSEs* for short) is considered as a known function of time,

$$P(t) = P_0\pi(t),\ \pi(0) = 1. \tag{2a}$$

and P_0 is a non-zero constant. Components FC and DP are functions of time $\pi(t)$ reflecting changes in business conditions; TAX and DIV depend on year returns and corresponding tax and dividend rates. Here we concentrate on the inflation effects and consider the development of the firm value *without taxes and dividends*, setting them to zero.

For $P = \delta_0 X$, $\delta_0 \geq 0$, EMM reduces to the structural model, or GBM:

$$dX/X = (\mu - \delta_0)dt + CdW. \tag{3}$$

A GBM solution ($0 \leq X < \infty$) is a lognormal distribution

$$U(x,t) = (2\pi\sigma^2)^{-1/2}X^{-1}exp\{-(lnX - H)^2/(2\sigma^2)\}, \tag{4}$$

$$H(t) = H_0 + Rt, \sigma^2(t) = \sigma_0^2 + C^2 t,\ R = \mu - \delta_0 - 0.5C^2.$$

In the original structural model introduced by Merton (1974), a firm comes to default only if its value turns less than the firm's outstanding debt at the debt maturity. Black and Cox (1976) improve it, introducing a threshold triggering default any time when the firm value hit the threshold (the default line). This version of the model with all its subsequent generalizations historically has got the name of structural model. Since that time, "the GBM or structural model becomes the workhorse for gaining insights in different fields of economics and finance" (Sunduresun, 2013).

For the firms described by the structural model with a linearly growing log-value mean (Eq. 4), a stock market proves to be a no-arbitrage market (Harrison & Kreps, 1979; Harrison & Pliska, 1981). Any firm has the same mean year returns R each year that makes the mean calibrated stock prices to stay constant, and the firm value is independent of the asset structure so far as the market remains perfect (there is no friction). In other words, the no-arbitrage market is consistent with the Modigliani-Miller theorem of irrelevance (Modigliani & Miller, 1958). The firm value distribution is almost insensitive to inflation. All these inferences from the distribution lognormality contradict the observable facts: firms do not have the same mean year returns each year, and inflation does affect the firm's development (see Keynes, 1920, p. 220). We emphasize here that the no-arbitrage pricing principle is specific for GBM and, therefore, is not universally valid in the more general EMM.

Equation (1) for the random variable $x = \ln(RX_0/P_0)$ with Ito's lemma transforms to

$$dx = R(1 - \pi(t)e^{-x})dt + CdW, \tag{5}$$

$$x(0) = x_0 = \ln(RX_0/P_0),\ R = \mu - 0.5C^2. \tag{6}$$

Writing a Fokker-Plank equation for the process (5), one comes to an equation for the probability distribution, $V(x, t)$; V_y is a partial derivative over variable y:

$$V_t + R(1 - \pi(t)e^{-x})V_x - 0.5C^2V_{xx} + R\pi(t)e^{-x}V = 0. \tag{7}$$

The initial condition is

$$V(x,0) = N(x; H_0, \sigma_0^2),\ H_0 = \langle x(0)\rangle,\ \sigma_0^2 = \langle (x(0) - H_0)^2\rangle, \tag{8}$$

where $N(x; H_0, \sigma_0^2)$ is a normal distribution. To study the credit risk problem, one must add a boundary condition implying that a firm comes to default when its value falls to an outstanding debt (Black & Cox, 1976). If the firm has debt X_D (an *exogenous* boundary),

$$V(DL,t) = 0,\ DL = \ln(RX_D/P_0) > 0. \tag{9}$$

If the firm has no exogenous obligations, there is another constraint. A share of BSEs in expected year returns is

$$P_0/(R\langle X_0\rangle) = \exp(-H_0 - 0.5\sigma_0^2).$$

For $H_0 \geq 0$ this share is less than one, while for $H_0 < 0$ it exceeds one, and the firm pays out more than it earns. The line $x = 0$ separates a profitable business from failure. In this case it is reasonable to introduce a soft *endogenous* boundary

$$V(DL,t) = 0,\ DL = 0, \tag{10}$$

and watch the probability of crossing this line. The general boundary condition is

$$V(DL,t) = 0,\ DL = \max(0, \ln(\mathrm{R}X_D/P_0). \tag{11}$$

A solution of the problem (7, 8, 11) is the log-value distribution for a firm in financial distress; it is denoted as $\hat{V}(x,t)$. If one knows a solution $V(x,t)$ in the open space, then a solution of the boundary problem (7, 8, 11) can be written as

$$\hat{V}(x,t) = V(x,t) - V(2DL - x,t). \tag{12}$$

The probability distribution turns to zero at the default line, $\hat{V}(DL,t) = 0$, and the intensity of default probability is

$$IPD(t) = 2\int_{-\infty}^{DL} V(x,t)dx. \tag{13}$$

The first three moments are calculated along with the probability distribution $\hat{V}(x,t)$:

$$\hat{H}(t) = \int_{DL}^{\infty} x\hat{V}(x,t)dx, \qquad \hat{V}ar(t) = \int_{DL}^{\infty}(x - \hat{H})^2\,\hat{V}(x,t)dx,$$
$$\hat{S}(t) = \int_{DL}^{\infty}(x - \hat{H})^3\,\hat{V}(x,t)dx. \tag{14}$$

$\hat{S}(t)$, proportional to skewness, shows the development of the distribution asymmetry.

The main objective of any credit risk analysis is estimation of the default probability over a chosen time interval (e.g. an interval of debt maturity)

$$PRD(t_s,T) = \int_{t_s}^{t_s+T} IPD(t)dt, \tag{15}$$

Here t_s is a moment when a credit is issued, $t_s + T$ is a debt maturity, and $PRD\ (t_s,T)$ is the probability of corporate default over the credit period. In this paper, t_s is set to zero: $t_s = 0$.

EMM Properties and No-arbitrage Pricing Principle

For R and P constant, $\pi(t) \equiv 1$, and BSEs paid continuously, Eq. (5) transforms to

$$dx = R(1 - e^{-x})dt + CdW,\ x(0) = x_0. \tag{16}$$

Stochastic Eq. (16) has no exact solution; we shall analyze the process behavior in the open space

$(-\infty < x < \infty)$ qualitatively using the Brownian motion model. Suppose that at $t = 0$ we have an ensemble of Brownian particles whose initial locations x_0 have a normal distribution $N(x_0;\ H_0,\ \sigma_0^2)$ with the mean H_0 and variance σ_0^2, and most of that ensemble is over the line $x = 0$, while the other is under it ($H_0 > 0$). For these conditions, the line $x = 0$ is the line of balance between the firm's mean year returns and yearly BSEs.

Due to the properties of the drift term, $R(1 - e^{-x})$, the particle diffusion to the left ($x < 0$) runs much faster than their diffusion to the right ($x > 0$). As a result, the distribution gains a heavy left tail, continuously growing. The initially normal distribution $N(x_0;\ H_0,\ \sigma_0^2)$ gradually turns into a leptokurtic and negatively skewed x-distribution. At that, the higher located the initial ensemble over the line $x = 0$, the more time the ensemble deformation will take, the lesser the distribution skewness for fixed time intervals. A rising mean value due to the positive drift can compensate to some extent the effect of diffusion mass transfer across the line $x = 0$ slowing down the skewness development. Vice versa, the closer the mean of the original ensemble to the line $x = 0$, the faster runs the distortion of the initially normal distribution. The described ensemble evolution explains the space-time development of x-distribution. The ever decreasing mean year returns together with an increasing volatility make the firm's stock price to decrease systematically. As a consequence, the stock price cannot be a martingale, or the martingale (risk neutral) measure does not exist. This fact together with the First fundamental theorem of asset pricing (see Shiryaev, 1998, p. 529; Financial Economics, 1998, p. 525) proves that no-arbitrage pricing principle is not a universal property of the perfect market. It can be applied to the stocks of a particular firm so far as its mean year returns remain (almost) constant (see for the details Shemetov, 2020).

To illustrate general properties of the distribution generated by EMM, we solve numerically the problem (7, 8, 11) with $\pi(t) \equiv 1$ evaluating the firm's default probability. We suppose that all of *the perfect market* assumptions hold. Examples of modeling of the x-distribution $V(x, t)$ and its statistical moments $H(t)$, $VAR(t)$ are presented in Figures 1-3. The evolution of the intensity of default probability $IPD(t)$, we present in Figure 4. Model parameters are: $R = 0.10$, $\sigma_0^2 = 0.03$, $C = 0.008$, $T = 10$, $DL = 0$.

Figure 1 shows the development of the mean value as a function of time and H_0. All $H(t)$-lines fall apart into two classes. The first class consists of the lines, first rising then falling, whereas the second includes $H(t)$-lines, falling from the start (the firm bears steady losses). We do not consider the second class because it makes no practical interest. The separation of H-lines between the classes is mainly controlled by H_0. A threshold value H_0 for the chosen problem parameters is about one. Figure 1 shows that there are lines whose evolution takes a long time (decades, lines 1-6). A slope of each $H(t)$-line declines from R specific for the asymptotic GBM-distribution to small values as H_0 descends from high to small values. Figure 1 proves that the GBM-mean is a poor approximation for the real mean returns $H(t)$. One must keep in mind that *all* $H(t)$ – H_0 lines are concave-down lines (as lines 7 and 8) and sooner or later come over their maximums and tend to zero. An approximately straight rise of $H(t)$, observed for high H_0 (lines 3-6), provides for almost constant mean year returns. Time-independent mean year returns make the firm's stock price a martingale. The no-arbitrage pricing principle holds for such stocks as far as they have this martingale property, and the mean year returns of the firm issued these stocks, remain constant. Thus, at the EMM-market, the no-arbitrage pricing principle is not a property of the market as whole as it is at the GBM-market. Now it is a feature of an individual stock and the firm standing behind it.

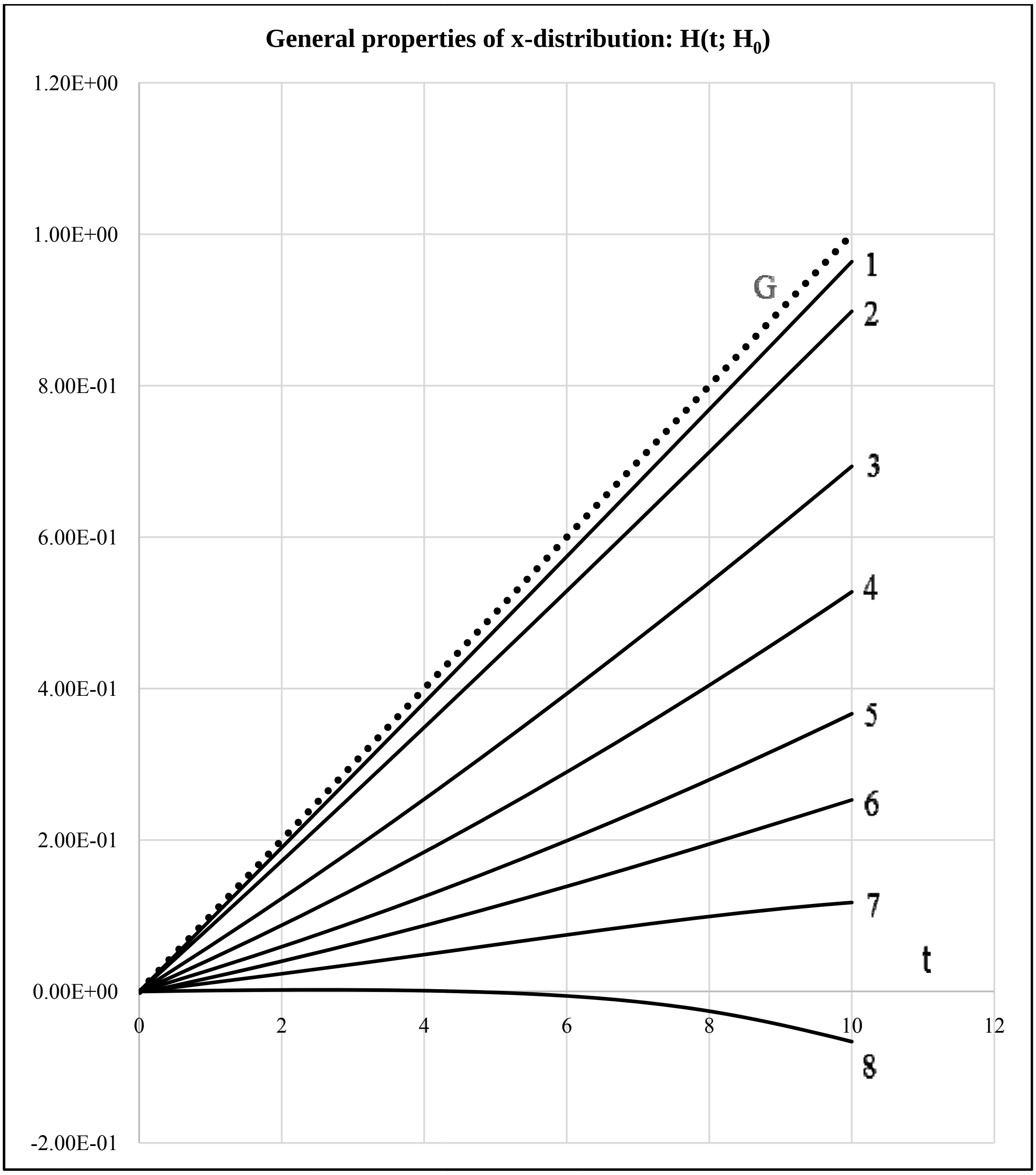

Figure 1. Drift of the mean value $H(t; H_0) - H_0$ as a function of time (years) and H_0 values: H_0 = 4.0 (1), 3.0 (2), 2.0 (3), 1.6 (4), 1.4 (5), 1.3 (6), 1.2 (7), 1.1 (8). The dot straight line (G) shows a drift of the asymptotic structural (GBM) solution $\Delta H = Rt$.

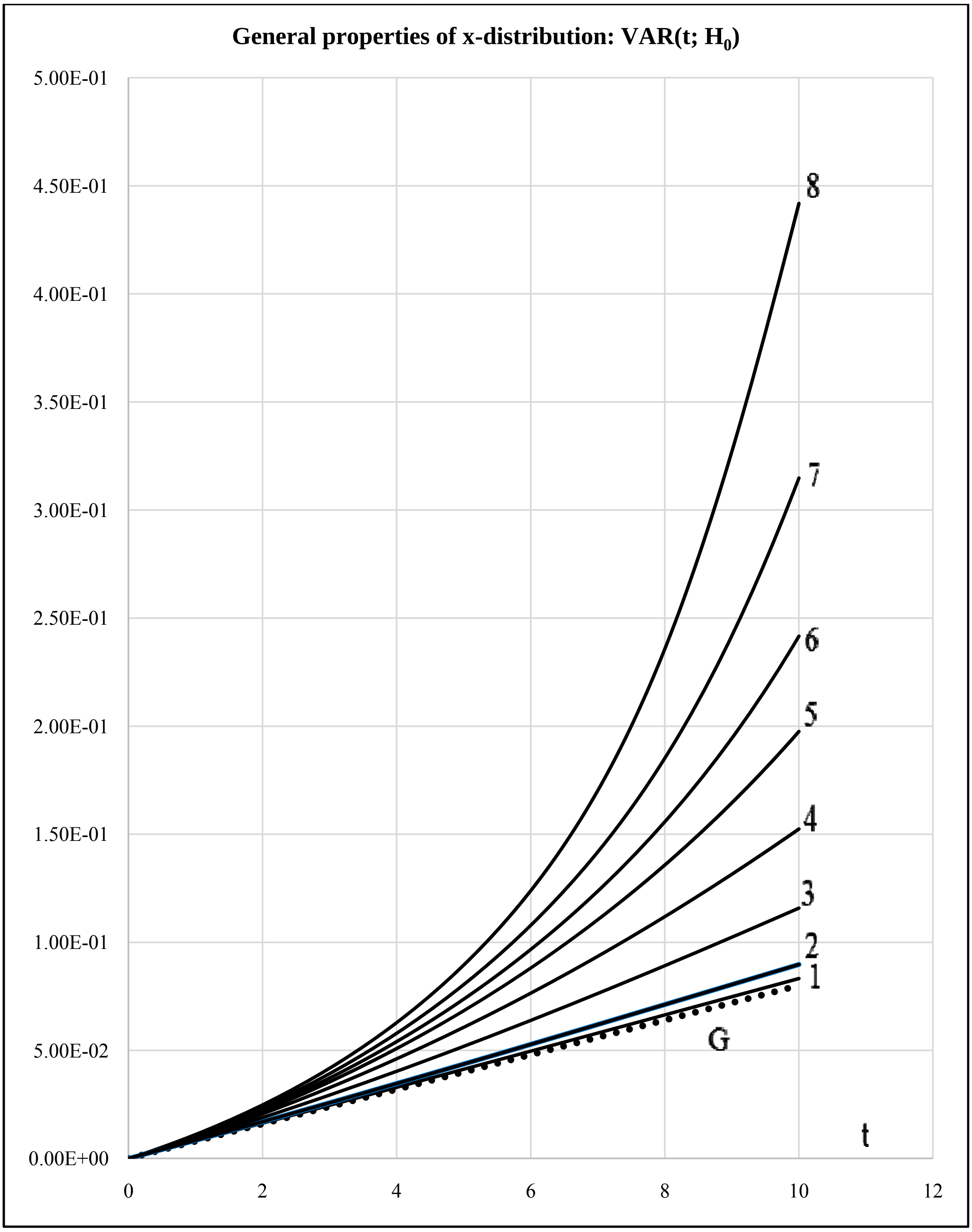

Figure 2. Development of variance $VAR(t; H_0) - VAR_0$ as a function of time (years) and values $H_0 = 4.0$ (1), 3.0 (2), 2.0 (3), 1.6 (4), 1.4 (5), 1.3 (6), 1.2(7), 1.1 (8). The dot straight line (G) shows the spread of the asymptotic structural (GBM) solution $\Delta VAR = Ct$.

Variance $VAR(t, H_0)$ (Figure 2) demonstrates the effect of distribution deformation: when H_0 descends from high to low values, the variance grows fast starting from the Ct-line specific for GBM. When x-distribution gains material skewness, its variance significantly exceeds GBM-variance. For x-distributions with temporarily low skewness (H_0 = 3.0 or more), their variance is close to GBM-variance. The excess of x-variance over GBM-variance is due to the distribution deformation. The EMM-volatility

$$C_E(t) = \left(\frac{d}{dt}VAR(t)\right)^{1/2} \tag{17}$$

is an increasing function to the contrast of GBM-volatility which remains constant, $C_G(t) = C$. Typical examples of x-distribution one can see in Figure 3; mark the development of heavy negative tails contributing to the default probability.

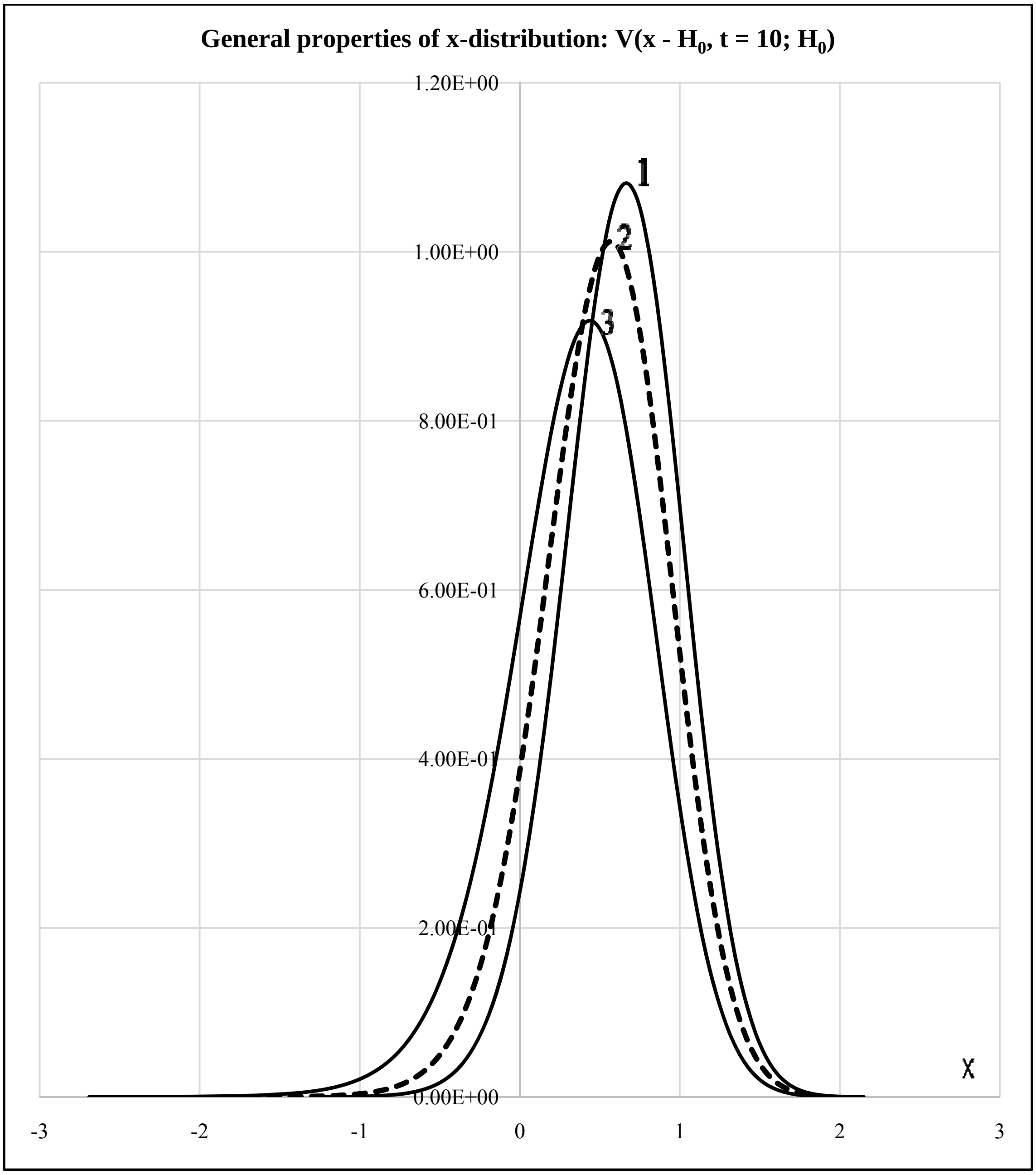

Figure 3. The x-distribution $V(x - H_0, t = 10)$ for H_0 = 1.8 (1), 1.6 (2), 1.4 (3). Observe development of a negative tail and increasing distribution skewness.

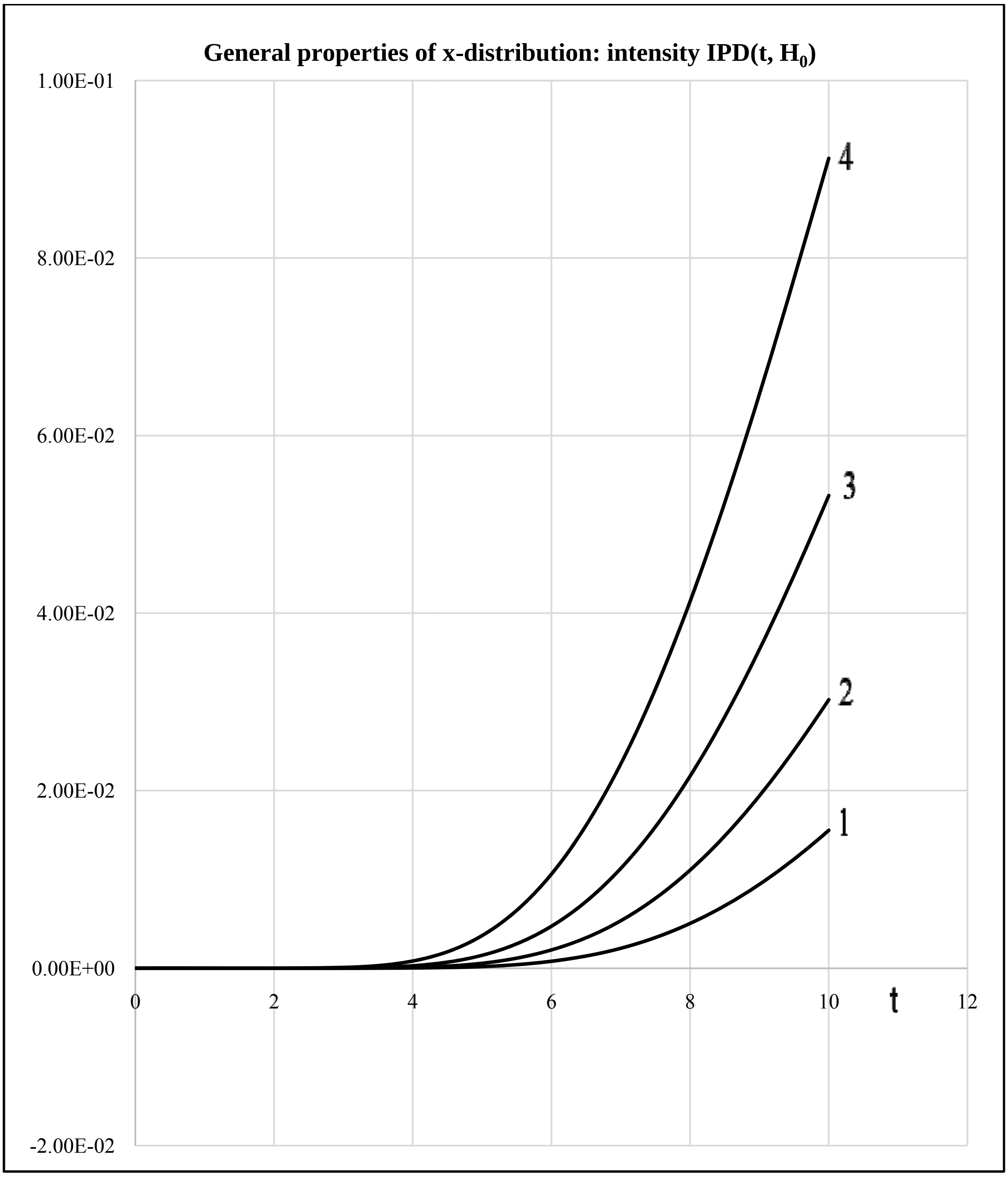

Figure 4. Development of the intensity of default probability *IPD*(t, H_0) as a function of time (years) and initial values H_0 = 1.35 (1), 1.30 (2), 1.25 (3), 1.20 (4). Observe the fast growth of IPD in the right part of the graph.

Figure 4 shows how the intensity of default probability *IPD*(t, H_0) depends on time and H_0. The figures demonstrate a fast *IPD* rise when H_0 decreases. Note that the *IPD*-development is very inertial: for the most time, *IPD* remains low, rising to relatively large values in the second half of the graph. This behavior of the *IPD*-function can put a false feeling of safety into heads of the managers using one-year-horizon models. Estimating credit risks, these models tall the alarm too late when a significant part of time so necessary for solving the crisis is already wasted.

The martingale property of stock prices and the time-independent and entire-market-valid no-arbitrage pricing principle follow from GBM neglecting BSE payments. For short-time deals (much less than a year), stock prices are independent of the firm mean returns and have the log-normal distribution. Thus for short-term dealers, the stock prices make a martingale, the whole market is no-arbitraging, and the structural model is an adequate means for describing a state of the firm. A long-term investor has to be cautious because the martingale characteristic of stock prices and no-arbitrage pricing principle are not universal, but depend on the mean year returns of the firm issuing the stocks. A rise in debt, taxes, dividends, or inflation will decrease the mean year returns of the firm inflicting losses to the long-term investor (Figures 8, 10).

Effects of Inflation

Inflation is an intrinsic factor of any economic environment, and as such, it must be taken into consideration when analyzing the development of a firm over a long period. Inflation affects the firm value through the firm mean year returns and BSEs. A price rise on the materials used in the manufacturing as well as a price rise on the firm's production, with a corresponding shift in demand, can change the expected rate of returns:

$$R_i = R + \Delta R_{inf}, \; R = \mu - 0.5C^2 \tag{18}$$

For low inflation rates and low demand elasticity, this change can be positive, $\Delta R_{inf} > 0$, when the expected return increase due to a higher price on the firm's goods is not offset by the decrease in demand. For higher inflation this change becomes negative, $\Delta R_{inf} < 0$, whereas the expected rate of returns drops, as a higher price on the firm's goods fails to compensate for the decrease in demand. Here we will consider a specific case when the increase in price caused by inflation is exactly offset by the decrease in demand. In this case, GBM does not see any change in the state of the firm (see Introduction).

Let us consider a levered firm F_L financed with a composition of equity E and debt X_D of leverage α at the initial moment, $\langle X_0 \rangle$ denotes the initial mean value:

$$\langle X_0 \rangle = \langle E \rangle + X_D = (1-\alpha)\langle X_0 \rangle + \alpha \langle X_0 \rangle.$$

For a *debt of infinite maturity*, the levered firm's BSE, $P_L = P(\alpha)$, is

$$P(\alpha) = P(0) + k\alpha \langle X_0 \rangle, \tag{19}$$

where k is the debt interest rate, and $P(0) = FC$ makes BSEs of the unlevered firm (taxes and dividends are not included).

Debt payments are paid in nominal dollars of the day when the debt has been issued, while all current FC payments are subject to inflation. For inflation of the expected rate i, the equation for the probability distribution of the levered firm is ($x = \ln(R_i X / P_0)$)

$$V_t + R_i[1 - (e^{it} + \beta)e^{-x}]V_x - 0.5C^2 V_{xx} + R_i(e^{it} + \beta)e^{-x}V = 0. \tag{20}$$

$$\beta = \frac{k\alpha}{R_i}\exp[H_0(0) + 0.5\sigma_0^2], \tag{21}$$

where $\pi(t) = e^{it} + \beta$, and $H_0(\alpha = 0) = \langle x \rangle_N$ is the mean value of the initial value distribution of the unlevered firm. The initial condition for the levered firm becomes

$$V(x,0) = N(x; H_0(\alpha), \sigma_0^2), H_0(\alpha) = H_0(0) - \ln(1+\beta). \tag{22}$$

The boundary condition is

$$V(DL, t) = 0, \; DL = max\,(0, H_0(\alpha) + 0.5\sigma_0^2 + ln\alpha), \tag{23}$$

here $N(x;\, H_0,\, \sigma_0^2)$ is a normal function. The problem (20)-(23) is solved numerically for various values of leverage α and inflation rate i. Here we present results for the unlevered firms only, $\alpha = 0$. Other model parameters are $VAR_0 = 0.03$, $R = 0.10$, $C = 0.008$, $DL = 0$. Results are presented in Figures 5-10. The figures show the influence of inflation on the firm's development and default risks when inflation makes a non-positive effect on the expected rate of returns ($\Delta R_{inf} \leq 0$). As inflation grows, more and more firms find out that inflation makes an negative effect on their returns. We present the results for the case $\Delta R_{inf} = 0$, for which GBM sees no difference between the cases with and without inflation. For steady firms ($H_0 = 1.8$), the effect of inflation is negative but small: the firm mean $H(t) - H_0$ increases a bit slower (Figure 5), the variance (Figure 6) grows a bit faster, but as a whole, the firm develops steadily, and the intensity of default probability *IPD* in the time interval of 10 years remains low (Figure 7).

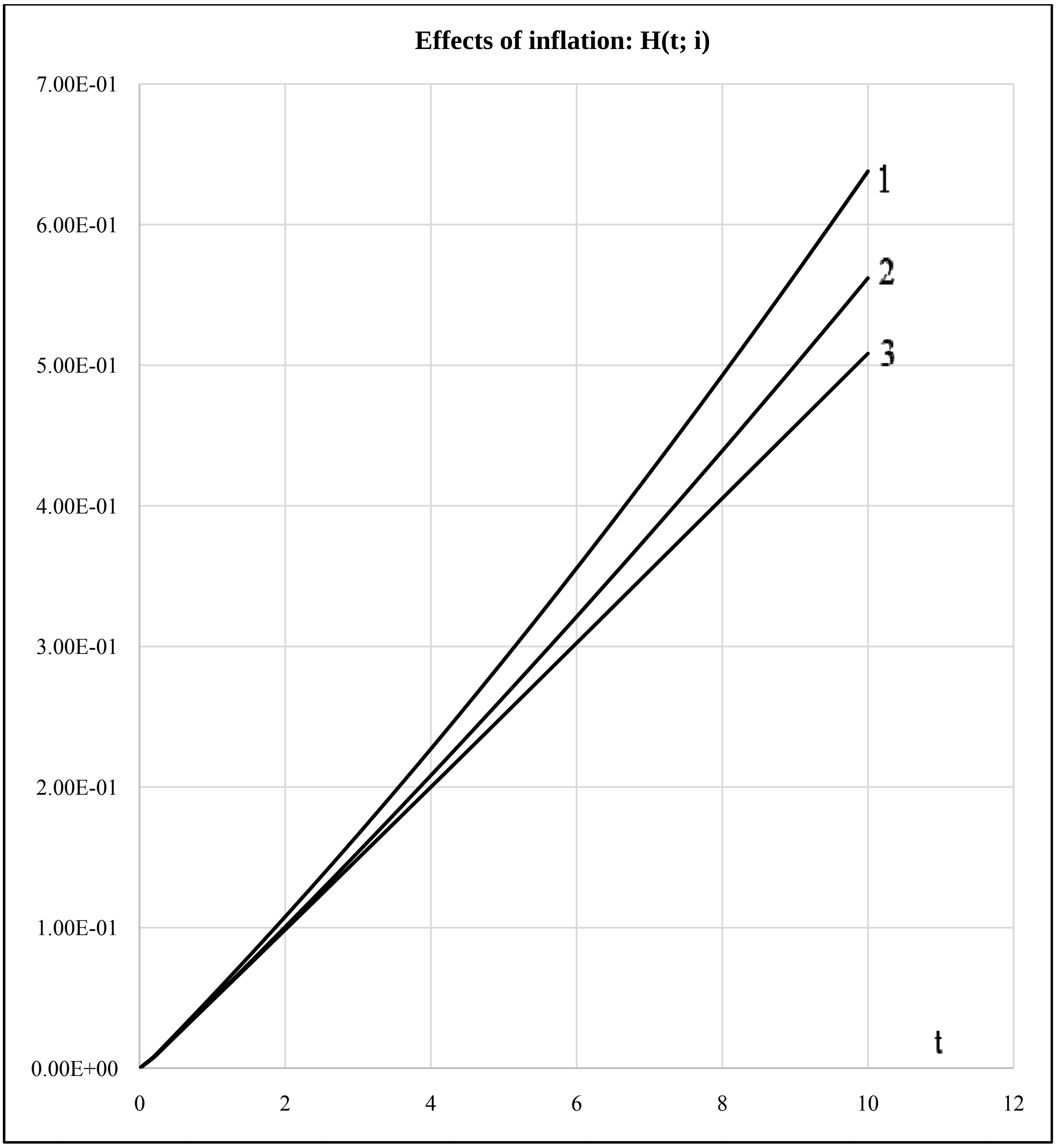

Figure 5. Development of the mean value $H(t) - H_0$ for a steady firm ($H_0 = 1.8$) for different expected rates of inflation $i = 0$ (1), 0.02 (2), and 0.04 (3) when the inflation makes no contribution to the expected rate of return: $\Delta R_{inf} = 0$.

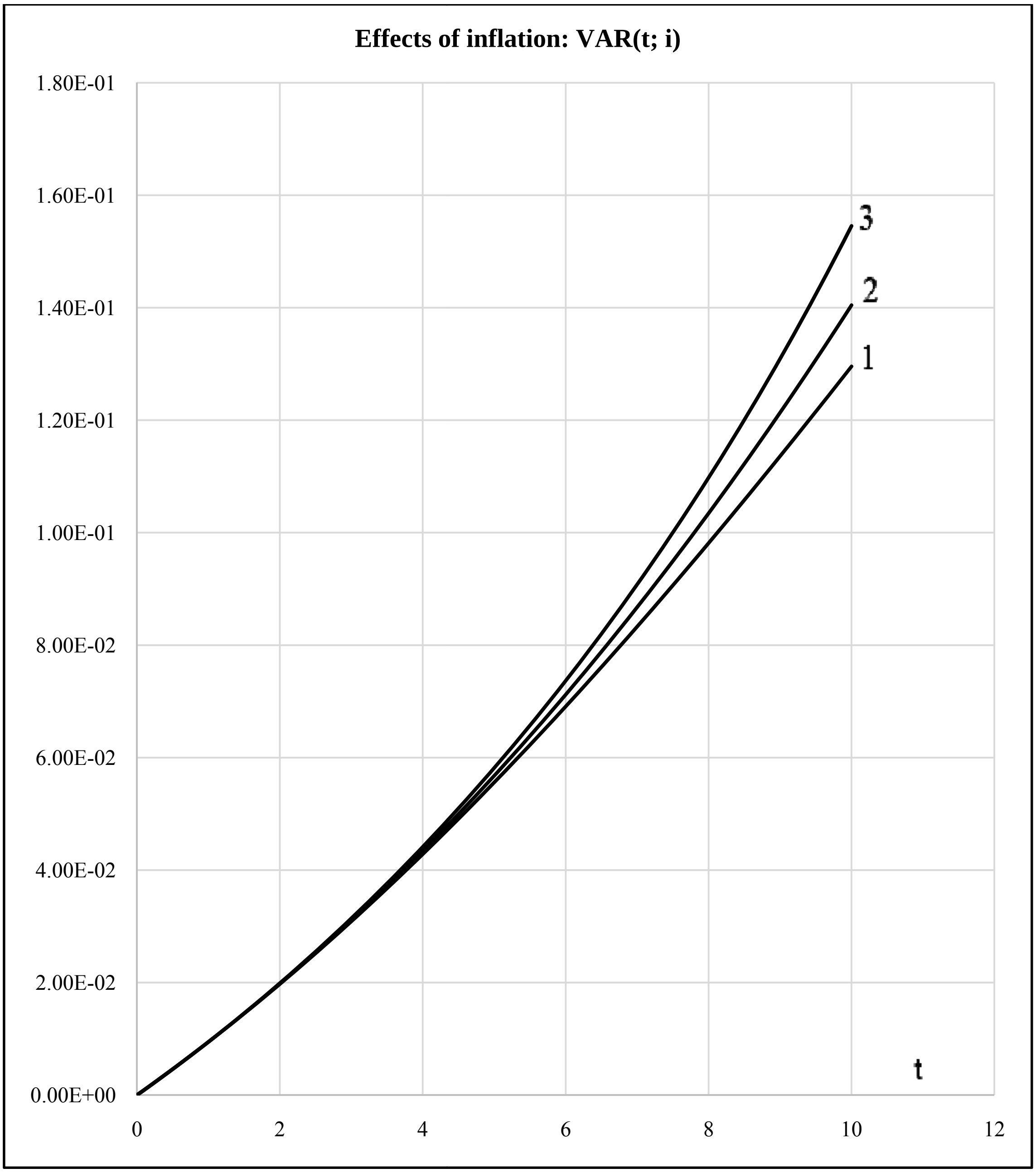

Figure 6. Development of variance $VAR(t) - VAR_0$ for a steady firm ($H_0 = 1.8$) for different expected rates of inflation $i = 0$ (1), 0.02 (2), and 0.04 (3) when the inflation makes no contribution to the expected rate of return: $\Delta R_{inf} = 0$.

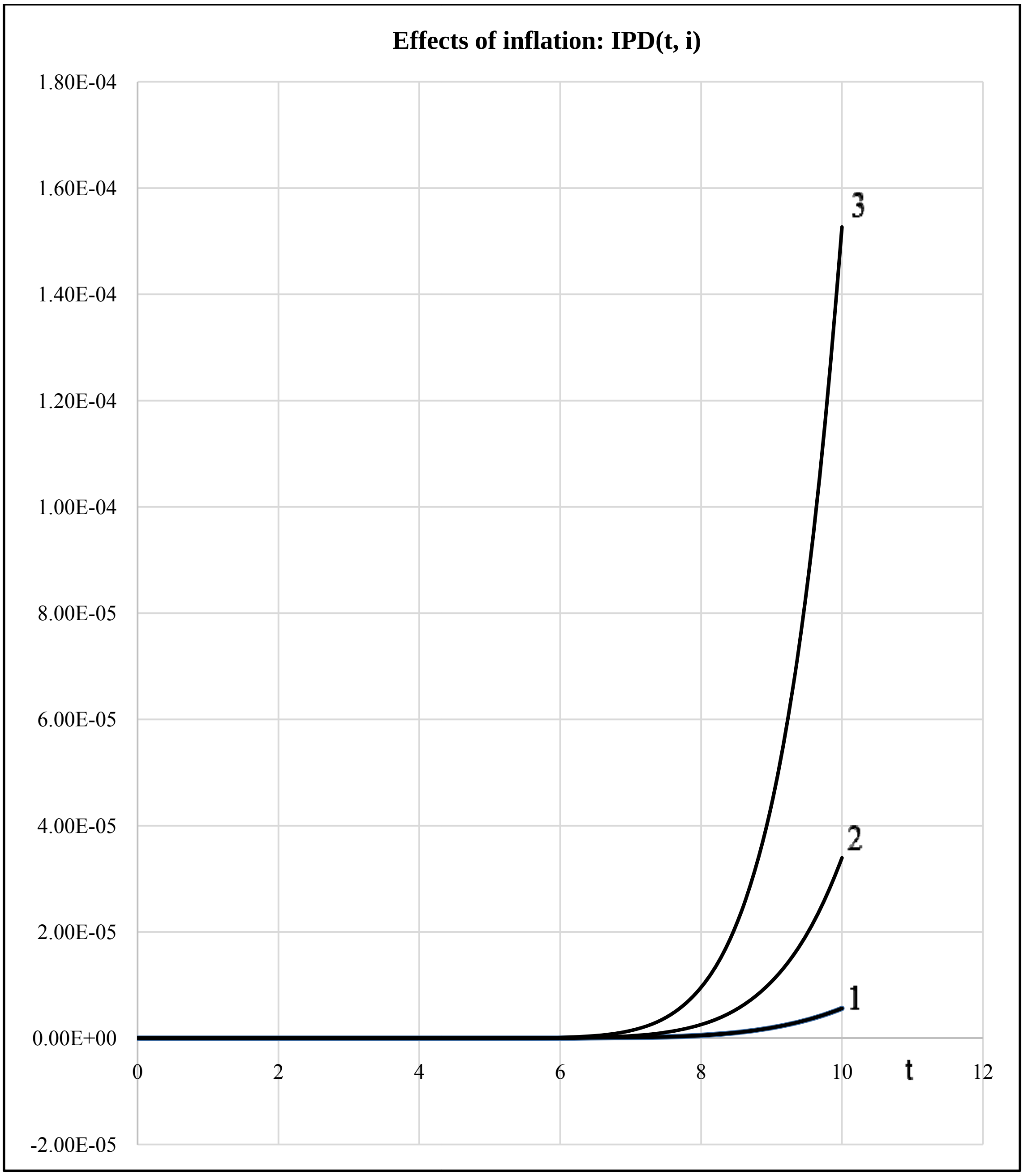

Figure 7. The intensity of default probability *IPD*(t) for a steady firm (H_0 = 1.8) as a function of time and expected inflation rate i = 0 (1), 0.02 (2), and 0.04 (3) when inflation makes no contribution to the expected rate of return: $\Delta R_{inf} = 0$.

For a weaker firm with a lower initial mean H_0 (H_0 = 1.4, VAR_0 = 0.03, R = 0.10, C = 0.008, DL = 0), the effect of inflation becomes dramatic (Figures 8-10). Development of the mean switches from a steady progress (Figure 8, line 1) to a downfall, the variance grows faster (Figure 9). The total effect is devastating: the firm's cumulative default probability over the ten-year interval grows from moderate 0.05 to dangerous 0.25 (Figure 10). A decrease in the expected return ($\Delta R_{inf} < 0$) potentiates the negative effect of inflation on the default probability. So, the inflation effect on corporate stability when $\Delta R_{inf} \leq 0$ is absolutely negative, but its magnitude depends on a state of the firm: for steady firms with high $H(t)$ the effect implies no dramatic consequences, but for a weak firm with low $H(t)$ it can be dangerous.

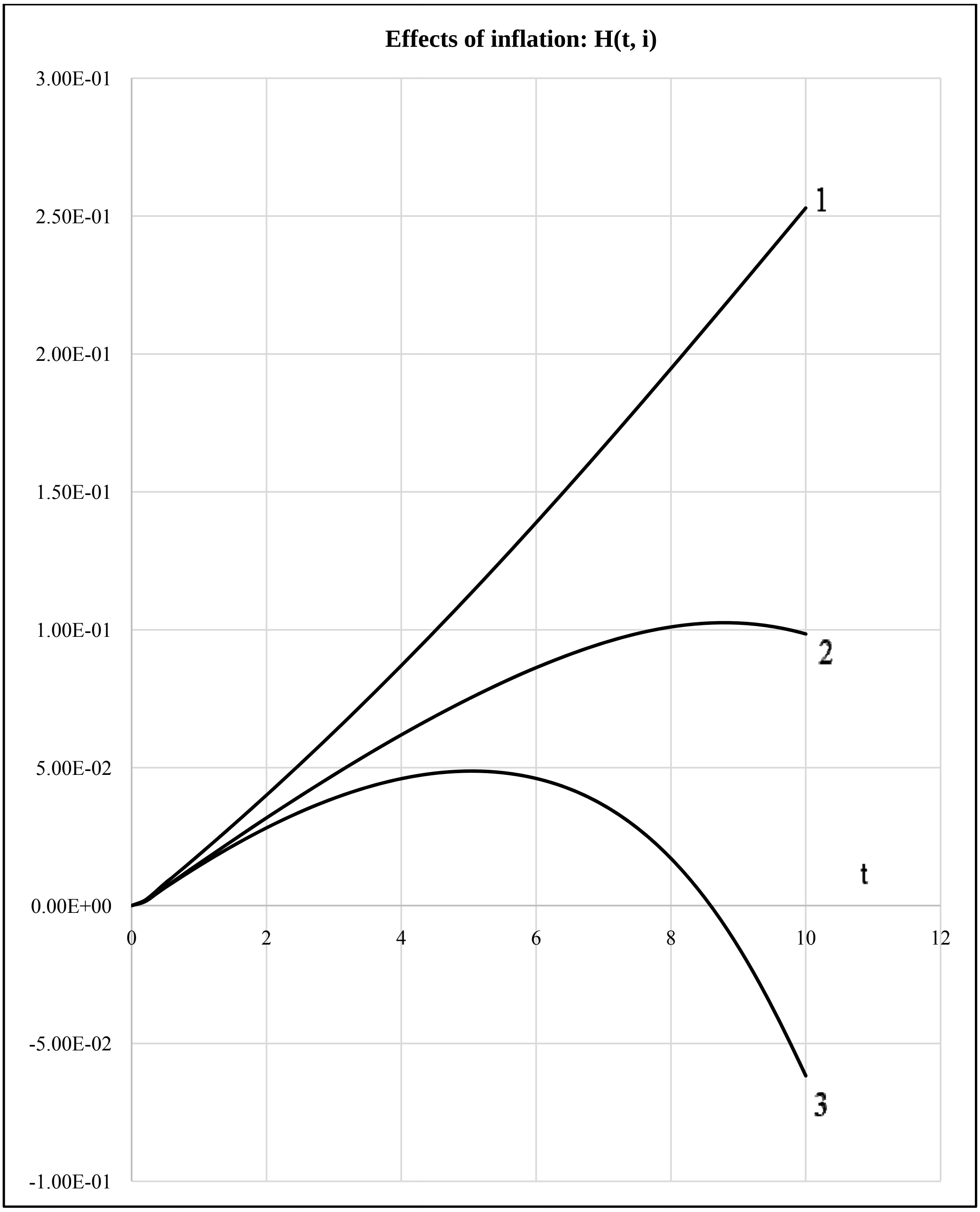

Figure 8. Development of the mean $H(t) - H_0$ for a weak firm (H_0 = 1.4) as a function of inflation: i = 0 (1), 0.02 (2), and 0.04 (3) when inflation makes no contribution to the expected rate of return: $\Delta R_{inf} = 0$.

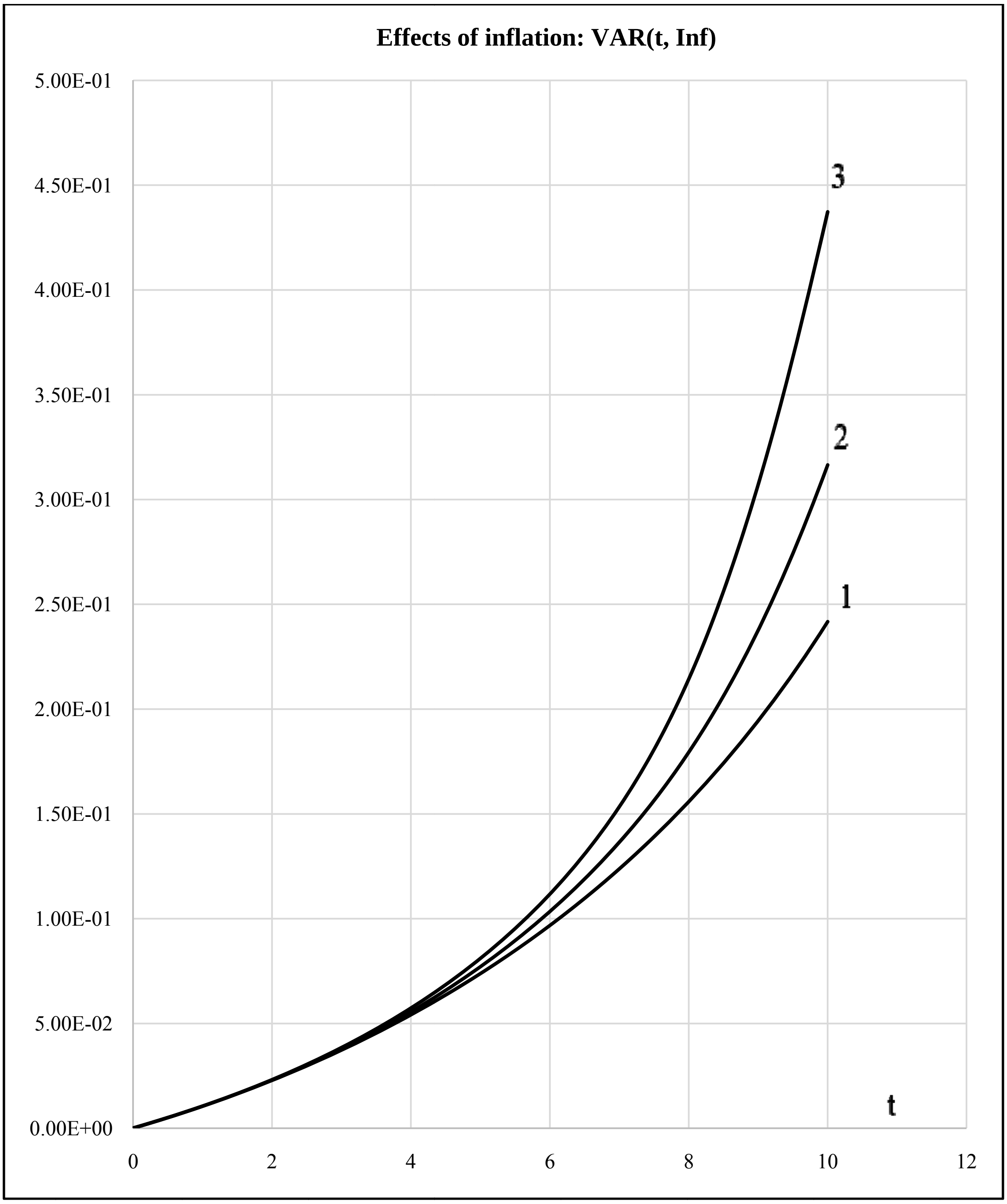

Figure 9. Development of the variance $VAR(t) - VAR_0$ for a weak firm (H_0 = 1.4) as a function of inflation: i = 0 (1), 0.02 (2), and 0.04 (3) when inflation makes no contribution to the expected rate of return: $\Delta R_{inf} = 0$.

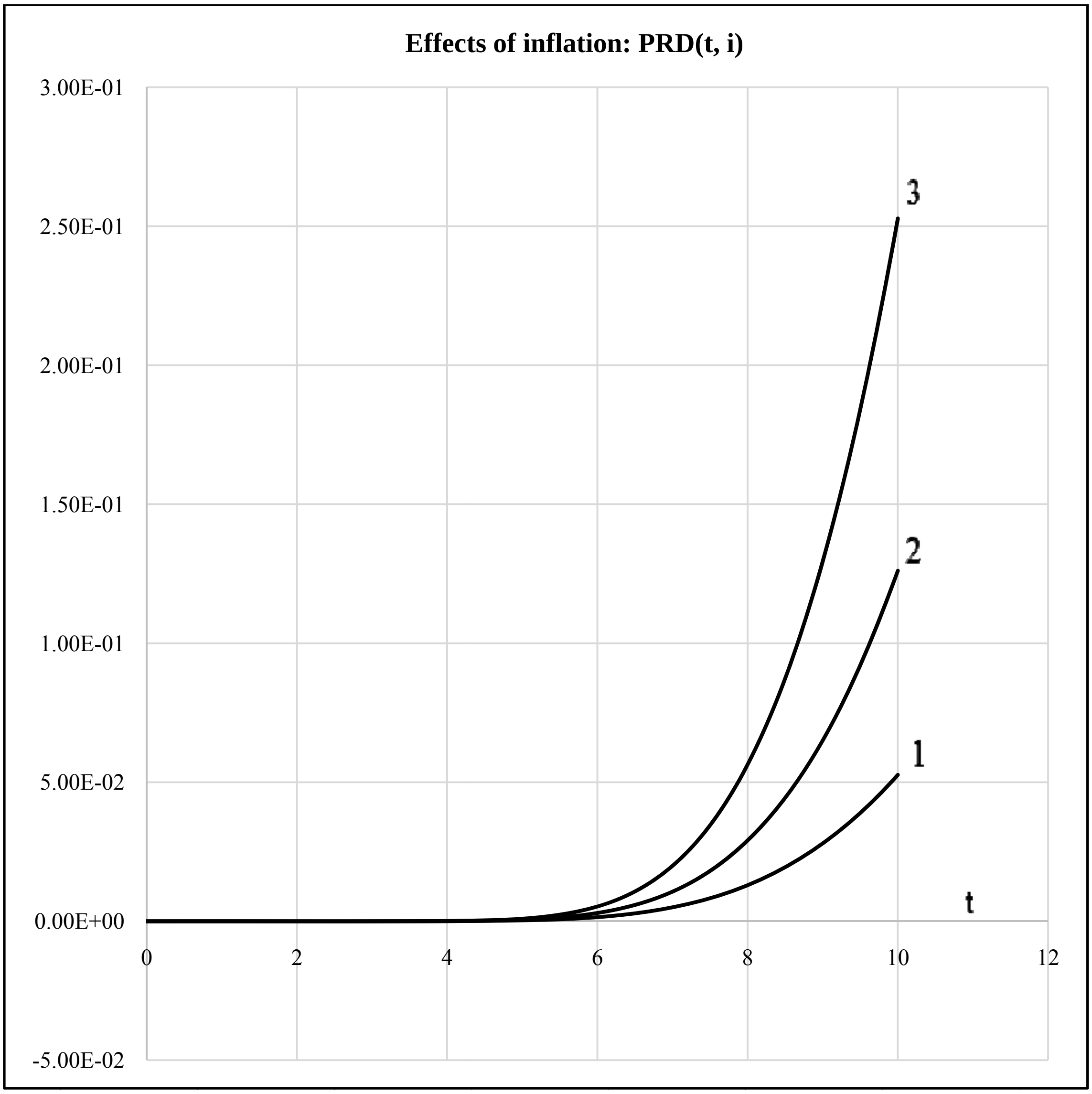

Figure 10. Development of the default probability *PRD*(*t*) (t_s = 0) for a weak firm (H_0 = 1.4) as a function of inflation *i* = 0 (1), 0.02 (2), and 0.04 (3) when inflation makes no contribution to the expected rate of return: $\Delta R_{inf} = 0$.

As one can see, in the case $\Delta R_{inf} = 0$ when GBM expects no effect of inflation on the firm state, EMM reveals a negative effect of inflation on the firm survival, which is the more significant, the weaker the firm (the lesser H_0).

Now we consider the case when the expected rate of returns grows with inflation, $\Delta R_{inf} > 0$. To be specific, we set $\Delta R_{inf} = 0.5i$. For a weak firm (H_0 = 1.3, VAR_0 = 0.03, *R* = 0.10, *C* = 0.008, *DL* = 0), inflation changes the dynamics of the mean from a slow growth at a zero-inflation rate to an energetic growth, the faster, the higher the inflation rate (Figure 11). The effect of inflation on the variance is negative: it grows the faster, the higher the inflation rate. However, the overall effect of moderate inflation on the corporate stability should be recognized as *positive*, because of the intensity of default probability *IPD* (Figure 12) and default probability *PRD* (Figure 13) decrease. This effect is the more significant, the greater ΔR_{inf}.

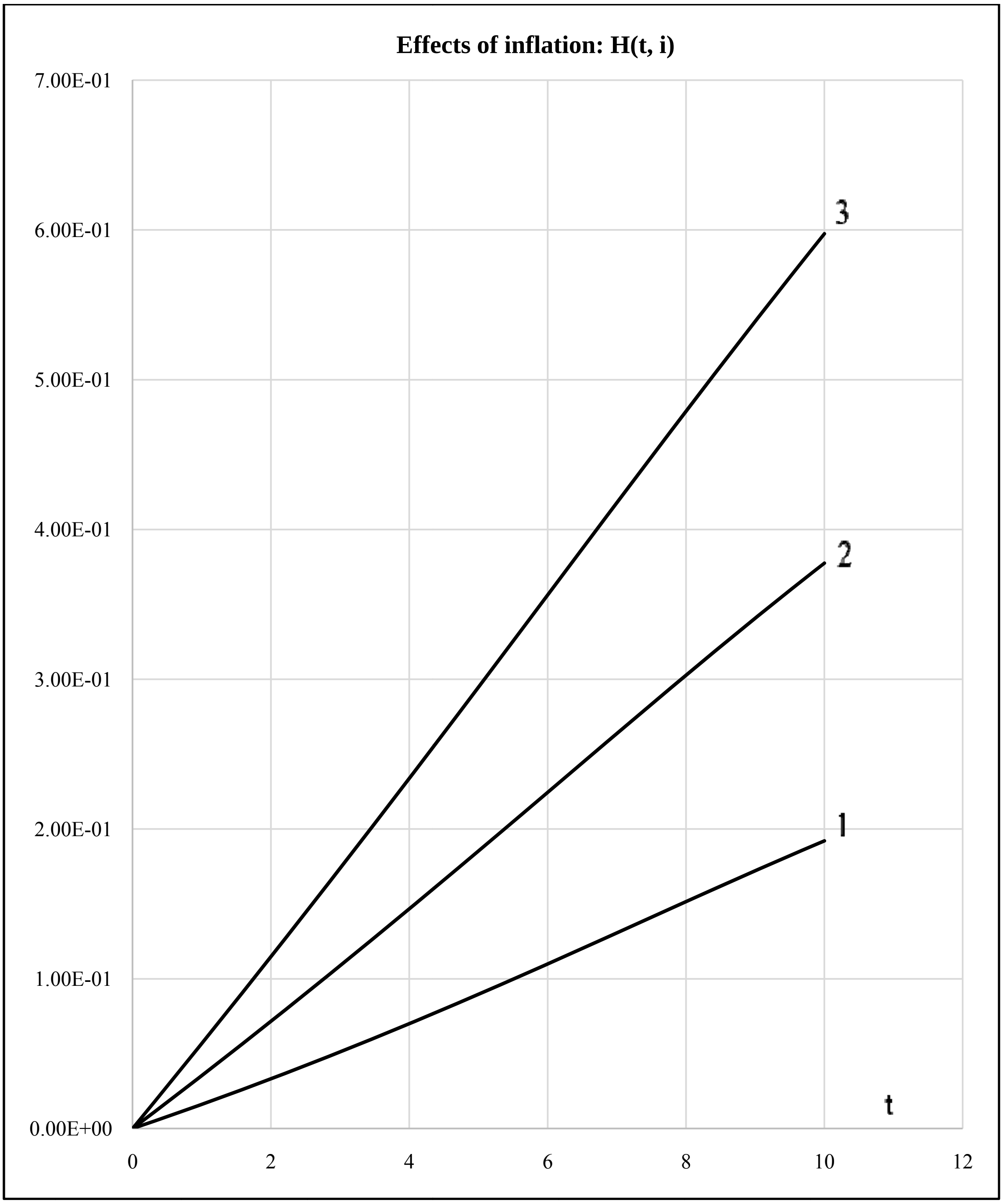

Figure 11. Development of the mean $H(t) - H_0$ for a weak firm ($H_0 = 1.3$) as a function of inflation i and effective rate of returns R_i affected by inflation: $i = 0$ and $R_i = 0.10$ (1), 0.02 and 0.11 (2), and 0.04 and 0.12 (3). $\Delta R_{inf} = 0.5i$.

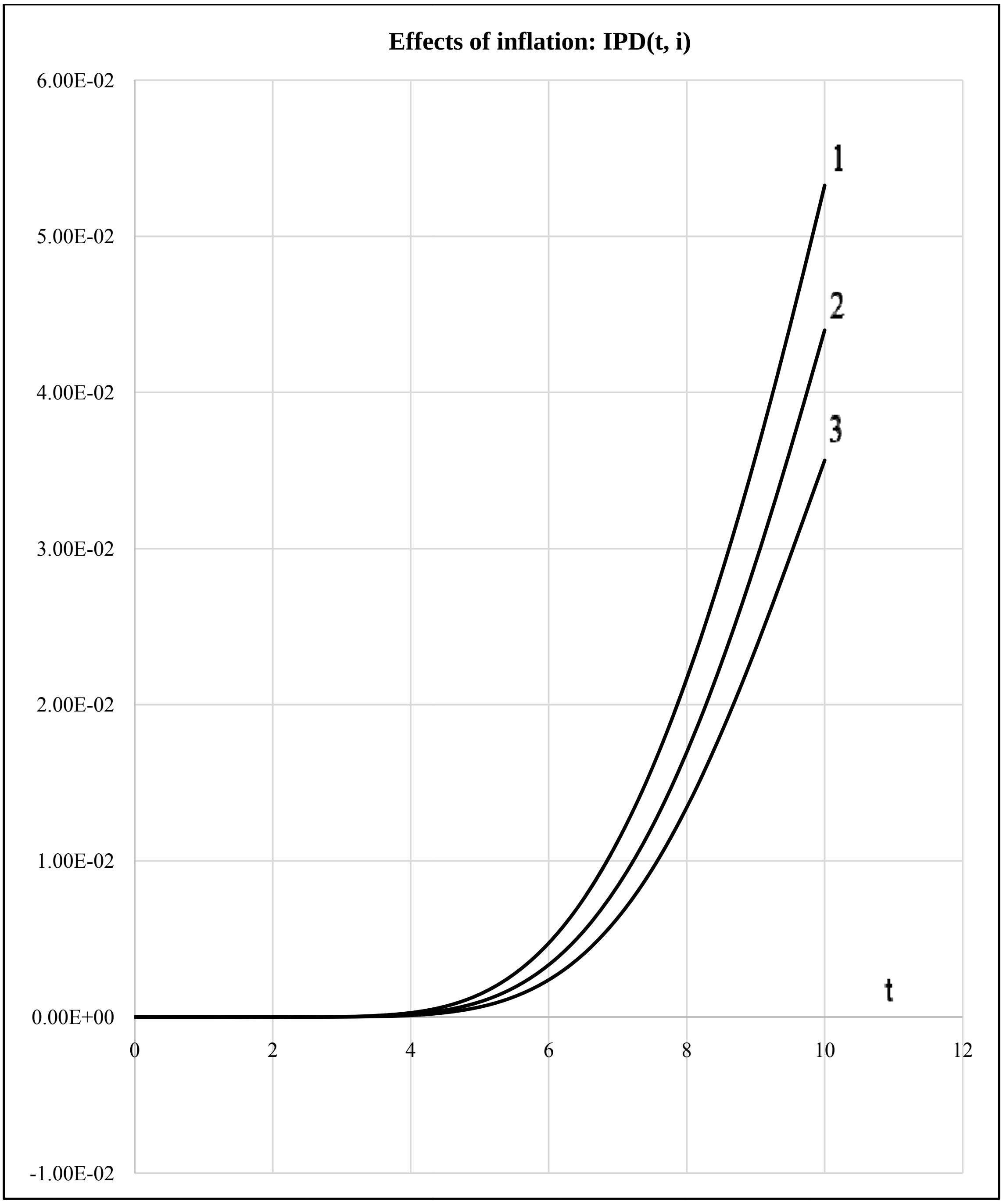

Figure 12. Development of the intensity of default probability *IPD*(*t*) for a weak firm (H_0 = 1.3) as a function of inflation *i* and effective rate of returns R_i affected by inflation: i = 0 and R_i = 0.10 (1), 0.02 and 0.11 (2), and 0.04 and 0.12 (3). $\Delta R_{inf} = 0.5i$.

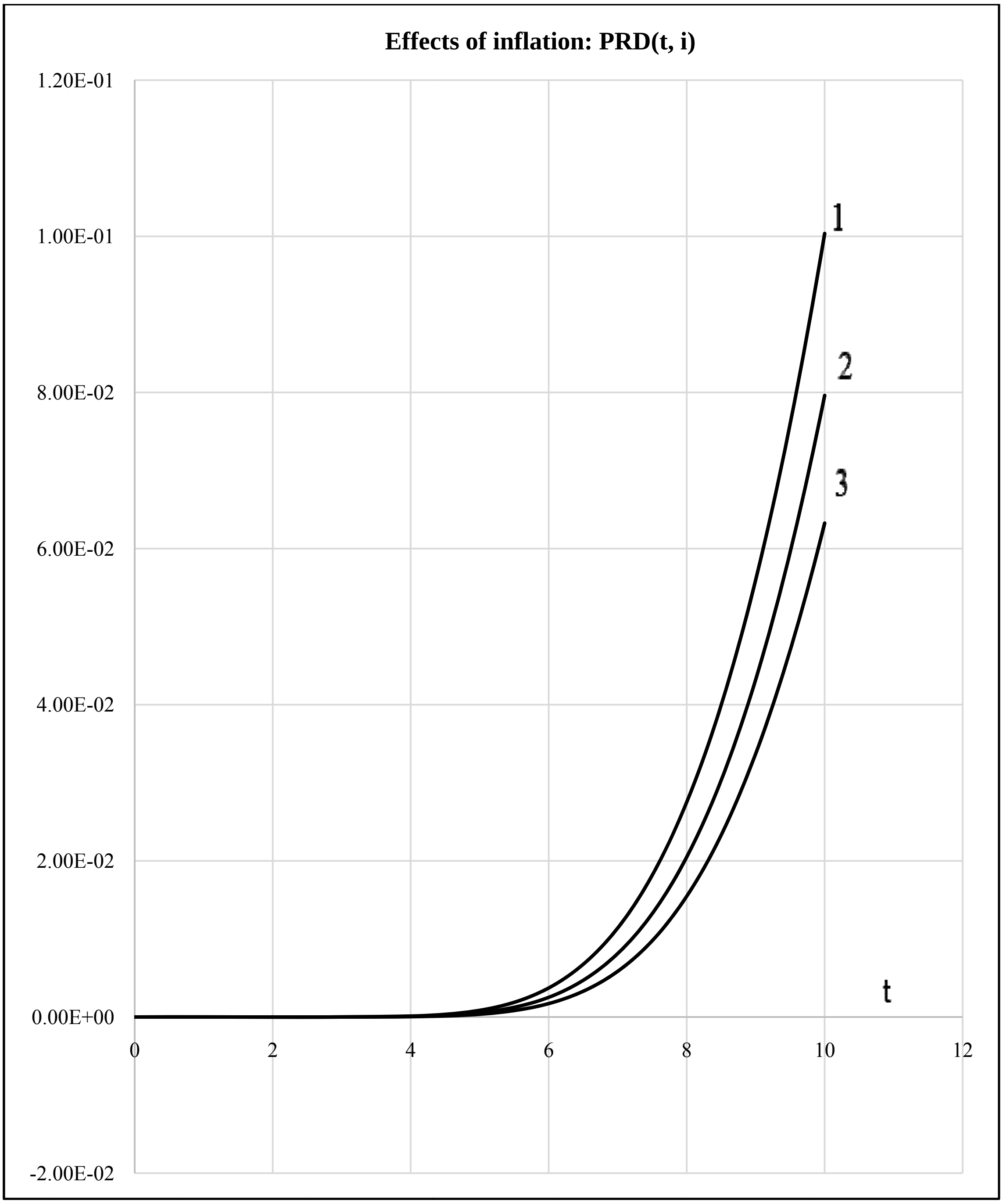

Figure 13. Development of the default probability $PRD(t)$ for a weak firm (H_0 = 1.3) as a function of inflation i and effective rate of returns R_i affected by inflation: i = 0 and R_i = 0.10 (1), 0.02 and 0.11 (2), and 0.04 and 0.12 (3). $\Delta R_{inf} = 0.5i$.

The model supports at the microeconomic level the New Keynesian findings that inflation can exert a dual effect on output growth: for lower inflation rates this effect is benign, whereas for higher inflation rates this effect is adverse. Mark that this macroeconomic effect has microeconomic roots and depends on a lot of parameters determining a state of the firm such as its mean value, BSEs, the asset structure, the elasticity of demand on the firm's goods, and volatility of the firm value depending both on the exogenous conditions and on the quality of the firm's team. The macroeconomic parameters directly affecting the inflation effect are the state of the firm's industry and the national economy as a whole (boost-recession), including the magnitudes of their volatilities, the interest rate, and the inflation rate. It seems that the inflation rate optimal for the entire economy does not exist, because two firms in the same industry can experience the opposite inflation effects at the same rate of inflation depending on the patterns of their microeconomic parameters. (The optimal rate is the value separating the interval of inflation rates making a positive effect on the national output growth from the region of inflation rates exerting an adverse influence on output growth.) All said above makes the problem of determination of the optimal inflation rate hardly solvable by macroeconomic methods only.

Conclusion

We use a continuous-time credit risk model (EMM) taking account of the firm's business securing expenses (BSEs) and inflation. In this problem setting, BSEs consist of fixed costs and debt payments. EMM computes a firm value distribution along with its mean, variance, and skewness as functions of time and parameters of the firm and its business environment. In an open space in variables (x, t), $x = \ln(RX/P)$, X—the firm value, R—the expected rate of returns, P—a BSE magnitude per time unit, t—time, the EMM-distribution transforms from an initially normal distribution to a negatively skewed one with its skewness and variance growing fast. The growing left tail in competition with the distribution right drift determines the development of mean returns $H(t)$. After an almost straight rise of $H(t)$ for period T_m depending on parameters of the firm and its business environment, the mean year returns begin to decline, causing a decrease in the firm's stock price and cancelling the no-arbitrage pricing principle since the end of T_m. The no-arbitrage pricing principle always holds for short-term deals ($t << T_m$) when traders buy and shortly resell stocks trying to gain profits from the price difference. The structural model adequately describes this short-term activity. The structural model does not hold for long-term investors, such as pension funds, mutual funds, banks, insurance companies, and big firms. For long times ($t \approx T_m$) when BSEs becomes essential, the no-arbitrage pricing principle is not a market characteristic anymore, but rather a characteristic of individual assets traded at the market. The stock value depends on the firm mean year returns which depend on the firm's business parameters (the BSE share in the mean year returns, asset leverage, inflation rate, etc.). At that, the investor must know the period when the mean year returns will remain about constant. All risks raising the firm's BSEs and decreasing its mean year returns can inflict losses to long-term investors.

The model proves that the New Keynesian nonlinear dependence of output growth on inflation has its roots in the microeconomic characteristics of the firm hardly visible from the macroeconomic level, such as its mean value, the mean year returns, business securing payments, the asset structure, the elasticity of demand on the firm's goods, and the volatility of the firm value depending both on the exogenous conditions and on a quality of the firm's team. Two firms in different microeconomic conditions but in the same macroeconomic conditions can experience the opposite effects of inflation on their mean values and stability. That means that in the search for the optimal inflation rate, macroeconomists must take account of the microeconomic conditions too.

At the microeconomic level, the model proves that:

- inflation is an important factor of the firm's business environment, and it must be taken into account when considering long-term corporate credit risks;
- low inflation increasing the expected rate of returns makes a benign effect on the firm value and provides for conditions of a sustainable development decreasing the default probability;

- high inflation leading to a decrease in the expected rate of returns make an adverse effect on the firm development: the mean value decreases, and the default probability increases. However, the magnitude of this effect depends on the firm health: for steady firms, whose mean value is high, this effect is insignificant, but for weaker firms, this effect can be fatal;
- the threshold between "low" and "high" inflation rates depends on the relation between inflation and the expected rate of returns, BSEs, and the elasticity of demand on the firm's goods. These relations are sensitive to the state of an economy (a developed or developing economy, a boost or decline stage in the economic cycle, etc.), making these thresholds time-, industry-, and country-varying.

The model can help the firm's management better understand the current state of their firm and its prospects, especially when planning long-term business operations. It can be helpful forthe long-term investors evaluating their risks. It can also be useful for banks and insurance companies estimating credit risks for a particular commercial borrower at a horizon of debt maturity.

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