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Probabilistic Project Duration Estimation Based on Uncertainty of Linkage between Activities

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Abstract: Generally, one of the most difficult works at scheduling is to estimate the duration of activities and linkages between them because the possibility that the duration and linkages could be exposed to the uncertainties is very high. When estimating project duration, therefore, the probabilistic estimation of the duration as well as the probabilistic estimation of the linkages between activities should be considered concurrently. The Project Evaluation and Review Technique (PERT) that is considered to be one of the most popular techniques applied for the probabilistic estimation of a project duration cannot consider the uncertainties of the linkages because it only estimates the probabilistic duration limited to "FS0" relationship. The purpose of this study is to propose the new method, the Probabilistic Linkage Evaluation Technique (PLET), for probabilistically estimating the project duration based on the probabilistic estimation of the BDM's relationships, and also provide more wide and various probabilistic information about the project duration.

Key words: PDM, BDM, linkage, overlap, probabilistic evaluation.

1. Introduction

The schedule of a construction project classifies the works that will occur in the future and logically expresses them in terms of time. However, most of the construction projects have numerous predictable or unforeseen risks that limit the project manager to predict actual performance [1], and it is virtually impossible to accurately estimate the duration of a construction project. Nevertheless, many researchers have proposed various methods for probabilistic estimation of the duration of the project schedule considering various risk factors in the construction project, and some of them are actively applied in the practice. In general, the most important considerations when estimating project duration are duration and linkage of each activity. This is because the potential risk factors in the project schedule are likely to expose uncertainty to duration and the linkage between activities. In other words, the uncertainty inherent in the activity duration not only makes it difficult to guarantee that the activity will be completed within the expected period, but also ensures that the uncertainty inherent in the linkage between activities cannot be determined within the expected period of time. Therefore, it is necessary to take into account not only the probabilistic estimation of the activity duration but also the probabilistic estimation of the linkage when probabilistic estimating of the project duration. Most of the techniques for probabilistic estimation of project duration focus on the uncertainty of activity duration, and the Project Evaluation and Review Technique (PERT) is a typical example. However, since the PERT is limited to the finish-to-start (FS) relationship as in the Arrow Diagramming Method (ADM), if there is an overlapping relationship between activities, probabilistic estimation of relationships between activities is impossible. Therefore, the PERT cannot satisfy it if probabilistic estimation of relationships between activities is required. Furthermore, until now there have been shortages of studies on probabilistic

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estimation of relationships between activities and evaluation of project duration based on them.

This study proposes a new technique, Probabilistic Linkage Evaluation Technique (PLET), which probabilistically estimates the relationship between activities based on the BDM technique, which can freely express overlapping relationship between activities, and probabilistically evaluate the project duration. The purpose of the PLET is to provide construction project management team, including the client, with broader and more diverse stochastic information about the project duration.

2. Research Scope and Methods

This study was carried out in the following way and in order. First, we consider the representative probabilistic project duration estimating methods proposed so far and analyze their restrictions and limitations. Second, it suggests the necessity of probabilistic evaluation of connection relation between activities. Third, we propose the basic concept and methodology of probabilistic linkage evaluation technique (PLET) in BDM. Fourth, PLET technique is verified through practical examples. Fifth, we show the expected effect of PLET technique and future studies. The scope of this study is limited to the probabilistic estimation of relations between activities and the integration of PERT and PLET techniques is not considered.

3. Review of Existing Probabilistic Project Duration Evaluation Methods

If the relationship between the activities and the activity duration is accurately represented in the CPM network, the schedule will have the highest reliability. However, the most difficult task in establishing a schedule in a project management practice is to estimate the relationship between the activities and the activity.

Among them, a general method for estimating the duration of activity is to utilize the past experience data, to construct the working crew and the input equipment for the specific activity, to derive the daily workable amount, and to apply this to the total work amount to estimate the duration of activity. However, the construction site has not only the non-repetitive characteristics that cannot apply the past experience data, but also it is very difficult to estimate the accurate activity duration due to numerous variables depending on the external working environment. Accurate estimations of duration are almost impossible, especially for unexperienced or new projects. If a client or decision maker requires more flexible and more probable project duration information than imprecise and undefined project duration, it is necessary to provide a methodology that satisfies this. The probabilistic duration estimation method stochastically evaluates the time to reach the entire project duration or major intermediate completion points, and various methods such as PERT technique, GERT technique, and application method that mixes PERT technique and Monte Carlo simulation are presented, some of them are actively applied in current practice.

3.1 PERT

The Project Evaluation and Review Technique (PERT), introduced in the US Navy's Polaris missile system program in 1958, is designed to support planning in the absence of empirical cost and schedule information for the overall project schedule and cost estimates [2]. The PERT has the similarity as the CPM in the concept of centrally locating and managing the main line, expressed in the form of ADM (Arrow Diagramming Method). The difference between the CPM and the PERT is that the CPM assumes schedule with very small dispersion, but the PERT defines schedule with a distribution with a relatively large dispersion. Therefore, the PERT analyses the network as a probability concept and is therefore suitable for use as a lack of experience or as a process management tool for new R&D projects [3].

Recently, uncertainty in the construction environment has increased [4], which is a method to analyze and manage the process uncertainty and risk of risk management.

The PERT network's activity duration has a probability distribution and is estimated with the following three estimated working hours: most likely time (*m*), most optimistic time (*a*), and most pessimistic time (*b*). Based on this, the average time (t_e), standard deviation (σ) and variance (σ^2) of the activity are calculated as follows.

$$\text{Average time:} t_e = \frac{a + 4m + b}{6} \quad (1)$$

$$\text{Standard deviation:} \sigma = \frac{b - a}{6} \quad (2)$$

$$\text{Variance: } \sigma^2 = \left(\frac{b - a}{6}\right)^2 \quad (3)$$

The most significant feature of the PERT is that it provides more information than the CPM, which estimates the schedule stochastically, by probabilistically evaluating the period to reach the main completion point in order to comply with the overall project schedule. However, the PERT is based on the ADM network format, so that the interconnection of relationship is limited to the finish-to-start (FS) logic. If the logic between activities is confined to the FS only, the reliability of the PERT network analysis will suffer because the logic between actual operations is not accurately reflected.

3.2 GERT

The Graphical Evaluation and Review Technique (GERT) was proposed in 1966 as a network analysis technique that can handle network logic and estimation of activity duration stochastically. The GERT approach provides a solution to the PERT/CPM limitations and allows loop logic between activities [5].

GERT is similar to PERT but allows deterministic and probabilistic branching unlike PERT. Fig. 1a shows the deterministic branching and Fig. 1b shows the probabilistic branching. In Fig. 1b, the sum of the probabilities of each branch should be 100%. Fig. 2 also shows that it is permissible to use iterative logic in the CPM/PERT [6]. The most fundamental flaw of the GERT is that the process for modelling the GERT system is very complex and rarely utilized in practice.

3.3 RDM

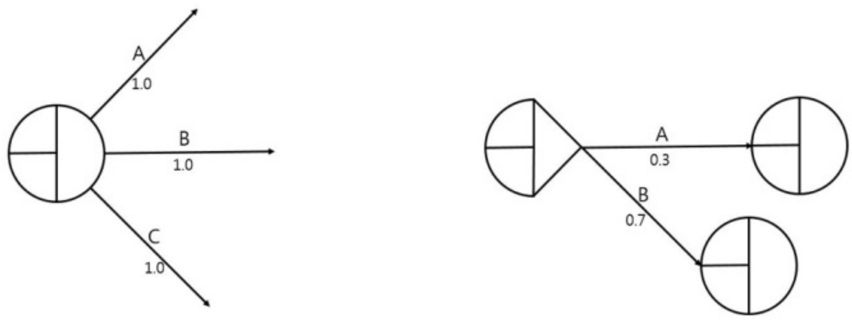
The Relational Diagramming Method (RDM) is proposed as a kind of CPM technique which focuses on the reason of overlapping and redundancy between activities. Fig. 3 shows an example of a network created by RDM. The RDM differentiates the existing PDM by adding the following key information to the PDM network [7]. First, add a node at the time the activity actually starts or changes. Second, add a reason/why restraint code that binds the connection between activities. Third, add a duration code that defines the basis of the activity duration. Fourth, add extension form of restraint between activities.

3.4 SAPA

The Stochastic Allocation of Project Allowance (SAPA) was proposed in 2011 to predict project time contingency and to allocate it to the activity level. The SAPA is based on the results of a project time simulation to predict the project planned duration (PPD) and the project target duration (PTD). The method used to predict PPD and PTD in the SAPA is the PERT and the Monte Carlo Simulation [8]. In general, the most widely applied methods for probabilistic estimation of project duration are a concoction of PERT and Monte Carlo Simulation like SAPA, and similar approaches to SAPA have been published by many researchers.

3.5 Restrictions and Limitations of Existing Methods

Table 1 compares the existing activity duration and linkage estimating methods described above.



(a) (b)

Fig. 1 Deterministic and probabilistic branching of GERT.

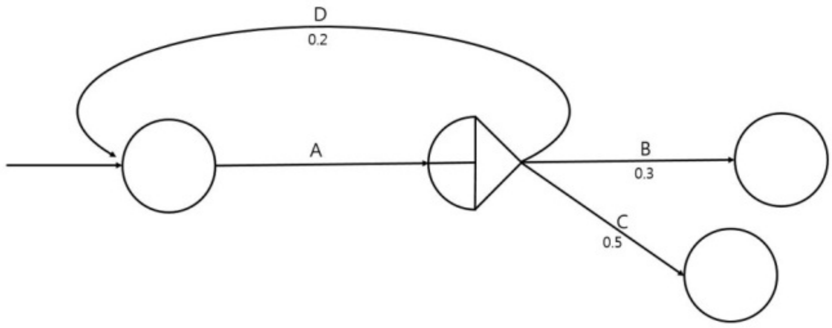


Fig. 2 Looping at GERT.

Table 1 Comparison of activity duration and linkage estimating methods.

No.	Comparison items	PERT	GERT	RDM	SAPA
1	Probabilistic time estimation	○	○	×	○
2	Overlapping relationship	×	×	○	×
3	Probabilistic linkage estimation	×	×	×	×

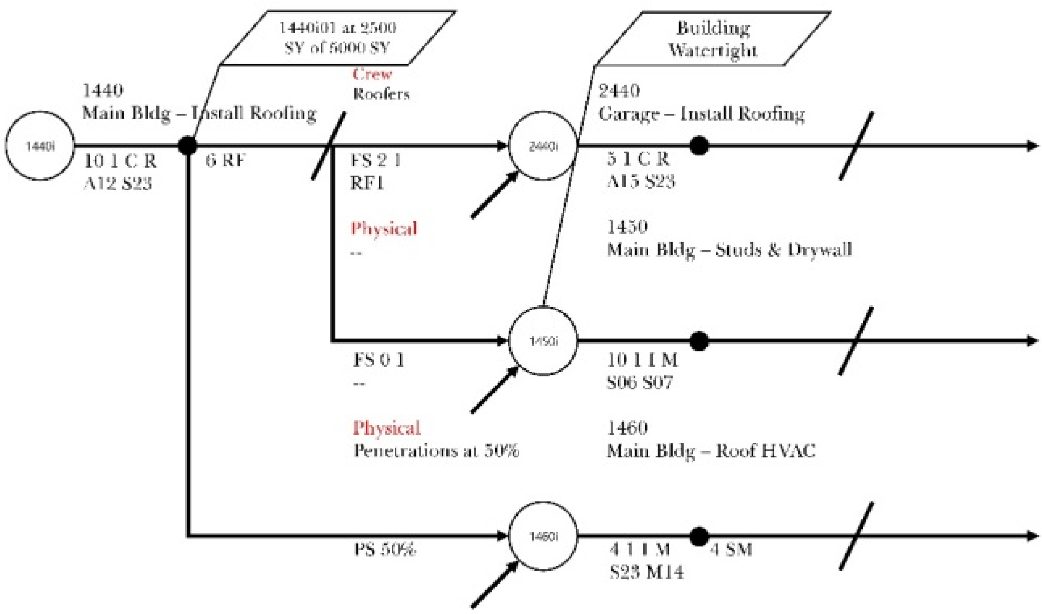


Fig. 3 Example of RDM.

The first is whether probabilistic estimation of activity duration is possible. PERT, GERT, and SAPA allow probabilistic estimation of the activity duration, since RDM does not provide such. Second, whether or not it is possible to express overlapping relationship between activities. PERT, GERT, and SAPA only express the relationship between activities as Finish-to-Start, so it is impossible to express overlapping relationship between activities. However, since RDM is based on PDM, it can not only represent overlapping relationship between activities, but also can describe overlapping relationship in detail. Third, it is whether or not the linkage between activities can be evaluated probabilistically. Since PERT, GERT, and SAPA have a linkage of Finish-to-Start between activities, probabilistic evaluation of the linkage is impossible in principle. RDM can explain the redundancy between activities in detail, but no probabilistic concept is introduced.

The results of the comparison of the existing activity duration and the linkage estimating methods are as follows. PERT, GERT, and SAPA allow probabilistic estimation of the activity duration, but probabilistic estimation of the linkage between activities is impossible.

4. Probabilistic Linkage Evaluation Technique (PLET)

4.1 Necessity of Probabilistic Linkage Evaluation

When establishing a schedule in the project management, estimating the linkage between the activities is as important and difficult as the estimation of activity duration. In general, the linkage between activities is judged based on past data such as the characteristics of the preceding or succeeding activities of similar projects or the speed of work. However, estimating the linkage between activities is not as easy as estimating the workspace and it is not very accessible to estimate the exact linkage between the preceding or succeeding activities due to the non-repetitive nature of the construction project and various variables depending on the external work environment.

In the ADM, since the logic between activities allows only a FS relationship, if all activities are decided in advance or afterward, it is only necessary to connect the relationship in terms of “FS0”. However, the overlapping relationships between the activities occur during the actual work of the construction project often because the succeeding activity is started even if the proceeding activity is not completed. In this case, the ADM cannot accurately represent overlapping relationships between activities, so the reliability of the relationship between activities cannot be reduced. On the other hand, since the PDM can accurately represent the overlapping relationships between the preceding and succeeding activities through the Start-to-Start (SS), Finish-to-Finish (FF), and Start-to-Finish (SF), it is possible to greatly improve the reliability. The PDM expresses the relationship between activities with an implicit number including “0”. However, it should also be considered that the relationship between activities is not simple or mechanical enough to be expressed as a definite number. For example, In the PDM, the relationship between the preceding and succeeding activities is expressed as “SS5” when the preceding activity is started and the succeeding activity is started 5 days later. However, this implies that it is likely to be more than a guarantee that it will be done. In this case, the linkage between activities also needs a probabilistic estimation like the activity duration. If we can probabilistically estimate the linkage between activities, not only will we provide additional options for estimating the project duration probability, but

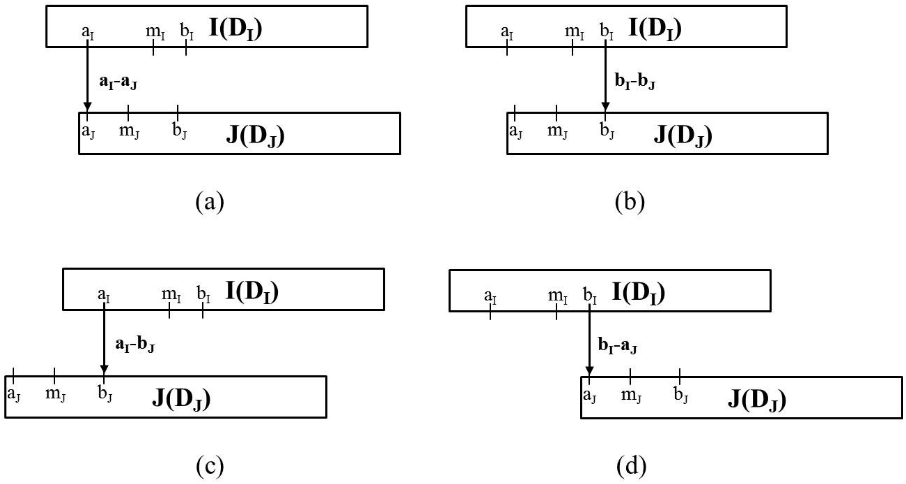


Fig. 4 Linkage of BDM under the uncertainties.

also the reliability of the probabilistic project duration estimates will be improved.

4.2Estimation of Uncertain Linkage in BDM

The BDM expresses the linkage between the preceding and succeeding activities as a straight line with the shortest distance from the intermediate point of the preceding activity to the arbitrary point of the succeeding activity.

In the BDM, the basic form for indicating the linkage is represented by "N-N". It is assumed that the number of days after the start of the preceding activity "I" and the number of days after the start of the succeeding activity "J" in the form of "N-N" are very uncertain.

And each uncertain elapsed day is expressed by three kinds of time points in a similar way to the activity duration estimation of the PERT. That is, the point at which the number of days elapsed since the start of the preceding activity "I" is referred to as "a_I" for minimum, "m_I" as the most probable point, and "b_I" for the maximum point. And, the point at which the number of days elapsed since the start of the succeeding activity "J" is referred to as "a_J" for minimum, "m_J" as the most probable point, and 'b_J' for the maximum point.

Fig. 4 depicts the uncertain elapsed days of the preceding activity "I" and the succeeding activity "J" as three types of points as described above, and expresses the four types of BDM linkages connecting the minimum and maximum points of the preceding and the succeeding activities.

Fig. 4a shows the linkage "a_I-a_J" connecting the time point (a_I) with the minimum elapsed days since the start of the preceding activity "I" and the time point (a_J) with the minimum number of days after the start of the succeeding activity "J". Fig. 4b shows the linkage "b_I-b_J" connecting the time point (b_I) with the maximum elapsed days since the start of the preceding activity "I" and the time point (b_J) with the maximum number of days after the start of the succeeding activity "J". Fig. 4c shows the linkage "a_I-b_J" connecting the time point (a_I) with the minimum elapsed days since the start of the preceding activity "I" and the time point (b_J) with the maximum number of days after the start of the succeeding activity "J", and Fig. 4d shows the linkage "b_I-a_J" connecting the time point (b_I) with the maximum elapsed days since the start of the preceding activity "I" and the time point (a_J) with the minimum number of days after the start of the succeeding activity "J".

If the uncertainty is inherent in the number of days elapsed in the preceding or succeeding activity through the above four kinds of linkage, the temporal change range of the linkage is from the minimum linkage (Fig. 4c) "a_I-b_J" to the maximum linkage (Fig.

4d) "b_I-b_J". Therefore, if there is uncertainty in estimating the linkage between the preceding and succeeding activities, the respective elapsed time points will change, and this change will directly affect the calculation of the schedule of the whole network.

4.3 BDM-Based Probabilistic Linkage Representation

This paper proposes "Probabilistic Linkage Evaluation Technique (PLET)" as a new method for probabilistic estimation of the linkage between activities containing uncertainty based on the BDM. Fig. 5 shows the probabilistic linkage representation format between the preceding and the succeeding activities in the PLET.

In Fig. 5, the "N-N" format of the BDM is changed to the form of the probabilistic linkage format "(a_I, m_I, b_I)-(a_J, m_J, b_J)" of the PLET, the detailed description is shown in Fig. 6.

Uncertain days elapsed after the preceding activity "I" was started are indicated by "(a_J, m_J, b_J)", "a" indicates the most optimistic elapsed days, "m" indicates the most likely elapsed days, "b" indicates the most pessimistic elapsed days, "$\bar{d}_I$" and "$\bar{d}_J$"in Fig. 5 indicates the average number of elapsed days in the preceding and the succeeding activity, respectively.

Charles E. Clark (1962), one of the founders of the PERT concept, estimates that the PERT activity duration is a beta distribution that is the closest to the natural phenomenon, and simplified mathematical equations calculating for the average duration, standard deviation, and variance are proposed as Eqs. (1)-(3) [3].

In the PLET, the probabilistic estimation method of elapsed days for the preceding and the succeeding activity is estimated to be the beta distribution closest to the natural phenomenon at three different time points as in the PERT. Therefore, the formula for estimating the elapsed days in the PLET can be directly applied to the equation of the PERT. Therefore, the formula for calculating the average number of days ($\bar{d}$), standard deviation (σ_d), and variance (σ_d^2) of the elapsed days in the PLET can be defined as Eqs.(4)-(6) using Eqs. (1)-(3).

$$\text{Average elapsed days: } \bar{d} = \frac{a + 4m + b}{6} \tag{4}$$

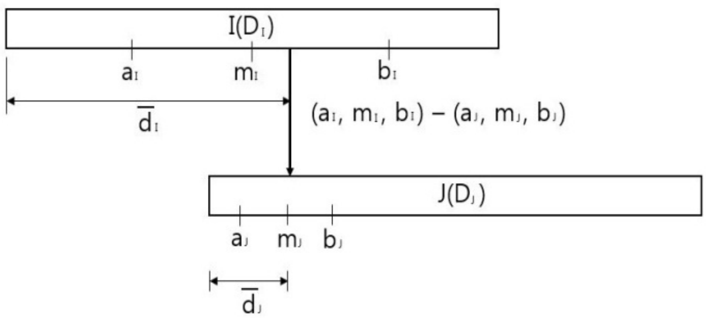


Fig. 5 Linkage representation of PLET.

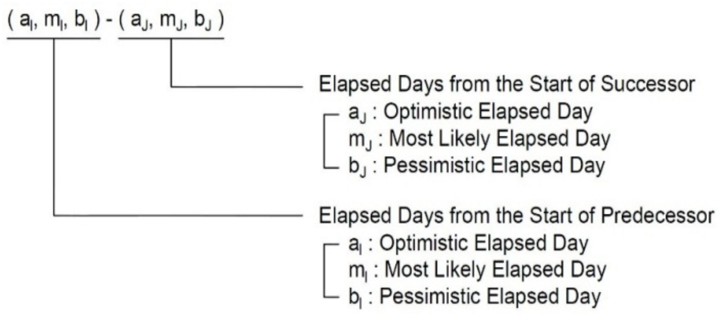


Fig. 6 Description of PLET linkage.

Standard deviation: $\sigma_d = \frac{b-a}{6}$ (5)

Variance: $\sigma_d^2 = \left(\frac{b-a}{6}\right)^2$ (6)

5. Schedule Computation of PLET

5.1 Forward Pass Computation

The forward pass computation of the PLET calculates the early start date (ESD) and the early finish date (EFD), which are early schedules of the succeeding activities, and the forward computation method of the BDM which is the basis of the PLET is applied as it is.

First, the formula for calculating the ESD_J and EFD_J of the succeeding activity in a single versus single relationship where the average elapsed days "$\bar{d}_I$" after the preceding activity "I" started and the average elapsed days "$\bar{d}_J$" after the start of the succeeding activity "J" is the same as Eqs. (7) and (8), where D_J is the average activity duration of the succeeding activity "J".

$$ESD_J = ESD_I + \bar{d}_I - \bar{d}_J \quad (7)$$

$$EFD_J = ESD_J + D_J \quad (8)$$

Next, when multiple preceding activities merge into a single succeeding activity, generalizing the forward calculation method of the PLET is equivalent to Eq. (9), where $\forall I$ means activity "I" as a whole.

$$ESD_J = \underset{\forall I}{Max}(ESD_I + \bar{d}_I - \bar{d}_J) \quad (9)$$

5.2 Backward Pass Computation

The backward pass computation of the PLET calculates the late start date (LSD) and the late finish date (LFD), which are late schedules of the preceding activities, and the backward computation method of the BDM technique which is the basis of the PLET is applied as it is.

First, the formula for calculating the LSD_I and LFD_I of the preceding activity in a single versus single relationship where the average elapsed days "$\bar{d}_I$" after the preceding activity "I" started and the average elapsed days "$\bar{d}_J$" after the start of the succeeding activity "J" is the same as Eqs. (10) and (11), where D_I is the average activity duration of the preceding activity "I".

$$LSD_I = LSD_J + \bar{d}_J - \bar{d}_I \quad (10)$$

$$LFD_I = LSD_I + D_I \quad (11)$$

Next, when a single preceding activity bursts into multiple succeeding activities, generalizing the backward calculation method of the PLET is equivalent to Eq. (12), where $\forall J$ means activity "J" as a whole.

$$LSD_I = \underset{\forall J}{Min}(LSD_J + \bar{d}_J - \bar{d}_I) \quad (12)$$

5.3 Computations of Free Float and Total Float

The free float (FF) is defined as the margin time of the preceding activity without affecting the early start date of the succeeding activity. There is a difference between the early start date (ESD) of the succeeding activity and the early completion date (EFD) of the preceding activity in the network forward computation process, which is called "Link Lag" [3]. In the PLET, Link Lag can be defined as the difference between the linking points of the preceding and the succeeding activities as shown in Eq. (13).

$$LAG_{IJ} = (ESD_J + \bar{d}_J) - (ESD_I + \bar{d}_I) \quad (13)$$

The free float (FF) is the minimum Link Lag value of the activity [3]. If a preceding activity "I" is associated with multiple succeeding activities "J", then the free float FF_I of activity "I" is the minimum LAG_{IJ} as shown in Eq. (14), where $\forall J$ means activity "J" as a whole.

$$FF_I = \underset{\forall J}{Min} LAG_{IJ}$$

$$= \underset{\forall J}{Min}((ESD_J + \bar{d}_J) - (ESD_I + \bar{d}_I)) \quad (14)$$

The total float (TF) is the amount of time that affects the early start date (ESD) of the succeeding activity but does not affect the completion time of the entire network. The total float is calculated as the difference between the forward and backward pass computations, which is the maximum amount of time an activity can have. In other words, the TF_I of activity "I" is the difference between LSD_I and ESD_I, or the difference between LFD_I and EFD_I as shown in Eq. (15).

$$TF_I = LSD_I - ESD_I = LFD_I - EFD_I \quad (15)$$

5.4 Calculations of Average Duration, Variance, and Standard Deviation of Schedule Path

Although the schedule computation of the PLET is very similar to the BDM, the calculation method of the average duration (t_e), variance (σ^2), and standard deviation (σ) of the schedule path, which is the sequential activities in the PLET network, differs greatly from the BDM, because the elapsed days of the PLET are assumed to be a probability distribution.

The process of calculating the average duration "$(t_e)_{Path}$", of the schedule path is as follows. First, the average duration of the schedule path is applied continuously Eq. (7) to calculate the ESD of each activity in the case of "N-N" linkage type. Second, if the linkage type "<0>" is included in the schedule path, it can be expressed as "N-0" in "N-N" format, so the average duration "(t_e)" of the preceding activity is added to the average duration of the schedule path. Thirdly, add the last average activity duration of the schedule path. The above process can be expressed as shown in Eq. (16).

$$(t_e)_{Path} = \sum Apply\ \text{“Eq.(7)” (if “N-N”)} + \sum \text{Predecessor's Average Duration (if “<0>”)} + \sum \text{Last Activity's Average Duration} \quad (16)$$

The variance "σ^2_{Path}" over the average duration of the schedule path is the sum of the variances over the average elapsed days for each activity shown in Eq. (17), and the standard deviation "σ_{Path}" for this is calculated as shown in Eq. (18).

$$\sigma^2_{Path} = \sum \text{Average Duration Variance } (\sigma^2) \quad (17)$$

$$\sigma_{Path} = \sqrt{\sigma^2} \quad (18)$$

Assuming a schedule path connecting activities "H", "I", "J", "K", and "L" in the PLET network, the average elapsed days and variances of each activity are shown in Fig. 7.

The average duration "$(t_e)_{HL}$" of the schedule path H-I-J-K-L is expressed as Eq. (19) by applying Eq. (16).

$$(t_e)_{HL} = \bar{d}_H - \bar{d}_{I1} + \bar{d}_{I2} - \bar{d}_{J1} + \bar{d}_{J2} - \bar{d}_{K1} + \bar{d}_{K2} - \bar{d}_L + D_I \quad (19)$$

The average duration variance "σ^2_{HL}" of the schedule path H-I-J-K-L is the sum of the variances of the average elapsed days of each activity as shown in Eq. (20) by applying Eq. (17), and the standard deviation "σ_{HL}" of this is expressed as Eq. (21) by applying Eq. (18).

$$\sigma^2_{HL} = \sigma^2_H + \sigma^2_{I1} + \sigma^2_{I2} + \sigma^2_{J1} + \sigma^2_{J2} + \sigma^2_{K1} + \sigma^2_{K2} + \sigma^2_L \quad (20)$$

$$\sigma_{HL} = \sqrt{\sigma^2_{HL}} \quad (21)$$

6. Verification of PLET

In order to verify the PLET proposed in this study, the PLET network is constructed as shown in Fig. 8 after the plastering work in the apartment unit finishing construction. The PLET network in Fig. 8 consists of a total of 10 activities and a total of 11 linkages, of which there are 8 probabilistic linkages as shown in Fig. 6.

For example, the preceding activity (Floor Tiles) and the succeeding activity (Painting) have a probabilistic linkage of "(7,9,10)-(0,1,3)". This means that they are linked from the optimistic elapsed days of "7", the most likely elapsed days of "9", and the pessimistic elapsed days of "10" after the start of the floor tiles work to the optimistic elapsed days of "0", the most likely elapsed days of "1", and the pessimistic elapsed days of "3" after the start of the painting work.

Table 2 is the results of the schedule computation for the PLET network in Fig. 8. It shows the results of calculating the average elapsed days ($\bar{d}$), variance (σ^2), standard deviation (σ), the early and late dates (ESD, EFD, LSD, LFD), and the float (FF, TF) of each activity applying Eqs. (7)-(15) proposed in this study.

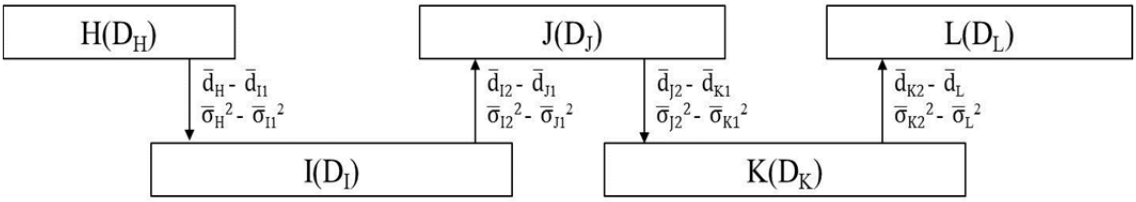


Fig. 7 Path's average elapsed days and variances of PLET.

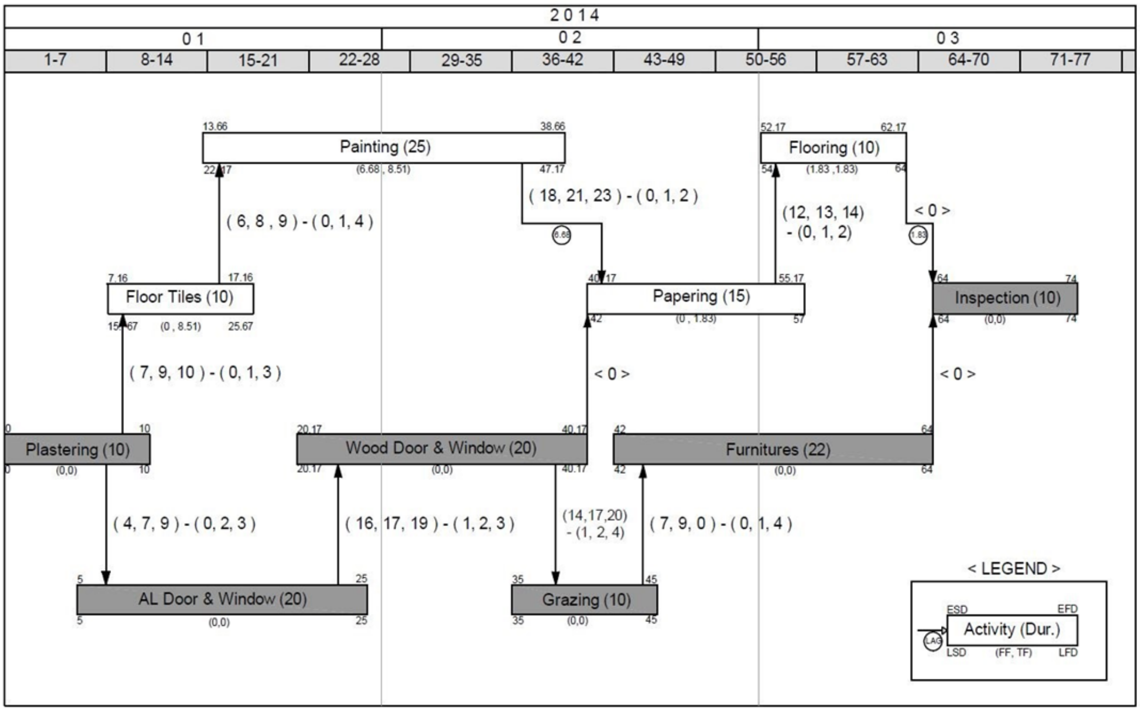


Fig. 8 A sample PLET network.

Table 2 Schedule computation results of sample PLET network.

No.	Activity Description └ Predecessors	Dur.	Prob. linkage	Avg.E. days	Var.	S.D.	Schedule computation dates and floats						
		days	(a,m,b)	$\bar{d}$	σ^2	σ	ESD	EFD	LSD	LFD	FF	TF	C.P.
1	Plastering	10	-	-	-	-	0	10	0	10	0	0	○
	└ none		-	-	-	-							
2	Floor tiles	10	(0,1,3)	1.17	0.25	0.50	7.16	17.16	15.67	25.67	0	8.51	
	└ plastering		(7,9,10)	8.83	0.25	0.50							
3	Painting	25	(0,1,4)	1.33	0.45	0.67	13.66	38.66	22.17	47.17	6.68	8.51	
	└ floor tiles		(6,8,9)	7.83	0.25	0.50							
4	AL door &window	20	(0,2,3)	1.83	0.25	0.50	5	25	5	25	0	0	○
	└ plastering		(4,7,9)	6.83	0.69	0.83							
5	WD door &window	20	(1,2,3)	2.00	0.11	0.33	20.17	40.17	20.17	40.17	0	0	○
	└ AL door &window		(16,17,19)	17.17	0.25	0.50							
6	Glazing	10	(1,2,4)	2.17	0.25	0.50	35	45	35	45	0	0	○
	└ WD door &window		(14,17,20)	17.00	1.00	1.00							

(Table 2 continues)

7	Papering	15	(0,1,2)	1.00	0.11	0.33	40.17	55.17	42	57	0	1.83	
	└ painting		(18,21,23)	20.83	0.69	0.83							
	└ WD door &window		0	0.00	0.00	0.00							
8	Furniture	22	(0,1,4)	1.33	0.45	0.67	42	64	42	64	0	0	○
	└ glazing		(7,9,10)	8,33	0.25	0.50							
9	Flooring	10	(0,1,2)	1.00	0.11	0.33	52.17	62.17	54	64	1.83	1.83	
	└ papering		(12,13,14)	13.00	0.11	0.33							
10	Inspection	10	-	-	-	-	64	74	64	74	0	0	○
	└ furniture		22	0.00	0.00	0.00							
	└ flooring		10	0.00	0.00	0.00							

Dur. = duration; Prob. = probabilistic; Avg. = average; E. = elapsed; Var. = variance; S.D. = standard deviation; C.P. = critical path.

As a result of schedule computations, the critical path (C.P.) where FF and TF are both “0” is “Plastering → AL Door & Window → WD Door & Window → Glazing → Furniture → Inspection”, and the average duration (t_e), deviation (σ^2), and standard deviation (σ) of the critical path are calculated by applying Eqs. (16)-(18) as follows:

$$t_e = 6.33 - 1.83 + 17.17 - 2.00 + 17.00$$
$$-2.17 + 8.33 - 1.33 + 22(< 0 > Linkage)$$
$$+10\ (LastActivity) \approx 74$$
$$\sigma^2 = 0.69 + 0.25 + 0.25 + 0.11 + 1.00$$
$$+0.25 + 0.25 + 0.45 \approx 3.25$$
$$\sigma = \sqrt{\sigma^2} = \sqrt{3.25^2} \approx 1.80$$

7. Conclusions

The increasing complexity and diverse interests of the construction environment are gradually causing it to be difficult to establish realistic and sophisticated schedules, as they dramatically increase the uncertainty and risks of construction projects.

In case of uncertain construction environment or lack of past data or inaccurate prediction about the future, the PERT is widely applied as a method of probabilistic estimation of the project duration. The PERT is basically a technique for probabilistically evaluating the project duration by probabilistically estimating the activity duration after confirming the relationship between the activities of the network to Finish-to-Start. In addition, GERT, PERT, and Monte Carlo simulations have been proposed to evaluate the overall project duration probability, some of which have been widely used in practice.

However, when establishing a schedule, it is not only impossible to display the interconnection of activity as Finish-to-Start, which means estimation of activity duration is not only difficult and uncertain task in establishing the schedule. It is also very difficult and uncertain to estimate the linkages between activities in situations that can express overlapping relationship.

In this study, we proposed Probabilistic Linkage Evaluation Technique (PLET), which is a new method for probabilistic estimation of total project duration by probabilistic estimation of the relationship between activities expressed by the BDM. In addition, the linkage representation and the schedule computation method of the PLET are also presented and they are verified through the sample PLET network.

This study is meaningful that extends to estimate the project duration based on the probabilistic linkage estimation between activities from estimating the project duration based on the probabilistic duration estimation of an activity of the PERT. And this new approach is expected to support more flexible project duration estimates when establishing a schedule under uncertain construction environments. Furthermore, it will be necessary to continue and further research on how to integrate existing PERT and new PLET techniques and how to implement it if integration is possible.

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