

Influence of Delayed Start on $1/f$ Noise in Vehicular Traffic Patterns

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Abstract: Previously we examined $1/f$ noise for a simple cellular automata model. For illustrative purposes we considered a specific case of approaching a city. The case involves a traffic light where one continues on the main road, into which additional cars are entering at the light. At this intersection an alternative route begins, which is longer but into which no additional cars are entering. In this paper we add a modified “Slow to move” model. We check the influence of different percentage of cars which have slow to continue values, on the overall velocity and flux. We calculate the Fourier transform of the average velocity for each traffic light cycle. All the cases can be written as $1/f^a$. We check by least squares the value of a . We compare qualitatively our results to experiments. When we do not assume cars which are “delayed”, the results differ from experiment, but when we introduce the delay mechanism, the results are similar to the experimental values. These values give a close to one, this is called pink noise. We consider too different densities of cars. There are different characteristics for low densities mid range and high densities. We wish to point out that when the autonomous cars, i.e. cars without a human driver, will enter into use, the simple case without delayed cars will become dominant and so the noise will have brown sections at some densities instead of pink sections.

Key words: Traffic noise, pink noise, $1/f$ noise, cellular automata, automatic driving, brown noise.

1. Introduction

Observations show that at high enough densities traffic behaviour becomes complex. Therefore, cellular automata, which was first studied by Ulam and von Neumann [1], is one of the most prevalent methods for evaluating traffic. This is because of their speed and complex dynamic behaviour. An important contribution to the field was that of S. Wolfram [2] who introduced classifications, used in this study. The elementary cellular automaton is a collection of cells arranged on a one dimensional array. Each cell can obtain just two possible numbers: one and zero. The “time” is discreet and at each time step all the cell values are updated synchronously. The value of each cell depends just on the values in the previous step of that cell and its two neighbours. Wolfram names each elementary cellular automaton with a binary numeral, which he calls: “rule”. This value results from reading

the output when the inputs are lexicographically ordered. This will become clearer when we will explain the rules which we use.

The most basic model, for traffic, was the paper by Biham et al. [3]. In that model a density is assigned to a one dimensional chain of sites wherein particles are distributed satisfying this density. At each time step each particle can move to the right provided that that site is not occupied. This model enabled to distinguish between free flow at low densities and a jammed region at high densities. A more realistic model was suggested by Nagel and Schreckenberg [4] who introduced several modes for the car velocity. A different approach was taken by Takayasu and Takayasu [5] who introduced the effect of delay by allowing a car to move only if the next and after the next site are empty. More recently Nassab et al. [6] introduced additional connection sites into what is essentially the Nagel and Schreckenberg model.

In a previous study [7], we examined a specific traffic problem. In this study we add to our previous

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calculations the “slow to start” model [8]. In our model each time a car moves a certain percentage is unable to move. We wish to stress that this gives us a delay just as in the two previously described models and seems to us simpler. In our opinion this justifies using our model.

To make our exposition clearer we describe in a shorter version our previous study. The rules we used are taken from the cellular automata model as proposed by Gershenson and Rosenblueth [9]. Our main interest in this paper is the power spectrum of the average velocity over a cycle which gives us the main contribution to the noise. All the cases can be written as $1/f^a$ where we check a using least squares. This will be clarified in the section dedicated to calculating the noise.

2. The Model

We will deal here only with the “microscopic” models that were we consider each individual vehicle. Our highways are represented by an array of cells, each cell has the values zero or one. One represents a vehicle and zero represents an empty portion of the highway. We assume that the magnitude of a cell corresponds to the average length of a vehicle. We assume that we have two traffic lights. At the first light we have two ways to proceed, straight or through a bypass. At the second light the two routes merge again. After the first light there is a possibility of additional cars to enter the main road. These cars are

removed when approaching the city. This procedure ensures that overall the number of vehicles is preserved. In Fig. 1, we show the layout of our model.

In this figure “IN 1” is the main car movement and we assume that the main outflow (“OUT 1”) of cars is the same as the number entering. The first junction is light 1 and after the intersection, additional cars enter (“IN 2”), this is the same number of cars as leaving at “OUT 2”. This procedure keeps the overall number of cars constant. At light 1 we have the possibility of cars to move to the bypass at light 2 and they again combine with the main road.

The rules, which are the same as used by Gershenson and Rosenblueth [9], are given in Table 1.

In the next figure (Fig. 2) we show schematically how the rules which are used in our calculations are implemented.

In this paper we have one modification, which is quite significant. Previously the model enabled a car to move if there was an empty space in front of it. In this study we assume that a certain part of the cars do not follow into the empty space in front of them. This is equivalent to having some cars which are slower to move than others. This is a more realistic model than the one we used in our previous study. We have a parameter telling us what part of cars do not move even when they could move according to our rules. We denote this parameter by crr , and it changes between zero and one.

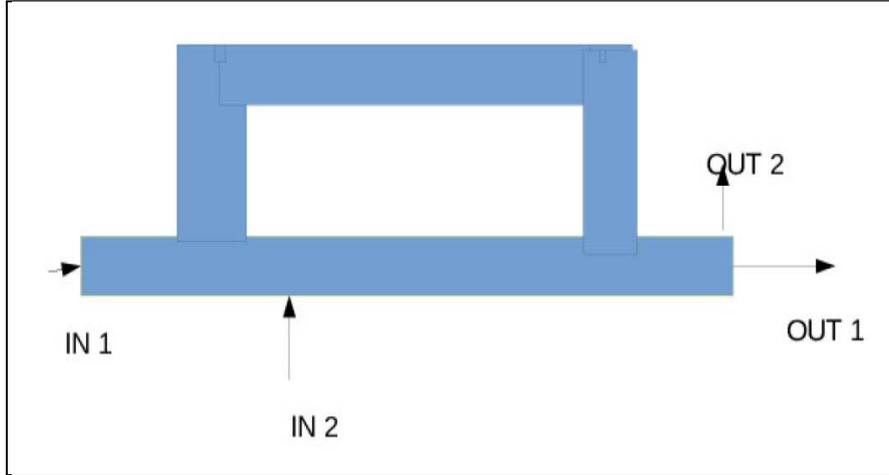


Fig. 1 The movement of vehicles.

Table 1 Wolfram rules used in this model.

$t-1$	t_{184}	t_{252}	t_{136}
000	0	0	0
001	0	0	0
010	0	1	0
011	1	1	1
100	1	1	0
101	1	1	0
110	0	1	0
111	1	1	1

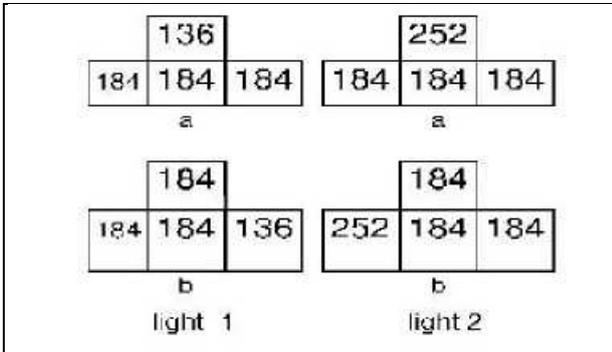


Fig. 2 The movement of vehicles.

In the discussion section we will point out that when in the future traffic will be dominated by self driven cars, we will return to our older model.

2.1 Measures

The density, p , is given by the number of “ones” (i.e. vehicles) divided by the general number of cells. Initially we take this value to be the same for the three sections. In this study we are interested only in the region beyond the second traffic light. Here we are

interested only in the equilibrium values. The velocities are given by the number of cells which change in one step from 0 to 1.

In our calculation, space and time are just abstract quantities. Still if concrete numbers are desired, one can quote [9] that one cell represents five meters, and a time step represents a third of a second, which gives us about 50 km/hour, roughly the speed limit within a city.

3. Noise

Traffic noise is one of the most important sources of noise pollution. It is well known that this is a health hazard. In this study we wish to check the frequency distribution of the noise. It was shown by Ref. [5] that we obtain 1/f noise. Let us explain here this term: “1/f noise” refers to the phenomenon of the spectral density, $S(f)$, of a stochastic process having the form:

$$S(f) = \text{const.}/f^a$$

When $a = 0$ we say that we have white noise. If $a = 1$ we say we have pink noise. If $a = 2$ we say we have brown noise. To understand better this term seen Refs. [10-11]. An Indian group [12] made measurements in a number of selected locations from busy roads of Aurangabad and obtained a mostly pink noise in a large range of frequencies. To obtain $S(f)$ we make the Fourier transform of the velocities. To perform our Fourier transform we take the averages over each light cycle and study the frequencies of these averages over

all the cycles taken in our calculations. We compare the results to the $1/f^a$ by a least square test.

4. Results and Discussion

We used a fixed grid: The main road was comprised of 1,200 cells, the “by pass” 300 cells and the distance between the two lights was 120 cells. We used the “green wave” regime. As we have just two lights it was shown by Gershenson and Rosenblueth [9] that in this case one does not get different results using the “self-organizing” regime.

The number of cycles we have to use in order to obtain significant results depends on the parameter that we wish to calculate. For calculating the velocity or the flux 32,000 cycles give quite accurate results but for the noise calculations we need 128,000 and sometimes even 256,000 cycles. We determine the number of cycles needed by taking a second sample, with different random numbers, and comparing the values thus obtained.

We introduce a vehicle on the first intersection for 40% of the steps and we eliminate the same number of vehicles on the last point of our main route, again per unit time.

On this first intersection we enable cars to go by the bypass for a longer time then on the main road. We denote by jw the additional time one can go on the bypass per light cycle. As the cycle is composed of 12 steps, the jw can have the numbers between 1 and 12.

In Fig. 3, we give a as function of jw for $crr = 0$ and $crr = 0.5$. The density which we use in this figure is $p = 0.55$. It is interesting to note that for $crr = 0.5$ and there is no change as we change jw .

In the next two figures we show the results for three cases of “delayed start”. We denote by crr , the part of vehicles which are delayed. The $crr = 0$ figures appeared in our previous paper, here we show the change with crr . The purpose of showing them here is to understand better the effect of delay mechanism. In Fig. 4 we show the change in velocity of the main road, after the second traffic light (vt), as function of

the car density. In this case we assume the same duration of the red and green lights at the first intersection.

We see in this figure that the average velocity changes from free flow to the jammed region at about $p = 0.6$, for the case that there is no part which is delayed. For higher crr the change is a little earlier and more abrupt. In the next figure (Fig. 5), we show the change of the appropriate a flux as function of the density for different crr .

In this figure we see very clearly the lowering of the flux for higher crr .

We averaged the velocities over a traffic light cycle and studied the power spectrum. The value we are interested in is a , in the expression $1/f^a$. In Fig. 6 we show the results for two cases, the case of no slow to start cars and in the case of 50 percent delayed to continue cars. The difference between these two cases is striking.

This is an interesting result. For the $crr = 0$ case, when we increase the density so that we reach the transition from free flow to the jammed region the noise shoots up from white noise to brown noise and then settles in the region of pink noise. This result does not correspond to a real situation where a is close to one [12]. We see clearly that for the $crr = 0.5$ the

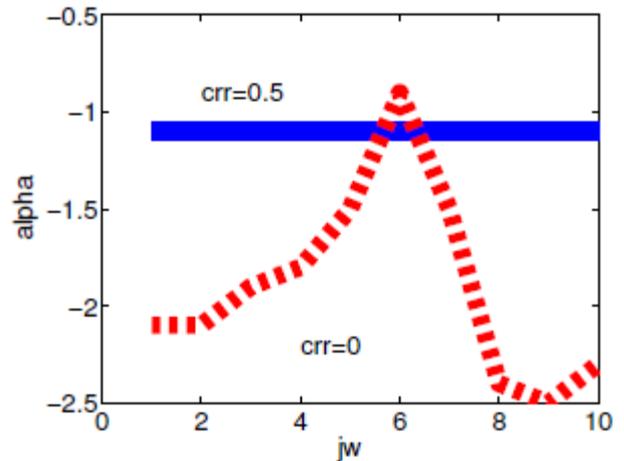


Fig. 3 The change in alpha as a change in jw .

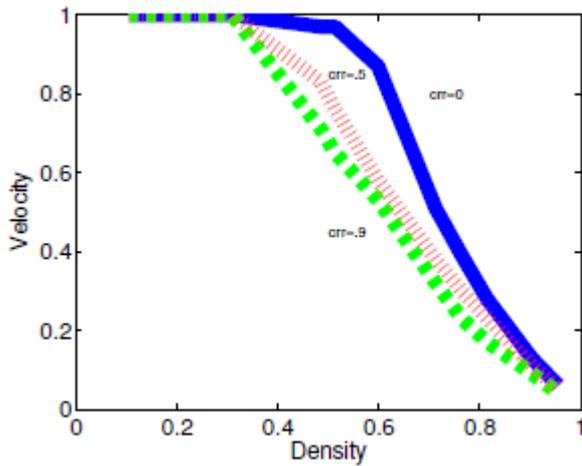


Fig. 4 The change in velocity as function of the density.

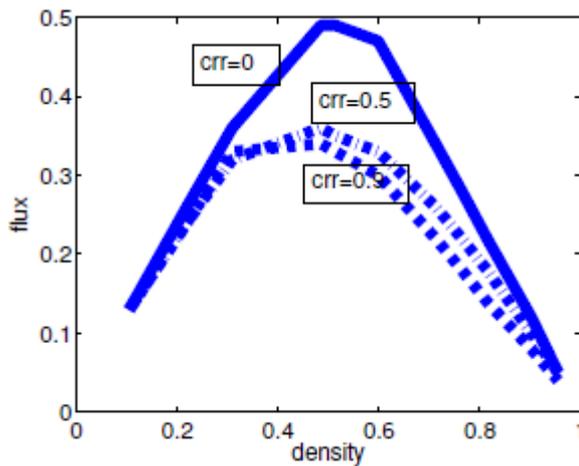


Fig. 5 The change in flux as function of the density.

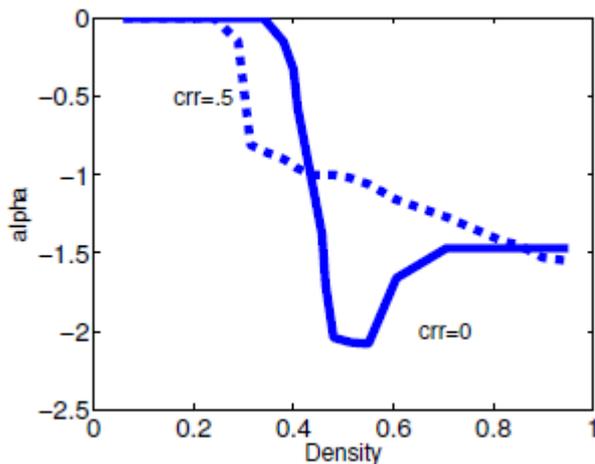


Fig. 6 The values of a as a function of density.

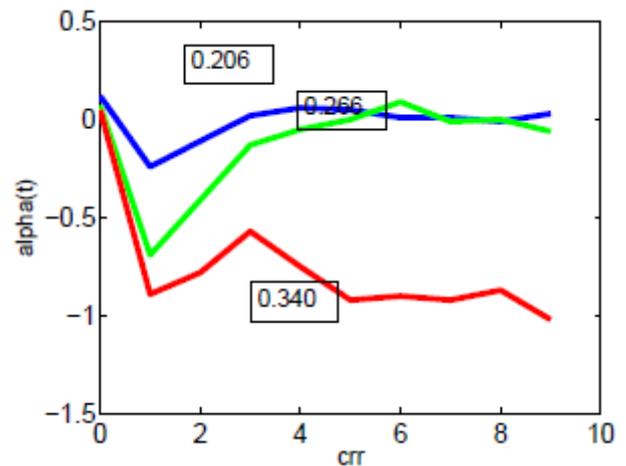


Fig. 7 a values as function of change in probability of slow to start values for low densities.

result is much more similar to the experimental results. In the next figures we examine the delayed effect in a different way. In Fig. 5 we show the values of α for the low densities. This is the region of free movement therefore there is no significant change for different delayed scenarios.

In Figs. 8 and 9 we present the results for a at mid range densities and for high densities.

We see in Fig. 8 that even a very small percentage of cars which are slow to start results in a being closer to the pink noise just as that given by the experimental results [12].

In Fig. 10 we present the fluxes for three densities, low, mid and high values.

Considering these figures, we can say that our calculations give us a wide range of information which can be applied for specific cases. It shows us the importance of modifying the simple rule by adding the “slow to move” rule. This rule enables us to avoid the complication of having different velocities. The advantage of the method and results given here is in the simplicity of the model. As we approach the jammed region the traffic load fluctuates widely and exhibits self similar structure. Therefore it is quite understandable that one obtains $a = 1$. Thus our results for a are consistent with those by Takayasu and Takayasu [5]. And also the paper by K. Fukuda et al. [13] which is concerned with data traffic, but

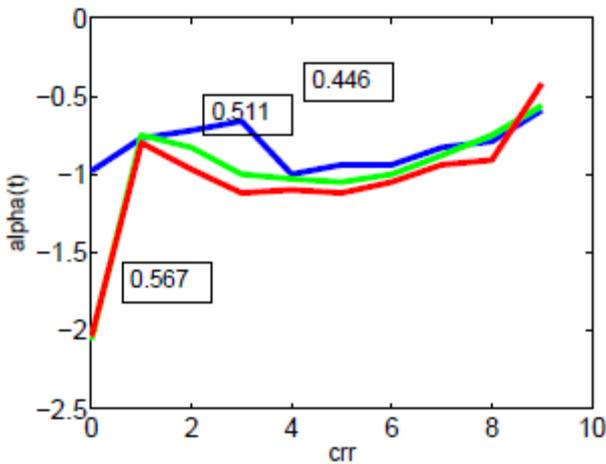


Fig. 8 The values of a as a function of change in probability of slow to start at mid range densities.

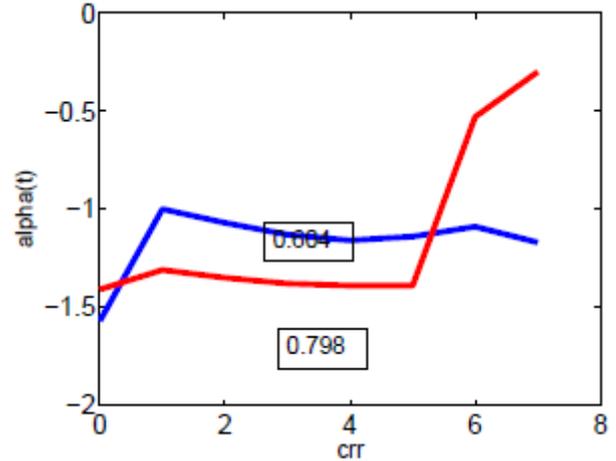


Fig. 9 The values of a as a function of change in probability of slow to start at high densities.

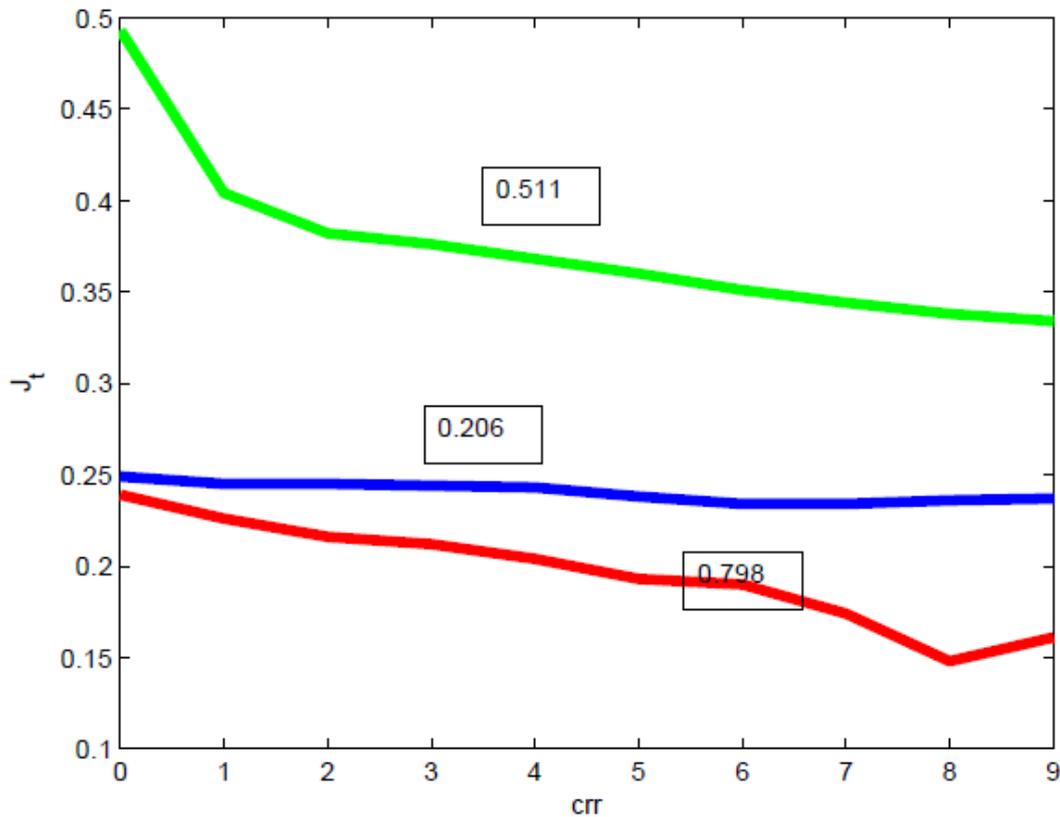


Fig. 10 The values of flux as a function of change in probability of slow to start at different densities.

according to F. Mayinger [14] that should give the same kind of results.

However, in conclusion, we would like to say that the method used here will give us the noise for the self driven cars if we assume that no delay to move cases will be found. That means that we should expect that

the self driven cars will change the noise spectra from pink noise to brown noise, in part of the densities.

Acknowledgments

We wish to thank Professor Charles Korn for valuable discussions.

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