Journal of Modern Accounting and Auditing, ISSN 1548-6583
May 2014, Vol. 10, No. 5, 551-559

Probabilistic Evaluation for Commercial Real Estate Investments

Lin Jeng-hsiang
Hwa Hsia Institute of Technology, New Taipei City, Taiwan

Throughout the decision-making process, prudent investors must address questions regarding the probabilities of future profit gain. In this study, the probability-based discounted cash flow (DCF) method with net present value (NPV) as the indicator was adopted as an analysis tool. A probabilistic framework for measuring exceeding probability of annual rate of return on a commercial real estate investment under a specified holding period was developed. Based on the framework, the relation curves of annual rate of return versus the corresponding exceeding probability of return for available financing schemes were constructed. These curves were used as a tool to prioritize the schemes and inform decision-making. An example case is presented to demonstrate the decision-making process developed in this study. Through the proposed process, investors are given basic information on the return, probability that profit gain will occur, and feasibility of financial schemes for commercial real estate investments.

Keywords: commercial real estate, investment, decision-making, probability

Introduction

There are difficulties inherent in the development of a complete financial analysis model for the evaluation of commercial real estate investments. Uncertainty is caused by a general lack of knowledge and poor or inaccurate information about the inputs that appropriate financial models require (French & Gabrielli, 2004; Byrne & Cadman, 1996). An appropriate model for the evaluation of real estate investments must consider the usual factors: vacancy rate, market rental rate, operating expenses, brokerage fees, loan interest rate, repayment model, tax rate, and length of holding period.

The discounted cash flow (DCF) method is well accepted by academics and broadly used by practitioners to evaluate whether the return on an investment is commensurate with the risk. The traditional DCF method is deterministic and does not take into account the wide year-to-year stochastic force variations that influence inflow and outflow. Future cash flow might therefore be better considered as a stochastic factor in simulations (French & Gabrielli, 2004). The Monte Carlo (MC) simulation method (Metropolis & Ulam, 1949; Rubinstein & Kroese, 2007) has been applied as a forecasting tool for future cash flows in order to improve long-term real estate investment decisions (Kelliher & Mahoney, 2000). The MC method is often used in risk and decision analysis to imitate uncertainties in market trends, fluctuations, and other indeterminate factors (Tucker, 2001; Hoesli, Jani, & Bender, 2006).

Profitability is one of the most important factors that must be considered in an investment decision-making process. For an investment to be worthwhile, the risk-adjusted return on capital must be higher than the cost of capital, and any appropriate model must incorporate not just projected returns but also the probabilities of those

Lin Jeng-hsiang, professor, Department of Assets and Property Management, Hwa Hsia Institute of Technology. Email: hsiang@cc.hwh.edu.tw.

projections. The general purpose of real estate investments is to gain profits under acceptable levels of risk. Throughout the decision-making process, prudent investors must address questions regarding the probabilities of future profit gain. Using appropriate financial models, an investor can assess the feasibility of candidate projects to select the optimal project. These issues are best answered with the utilization of modern analysis tools. The primary objective of this study is to provide an analytic method to explore these issues.

Methodology

In this study, the probability-based DCF method (Nygard & Razaire, 1999) with net present value (NPV) as the indicator was adopted as an analysis tool. A probabilistic framework for measuring exceeding probability of annual rate of return on a commercial real estate investment under a specified holding period was developed. The sources of cash inflows include the loan amount, the rental incomes over the holding period, and the liquidation value at the end of the holding period. Capital expenses, operational expenses, brokerage costs, principal and interest payments, and taxes over the holding period are the major sources of cash outflows. Annual potential gross rental income was considered as a random process; NPV was considered as a random variable. Thus, the probability-based DCF analysis does not merely yield a point estimate of the property value but rather an entire distribution of values.

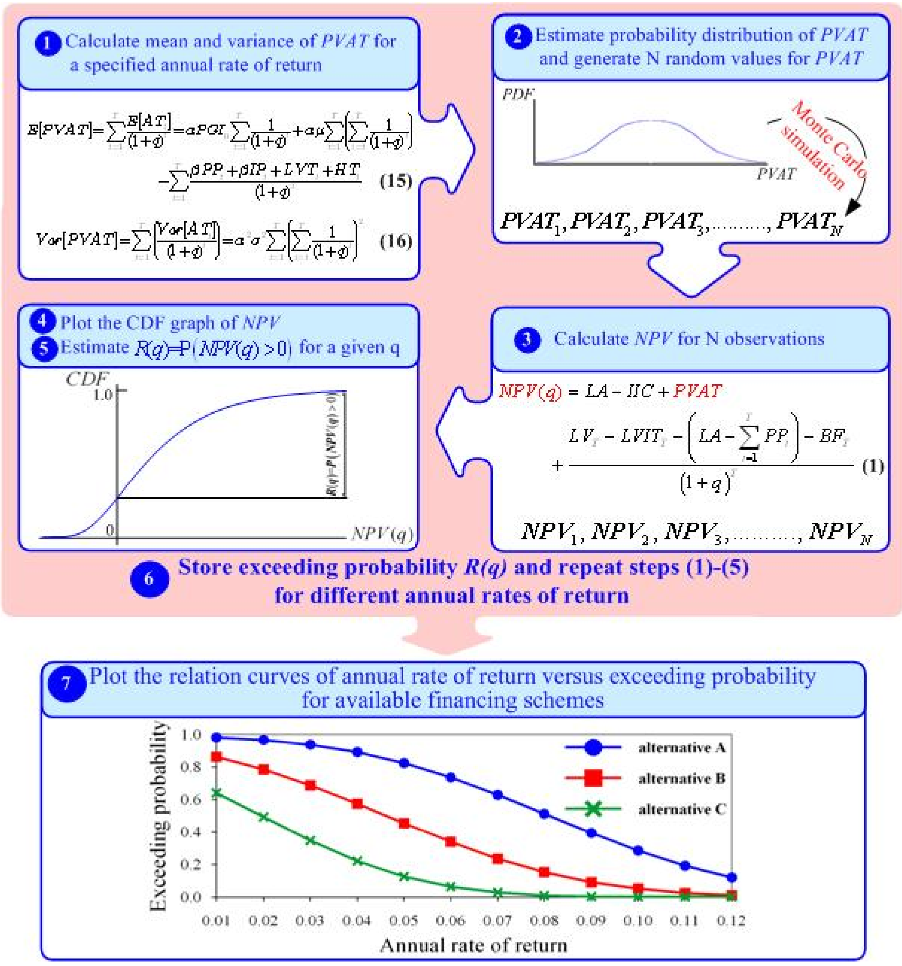

Figure 1. Flowchart showing the procedure for constructing the relation curves of annual rate of return versus corresponding exceeding probability (reliability) of return for available financing schemes.

Using the MC simulation technique, this framework repeatedly runs a sampling process to construct relation curves measuring annual rates of return versus the corresponding exceeding probability for all available financing schemes for the property. Figure 1 is a flowchart showing the procedure for constructing these curves. From a cost-benefit viewpoint, the curves can not only be used to assess investment project feasibility, but can also be used as a tool to select the optimum financing schemes for the property. Investment decision-making recommendations and feasibility evaluations are based on two criteria set by the investor: required rate of return and the corresponding exceeding probability of return. The exceeding probability of return on investment represents the reliability that the investor will gain returns on an investment.

NPV and Internal Rate of Return (IRR)

NPV and IRR are two commonly used indices that take into account the timing and the magnitude of yearly cash flow, helping investors to select projects that will generate maximum return. The two indices were used in the model developed in this study.

Commercial real estate purchases begin with huge capital outlays. Then, debt repayment, depreciation of any buildings on the property, and appreciation of land take place over time, which cause annual changes in the net worth of the property. Starting from the loan drawdown date, repayment will be amortized evenly every one-year period. By using weighted averages of the costs of debt and equity capital, NPV combines multiple cash flow streams into one key output variable that more accurately reflects future returns on multi-year investments. NPV takes inflation and estimated future returns into account to discount the present value of an investment, while also taking opportunity costs of the capital into consideration. As such, NPV helps to answer the question of whether a possible investment project will give the investor a higher return than if that investor devoted the same capital elsewhere. If NPV is greater than zero, the project is acceptable.

In this study, the NPV formula can be expressed as follows:

$$NPV = LA - IIC + \sum_{t=1}^{T}\left(\frac{AT_t}{(1+q)^t}\right) + \frac{LV_T - LVIT_T - (LA - \sum_{t=1}^{T} PP_t) - BF_T}{(1+q)^T} \tag{1}$$

$$IIC = IPP + DT + BF_0 \tag{2}$$

$$AT_t = BT_t - ICT_t - LVT_t - HT_t \tag{3}$$

$$ICT_t = \chi(BT_t - DE_t) \tag{4}$$

$$BT_t = NI_t - PP_t - IP_t \tag{5}$$

$$NI_t = (1 - \nu_t)PGI_t - OE_t \tag{6}$$

$$OE_t = k(1 - \nu_t)PGI_t \tag{7}$$

where *NPV* is the net present value; *LA* is the loan amount; *IIC* is the initial investment cost; *IPP* is the initial purchase price; DT is the deed tax; BF_0 and BF_T are the brokerage fees paid upon the purchase and sale, respectively; T is the length of the holding period; AT_t is the after-tax cash flow at the t^{th} year ($t = 1, 2, \ldots, T$); LV_T is the liquidation value or the nominal net worth of the property in the last year of the planning horizon; $LVIT_T$ is the land value increment tax; PP_t is the principal payment; IP_t is the interest payment; q is the annual discount rate; BT_t is the before-tax cash flow; ICT_t is the income tax; χ is the income tax rate; DE_t is the deductible expenses which include costs associated with running a rental business, costs of improvement and maintenance of the property, depreciation, and interest payment. These expenses are usually deductible if the business is operated to make a profit. DE_t is taken as $0.43BT_t$. LVT_t and HT_t are the land value tax and the house tax at the t^{th} year, respectively; NI_t is the annual net return or the net income stream withdrawn from the rental business; ν_t is the vacancy rate; OE_t is the operating expense; and k is the operating expense ratio. Vacancy conditions vary considerably from area to area. PGI_t is the potential gross rental income at the t^{th} year.

In this study, the potential gross rental income is modeled as a random process and the NPV is expressed as a random variable. The probability distribution of NPV is obtained using the MC simulation technique. MC simulation is categorized as a sampling method, because the inputs are randomly generated from probability distributions to simulate the process of sampling from an actual population. In this study, MC simulation was repeatedly applied to the deterministic model (see Equation (1)). This was done using a set of random numbers and the probability distribution function representing the total present value of the after-tax cash inflows, $PVAT$. A simulation typically involves over 10,000 evaluations of the model. The final summary of the random values of NPV is thus a sample of 10,000 or more NPVs in a cumulative distribution function (CDF) graph.

The discount rate q is the rate of return that an investor expects from an investment. Each investment has a different discount rate, based on expected future cash flow. Generally, the higher the risk, the higher the discount rate that an investor will demand before making an investment. IRR, defined as a discount rate that generates a zero NPV from the project’s cash flows, is a measure of the annual effective compounded return rate of an investment. IRR is commonly used by investors to decide whether they should make investments. If the IRR equals or exceeds the opportunity cost of capital, the project is acceptable. With reference to the NPV formula presented in Equation (1), IRR can be defined as follows:

$$NPV(IRR = q) = 0 \tag{8}$$

Brokerage Fees

An agent or agent’s company charges brokerage fees or commissions for negotiations, appraisals, sales, purchases, and any other services that facilitate transactions between buyers and sellers. Brokerage fees are usually based on either a fixed percentage of the sales price or on a sliding scale.

Taxes

Taxes are generally categorized as either central taxes or local taxes, according to the constitution and the law governing the allocation of government revenues and expenditures. Central taxes include individual income tax, estate tax, commodity tax, future transactions tax, and business tax and are collected by the national tax administration of the Ministry of Finance. Local taxes include stamp tax, deed tax, house tax, land value tax, and land value increment tax, among others, and are collected by the county tax collection office. In this study, deed tax, land value increment tax, land value tax, house tax, and individual income tax are considered in the property valuation procedure.

Deed tax, DT, on a sale is reported and paid by the purchaser and set as a percentage of the value of the deed. Deed tax is collectable only upon immovable properties: houses, buildings, and other fixtures on land.

Land value increment tax, $LVIT_T$, was designed to curb speculation and land monopolies. Its development was based on the concept that the natural value of land is attributable to social development rather than the result of labor or capital investment, and therefore, should be shared by the general public through the mechanism of the land value increment tax. $LVIT_T$ is calculated using the following formula:

$$LVIT_t = VI_t * LVITR \tag{9}$$

where VI_t is the total amount in excess of the original decreed value or the assessed value at the time of the last transfer (after adjustments are made pursuant to consumer price index), and $LVITR$ is the land tax rate.

Land value tax, LVT_t, is paid by the land title owner and set at a percentage of the land value. A progressive tax rate may be used to calculate the tax payable on regular land.

House tax, HT_t, is collected from home owners as an asset tax and calculated using the following formula:

$$HT_t = SP_t * A * (1 - DR * Y_t) * AR * HTR \tag{10}$$

where SP_t is the standard house price per unit area; A is the building area; DR is the annual depreciation rate; Y_t is the years of depreciation; AR is an adjustment rate based on the level/class of street or road; and HTR is the house tax rate.

The Potential Gross Rental Income

The potential gross income earned by businesses varies widely from year to year due to stochastic forces. In this study, PGI_t is modeled using a random walk process. The potential gross income at the t^{th} year is dependent on the potential gross income at the t-1^{th} year. The relationship between PGI_t and PGI_{t-1} can be expressed as:

$$PGI_t = PGI_{t-1} + Z_t \tag{11}$$

The increments Z_t (t = 1, 2, …, T) between rental income at the t^{th} year and rental income at the t-1^{th} year for any t over the holding period T are mutually independent Gaussian variables with mean μ and variance σ^2. Both μ and σ^2 are constant. In other words, over time, the growth of PGI_t is assumed to be normally distributed. The mean and the standard deviation of the growth rate are crucial. Growth will depend on macroeconomic factors such as growth of the gross domestic product (GDP), expected inflation and/or demographic phenomena. There are also property-specific characteristics to consider, such as the condition of the building (construction age, structural type, and location). The actual collectible rent partly reveals the values of these variables.

Equation (11) can be rewritten as:

$$PGI_t = PGI_0 + \sum_{i=1}^{t} Z_i \tag{12}$$

where PGI_0 is the initial value of potential gross income, assumed to be a given value.

Mortgage Loan

A mortgage entails pledging a property to a lender as security for a mortgage loan. Arranging a mortgage is seen as the standard method by which individuals and businesses in many countries purchase residential and commercial real estate without the need to pay the full value immediately.

There are various ways to repay a mortgage loan. The most common is regular payments of principal and interest over a set term. The most basic arrangement would require a fixed monthly or yearly payment over the term of the loan. The principal component of the original loan would be slowly paid down through amortization. The interest may be compounded annually.

Features of mortgage loans such as the size of the loan, interest rate, and method of paying off the loan can vary considerably. However, for the sake of simplicity, in this study, fixed-rate mortgages (FRM) were selected. In other words, the annual interest rate and the yearly principal payment remain constant over the term of the loan. The loan to value (LTV) ratio is the size of the loan in relation to the purchase price of the property. The higher the LTV, the higher the risk the value of property will be insufficient to cover the remaining principal of the loan. A standard mortgage generally has no more than 80% LTV.

Probability Distribution of Total Present Value of the After-Tax Cash Inflows

Total present value of the after-tax cash inflows (*PVAT*) referred to in Equation (1) is given by:

$$PVAT = \sum_{t=1}^{T}\left(\frac{AT_t}{(1+q)^t}\right) \tag{13}$$

Assuming $\nu_t = \nu$ for any t, substituting Equations (4)-(7) and (12) into Equation (3), followed by substituting Equation (3) into Equation (13) yields the following:

$$PVAT = \alpha PGI_0 \sum_{t=1}^{T}\frac{1}{(1+q)^t} + \alpha\sum_{t=1}^{T} Z_t\left(\sum_{i=t}^{T}\frac{1}{(1+q)^i}\right) - \sum_{t=1}^{T}\frac{\beta PP_t + \beta IP_t + LVT_t + HT_t}{(1+q)^t} \tag{14}$$

where $\alpha = (1-k)(1-\nu)\beta$ and $\beta = 1 - 0.57\chi$. Note that the increments of Z_t (t = 1, 2, …, T) are mutually independent Gaussian variables with mean μ and variance σ^2. Thus, *PVAT* is normally distributed. The mean and the variance of *PVAT* are given as follows:

$$\begin{aligned} E[PVAT] = \sum_{t=1}^{T}\frac{E[AT_t]}{(1+q)^t} &= \alpha PGI_0\sum_{t=1}^{T}\frac{1}{(1+q)^t} + \alpha\mu\sum_{t=1}^{T}\left(\sum_{i=t}^{T}\frac{1}{(1+q)^i}\right) \\ &- \sum_{t=1}^{T}\frac{\beta PP_t + \beta IP_t + LVT_t + HT_t}{(1+q)^t} \end{aligned} \tag{15}$$

$$Var[PVAT] = \sum_{t=1}^{T}\left(\frac{Var[AT_t]}{(1+q)^t}\right) = \alpha^2\sigma^2\sum_{t=1}^{T}\left(\sum_{i=t}^{T}\frac{1}{(1+q)^i}\right)^2 \tag{16}$$

Analytical Procedures

The procedures for constructing a representative relationship between annual rate of return q and its exceeding probability $R(q)$ for available financing schemes for the property in a specified holding period can be summarized as follows:

(1) Specify a discount rate or annual rate of return q;

(2) Calculate mean and variance of *PVAT* from Equations (15) and (16);

(3) Generate a set of random values to represent total present value of after-tax cash inflows ($PVAT_t$), where i is 1 to N, from a normal distribution, with the mean and standard deviation obtained from Step (2);

(4) Calculate NPV_i from Equation (1);

(5) Sort NPV_i, where i is 1 to N, into an ascending order;

(6) Plot the simulated empirical distribution function (CDF) graph for NPV from data consisting of N observations and construct the relation curve of NPV and its exceeding probability;

(7) Estimate the probability of $NPV > 0$ or the value of $R(q)$ from the results of Step (6);

(8) Repeat Steps (1)-(7) for different annual rates of return;

(9) Plot the relation curve of annual rate of return versus the corresponding exceeding probability of return;

(10) Repeat Steps (1)-(9) for all available financing schemes for the property.

Figure 1 is a flowchart showing the concise procedure for constructing these relation curves.

Criteria of Decision-Making

Before making a decision regarding real estate investments, it is imperative that the investor determine the required rate of return and the corresponding exceeding probability of return. An investment project is deemed feasible if the property satisfies the two requirements or criteria. The exceeding probability of return on investment represents the reliability that the investor will gain returns on an investment. In this study, the exceeding probability (or reliability) of rate of return is measured using the probability that the NPV of a property is greater than zero.

Case Study

An investment project involving a 3-story reinforced concrete commercial building in Taichung City, which is one of the major cities located in central Taiwan with a population of just over 2.6 million people, was examined using the proposed framework. The intended holding period was three years. Three financing options were available for the property, including: (A) no mortgage loan; (B) a mortgage loan of 60% of the initial purchase price; and (C) a mortgage loan of 80% of the initial purchase price. A 20-year mortgage loan was adopted for alternatives B and C.

The initial purchase price of the property was 24 million NT dollars, consisting of 13.2 million NT dollars for the building and 10.8 million NT dollars for the land. The current tax rates of Taiwan were used in the case studies. The deed tax, income tax, land value tax, house tax, and land value increment tax were 6%, 12%, 1%, 3%, and 20%, respectively. The annual depreciation rate was 1%. The adjustment rate was 100%. The interest rate and the yearly principal payment remained constant over the term of the loan. The operating expense ratio and vacancy rate were assumed to be 0.5% and 10%, respectively. The initial potential gross rental income was 10% of the initial purchase price of the property. Closing costs, private mortgage insurance premiums, and related costs are generally considered in the calculation of NPV. However, in this study for the sake of simplicity, these were not included. Brokerage fees paid upon the purchase or sale were 5% of the sales price. The buyer paid half of the brokerage fees; the seller paid the other half.

Figure 2 shows the relationship between annual rate of return and the corresponding exceeding probability of return for alternatives A, B, and C when examined under an annual interest rate of 3%. As shown in Figure 2, exceeding probability decreases with increases in annual rate of return and increases with increases in mortgage rate for this investment project under the conditions set for the property in this study. The two requirements set by the investor for investment feasibility evaluation (criterion No. 1 being the annual rate of return of 5%; criterion No. 2 being exceeding probability of 0.7 or above) were marked as point *P* in Figure 2. The curves of alternatives B and C lie above point *P* and the curve of alternative A lies below point *P*. As annual rate of return was fixed at 5%, the exceeding probabilities of alternatives B and C were greater than 0.7, and only the exceeding probability of alternative A was lower than 0.7. This means that alternatives B and C satisfied the requirements of the investor which alternative A was unable to do. It is concluded that alternatives B and C are feasible and profitable, while alternative A is not feasible. Furthermore, for a specified rate of return, the corresponding exceeding probability of alternative C was greater than that of alternative B, meaning that the better choice between alternatives B and C is alternative C.

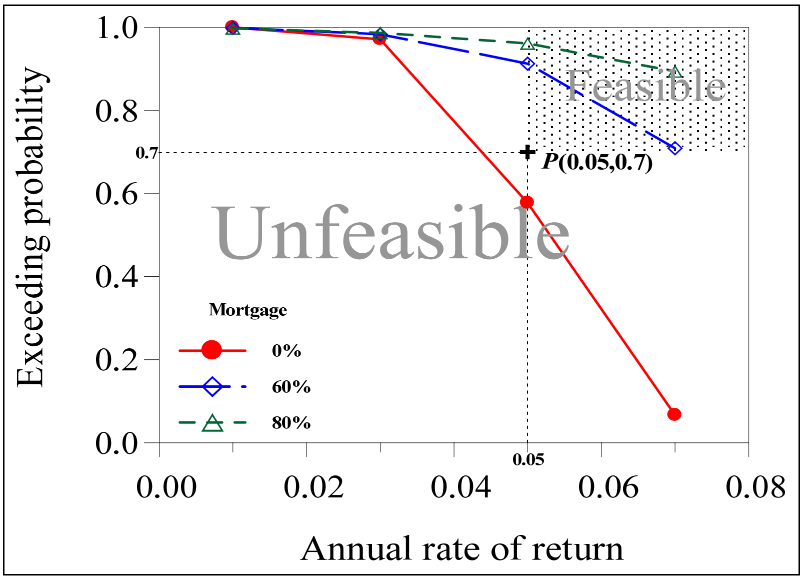

Figure 2. Exceeding probability of annual rate of return for different mortgage rates (holding period = three years; annual interest rate = 3%).

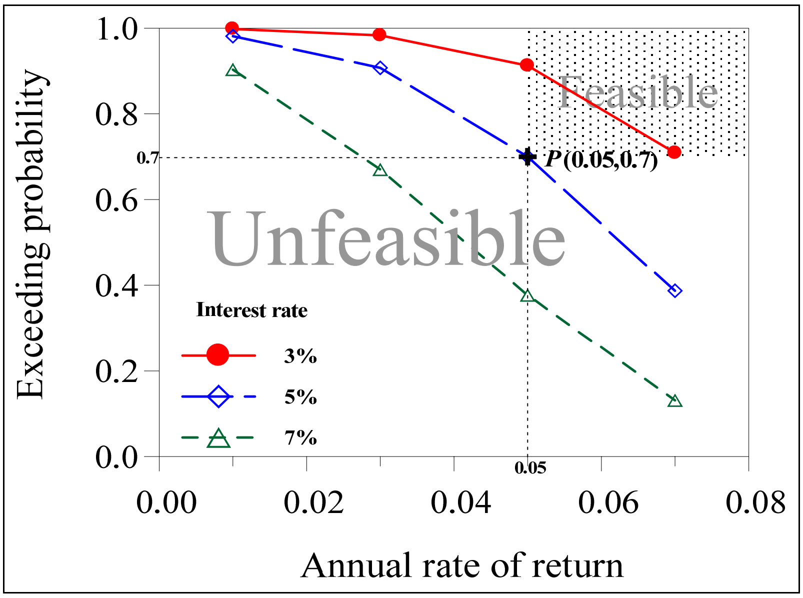

Figure 3. Exceeding probability of annual rate of return for discrete annual interest rates under 60% mortgage (holding period = three years).

Figure 3 shows the curves of annual rate of return versus exceeding probability for alternative B at discrete interest rates of 3%, 5%, and 7%. The requirements set by the investor were also marked as point *P* in Figure 3. As shown in Figure 3, the curve for interest rate of 3% lies above point *P* and the curve for interest rate of 7% lies below point *P*. It is noted that the curve for interest rate of 5% lies on point *P*. This means that the financing scheme of a 60% mortgage with an interest rate greater than 5% was not feasible. As a result, the investor could choose alternative B as the interest rate was 5% or below.

Conclusions

Commercial real estate investors are interested in the following issues: possible profit gain and the probability that profit gain will occur. This study proposed an analytic method to explore these issues. A probabilistic framework for measuring exceeding probability of annual rate of return was developed. Through the framework and the proposed processes, investors are given basic information on the return, probability that profit gain will occur, and feasibility of financial schemes for commercial real estate investments. The framework also has the potential for applicability to similarly hard-to-predict issues of investments. In addition, this study presumed that the time series model of potential gross rental income followed a random walk process. This presumption may not reflect reality, but it is still possible to gain a deeper understanding through numerical simulation as long as a time series model of actual potential gross income is given.

References

Byrne, P., & Cadman, D. (1996). *Risk, uncertainty, and decision-making in property development*. London: E & FN Spon.

French, N., & Gabrielli, L. (2004). The uncertainty of valuation. *Journal of Property Investment and Finance, 22*(6), 484-500.

Hoesli, M., Jani, E., & Bender, A. (2006). Monte Carlo simulations for real estate valuation. *Journal of Property Investment and Finance, 24*(2), 102-122.

Kelliher, C. F., & Mahoney, L. S. (2000). Using Monte Carlo simulation to improve long-term investment decisions. *The Appraisal Journal, 68*(1), 44-56.

Metropolis, N., & Ulam, S. (1949). The Monte Carlo method. *Journal of the American Statistical Association, 44*(247), 335-341.

Nygard, W., & Razaire, C. (1999). Probability-based DCF: An alternative to point-value estimates. *The Appraisal Journal, 67*(1), 68-74.

Rubinstein, R. Y., & Kroese, D. P. (2007). *Simulation and the Monte Carlo method* (2nd ed.). New York, NY: John Wiley & Sons.

Tucker, W. F. (2001). A real estate portfolio optimizer utilizing spreadsheet modeling with Markowitz mean-variance optimization and Monte-Carlo simulation. Working Paper, John Hopkins University, Maryland.