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Node Degree Distribution and Interrelationships of Beijing Subway, Bus, and Composite Network

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Abstract: This study examines Beijing subway network, bus network and the composite network of the two in 2022. Using stations as nodes and the lines between them as edges, the node degree probability distribution of the resulting complex networks is calculated. Through regression analysis, it is found that the node degree probability distributions for all three transportation modes follow a negative exponential function, and all three distribution curves decrease as the node degree increases. Among the three probability distribution curves, the node degree probability distribution curve of subway network is at the bottom, the probability distribution curve of bus network is in the middle, and the probability distribution curve of composite network is at the top.

Key word: Subway network, bus network, composite network, regression analysis, degree distribution.

1. Introduction

The air transportation network is a typical complex network and exhibits small-world characteristics [1-4]. As a type of transportation network, the air network uses cities with airports as nodes and flight routes as edges. The degree probability distribution of nodes in this complex network follows a power function. However, whether complex networks formed by other types of transportation also exhibit power function characteristics in their node degree distributions needs to be gradually verified through research, especially for composite networks formed by two types of transportation. It is necessary to investigate the characteristics of the node degree probability distribution in such networks and how it relates to the node degree distributions of the composite networks. This study examines the subway network, bus network, and a composite network of the two of Beijing in 2022, using stations as nodes, to analyze the probability distribution of node degrees and the relationships among the three. The research shows that the node degree probability distributions of the subway network, bus network, and the composite network all follow a negative exponential function, with the distribution curves positioned such that the curve of subway network is at the bottom, the curve of bus network is in the middle, and the curve of composite network is at the top.

2. Probability Distribution of Node Degree of Beijing Subway Network

In complex networks, the degree of a node is the number of edges connected to that node. As nodes of the subway network, the degree of a subway station is the number of subway lines it connects. According to statistical data, in 2022 [5], the Beijing subway network had a total of 27 routes and 384 stations, with a statistical table showing the distribution of node degrees in Table 1. The median node degree against the probability as scatter points were drawn in Fig. 1. Let the Beijing subway network be G_S. Let the adjacent matrix of Beijing subway network be A_S. The node quantity of Beijing subway network is $N_S = 384$. For network $G_S = (V_S, E_S)$, where $v_i \in V_S$ is the node of G_S.

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V_S is the set of nodes. E_S is the set of edges, $(v_i,v_j)\in E_S$. Matrix $A_S=(a_{i,j})_{N_S\times N_S}$ was constructed, where:

$$a_{i,j}=\begin{cases}1,(v_i,v_j)\in E_S\\ 0, otherwise\end{cases}$$

Let the node degree be x axis and the probability be y axis in Fig. 1. Let v = lny in Fig. 2. The points in Fig. 2 were calculated by the points in Fig. 1. The correlation coefficient r of the points in Fig. 2 was calculated by Eq. (1).

Table 1 Node Degree Distribution Frequency and Probability of Beijing Subway Network in 2022.

Interval of node degrees	[1,2]	[3,4]	[5,6]
Median	1.5	3.5	5.5
Times	311	68	5
Probability	0.810	0.177	0.013

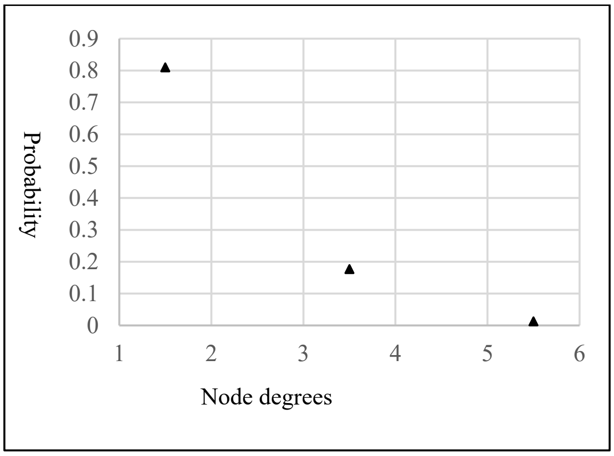

Fig. 1 Probability distribution of node degrees of Beijing subway network in 2022.

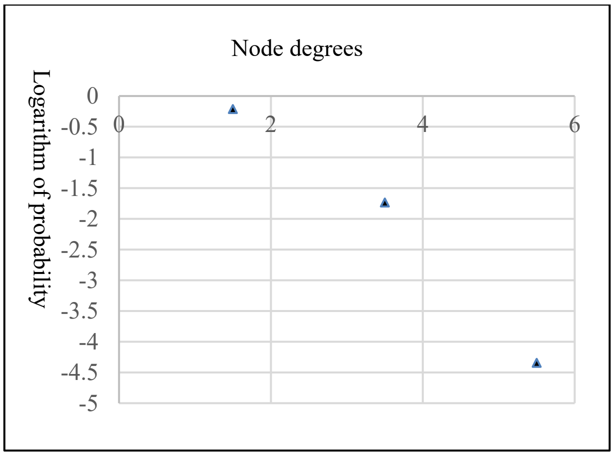

Fig. 2 Relationship between node degree and the logarithm of probability.

$$r=\frac{L_{xv}}{\sqrt{L_{xx}L_{vv}}}=\frac{\sum_{i=1}^{n}(x_i-\bar{x})(v_i-\bar{v})}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2\sum_{i=1}^{n}(v_i-\bar{v})^2}} \quad (1)$$

Here, n = 3. The value of correlation coefficient r was calculated, r = -0.998. The critical value of $r\big|_{\substack{\alpha=5\%\\ f=1}}$ was 0.997 found in critical value table [6] at degree of freedom f = n - 2 = 1 and level of significant α of 5%. Since $|r|=0.998>r\big|_{\substack{f=1\\ \alpha=5\%}}=0.997$, the scattered points in Fig. 2 had relatively significant linear correlation. Least square method [6] was used as an approach in Eq. (2) to fit the line with points in Fig. 2.

$$\begin{cases}\hat{\beta}_0=\bar{v}-\hat{\beta}_1\bar{x}=1.52\\ \hat{\beta}_1=\dfrac{L_{xv}}{L_{xx}}=-1.03\end{cases} \quad (2)$$

The linear equation:

$$\hat{v}=1.52-1.03x \quad (3)$$

The fitting line Eq. (3) was drawn with the sample points in Fig. 3 with good fitting effect.

To take t test [6] of Eq. (3), test hypotheses is: H_0:β_1 = 0,

When the hypotheses was true, there is:

$$\hat{\beta}_1 \sim N(0,\frac{\sigma^2}{L_{xx}}) \quad (4)$$

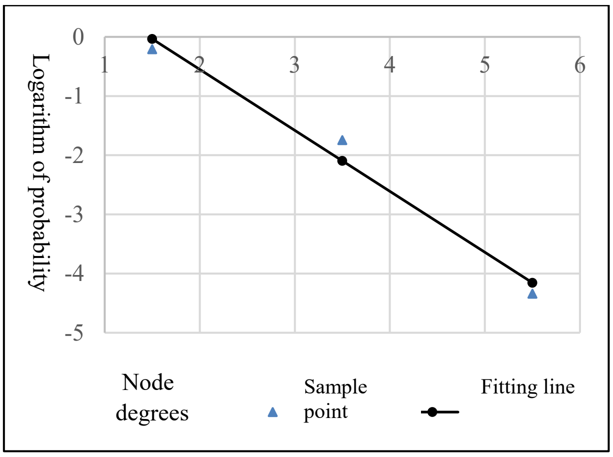

Fig. 3 Fitting linear relationship between node degree and logarithm of probability.

Here, $\hat{\beta}_1$ fluctuate near zero, statistic t is build.

$$t = \frac{\hat{\beta}_1}{\sqrt{\frac{\hat{\sigma}^2}{L_{xx}}}} = \frac{\hat{\beta}_1 \sqrt{L_{xx}}}{\hat{\sigma}} \tag{5}$$

Where:

$$\hat{\sigma}^2 = \frac{1}{n-2}\sum_{i=1}^{n}(v_i - \hat{v}_i)^2 \tag{6}$$

Calculated by statistic data: $t = -6.54$

To check the t distribution table [6], at significant level α of 0.05 and degree of freedom $f = n - 2 = 1$, the value of $t\big|_{\alpha=0.05, f-1}$ in table is 6.314. So, $|t| = 6.54 > t\big|_{f=1, \alpha=0.05} = 6.314$, null hypotheses H_0 is refused. The linear correlation of Eq. (3) is relatively significant. The fitting curve Eq. (7) of nonlinear relationship between node degree and probability was deduced from Eq. (3):

$$\hat{y} = 4.57e^{-1.03x} \tag{7}$$

The points of the fitting curve Eq. (7) and the sample points were drawn in Fig. 4 with good fitting effect. It showed that the node degree in Beijing subway network in 2022 had a negative exponential relationship with the probability.

3. Probability Distribution of Node Degree of Beijing Bus Network

In complex networks, the degree of a node is the number of edges connected to that node. As nodes of the bus network, the degree of a bus station is the number of bus lines it connects. According to statistical data, in 2022, the Beijing bus network had a total of 1191 routes and 9243 stations, with a statistical table showing the distribution of node degrees in Table 2. The median node degree against the probability as scatter points were drawn in Fig. 5. Let the Beijing bus network be G_B. Let the adjacent matrix of Beijing bus network be A_B. The quantity of node degree of Beijing bus network is $N_B = 9243$. For network $G_B = (V_B, E_B)$, where $v_i \in V_B$ is the node of G_B. V_B is the set of nodes. E_B is the set of edges, $(v_i,v_j)\in E_B$. Matrix $A_B = (a_{i,j})_{N_B \times N_B}$ was constructed, where:

$$a_{i,j} = \begin{cases} 1, (v_i, v_j) \in E_B \\ 0, otherwise \end{cases}$$

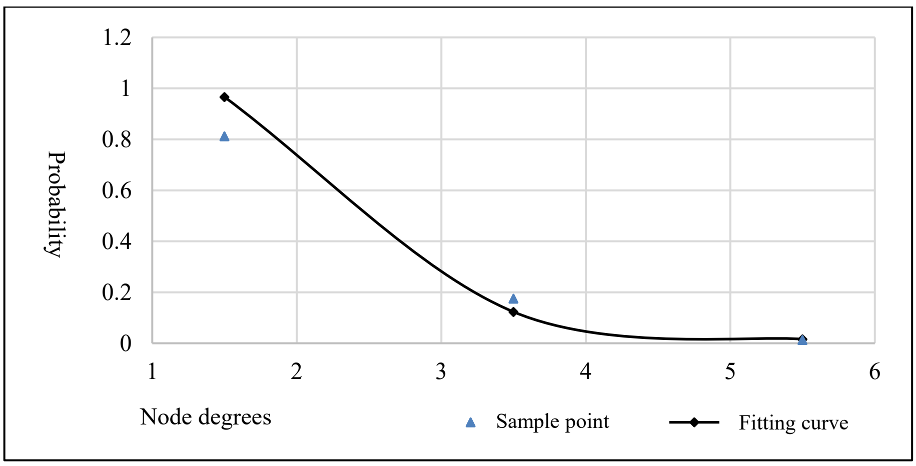

Fig. 4 Fitting nonlinear relationship between node degree and probability.

Table 2 Node degree distribution frequency and probability of Beijing bus network in 2022.

Interval of node degree	[1,2]	[3,4]	[5,6]	[7,8]	[9,10]	[11,12]	[13,14]	[15,16]
Median	1.5	3.5	5.5	7.5	9.5	11.5	13.5	15.5
Times	5515	2630	792	221	54	24	5	2
Probability	0.5967	0.2846	0.0857	0.0239	0.0058	0.0026	0.0005	0.0002

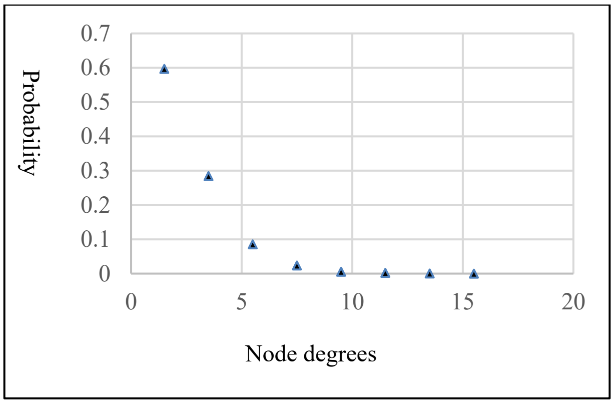

Fig. 5 Probability distribution of node degrees of Beijing bus network in 2022.

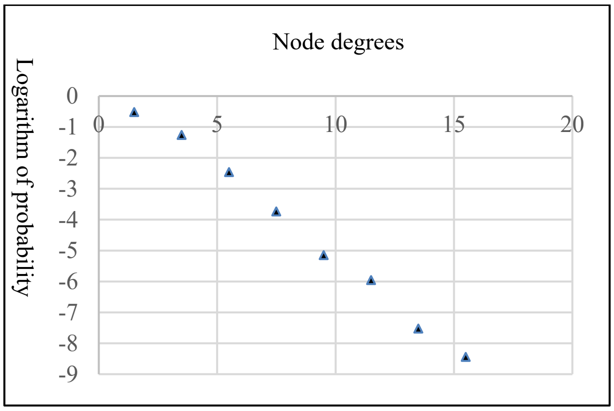

Fig. 6 Relationship between node degree and the logarithm of probability.

Let the node degree be x axis and the probability be y axis in Fig. 5. Let $v = \ln y$ in Fig. 6. The points in Fig. 6 were calculated by the points in Fig. 5.

Here, n = 8. Using the data in Table 2, the value of correlation coefficient r was calculated by Eq. (1), r = -0.997. The critical value of $r_{\substack{\alpha=1\% \\ f=6}}$ was 0.834 found in critical value table at degree of freedom f = n - 2 = 6 and level of significant α of 1%. Since $|r| = 0.997 > r_{\substack{f=6 \\ \alpha=1\%}} = 0.834$, the scattered points in Fig. 6 had significant linear correlation. Least square method was used as an approach in Eq. (8) to fit the line with points in Fig. 6.

$$\begin{cases} \hat{\beta}_0 = \overline{v} - \hat{\beta}_1 \overline{x} = 0.536 \\ \hat{\beta}_1 = \dfrac{L_{xv}}{L_{xx}} = -0.578 \end{cases} \tag{8}$$

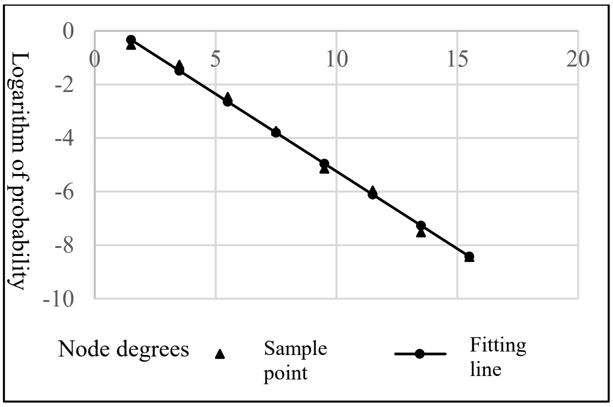

Fig. 7 Fitting linear relationship between node degree and logarithm of probability.

The linear equation:

$$\hat{v} = 0.536 - 0.578x \tag{9}$$

The fitting line Eq. (9) were drawn with the sample points in one diagram of Fig. 7 with good fitting effect.

To take t test of Eq. (7), test hypotheses is: H_0:$\beta_1 = 0$.

When the hypotheses were true, there are Eq. (4), Eq. (5) and Eq. (6).

Statistic t was calculated by data in Table 2 and Eq. (4), Eq. (5) , Eq. (6).: t = -36.5

To check the t distribution table, at significant level α of 0.01 and degree of freedom f = n – 2 = 6 the value of $t_{\substack{\alpha=0.01 \\ f=6}}$ in table is 3.143. So, $|t| = 36.5 > t_{\substack{f=6 \\ \alpha=0.01}} = 3.143$, null hypotheses H_0 is refused. The linear correlation of Eq. (9) is significant. The fitting curve Eq. (10) of nonlinear relationship between node degree and probability was deduced from Eq. (9):

$$\hat{y} = 1.709e^{-0.578x} \tag{10}$$

The points of the fitting curve Eq. (10) and the sample points were drawn in Fig. 8 with good fitting effect. It showed that the node degree of Beijing bus network in 2022 had a negative exponential relationship with the probability.

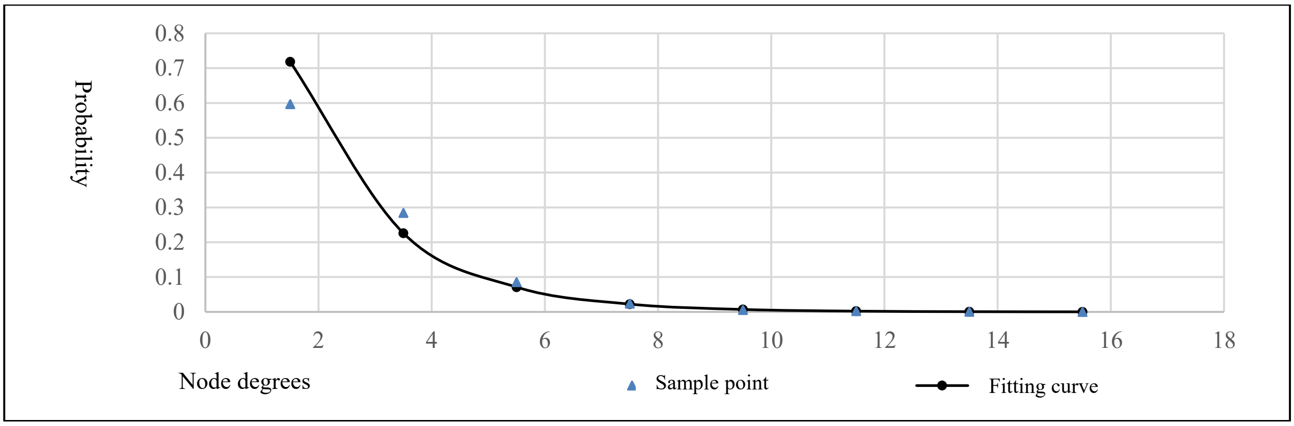

Fig. 8 Fitting nonlinear relationship between node degree and probability.

4. Probability Distribution of Node Degree of Beijing Composite Network

The total number of stations in the composite network equals the sum of subway and bus stations, which is 9,627. Since the number of subway station are far less than the number of bus stops, so the bus stops located within 200 meters of a subway station are considered transfer points for that subway station. These bus stops are considered as connected stations to that subway station. That means there are edges between these bus stops with that subway station in composite network. The subway network, bus network and the connections between transfer stations consists a Beijing composite public transit network. Based on statistical data, the degree distribution of nodes of Beijing composite network in 2022 is summarized in Table 3. The median node degree against the probability as scatter points were drawn in Fig. 9. Let the Beijing composite network be G_c. Let the adjacent matrix of Beijing composite network be A_C. The quantity of node degree of Beijing composite network is $N_C = N_S + N_B = 9627$. For network $G_C = (V_C, E_C)$, where $v_i \in V_C$ is the node of G_C. V_C is the set of nodes. E_C is the set of edges, $(v_i,v_j) \in E_C$. Matrix $A_C = (a_{i,j})_{N_C \times N_C}$ was constructed, where:

$$a_{i,j} = \begin{cases} 1, (v_i, v_j) \in E_C \\ 0, otherwise \end{cases}$$

Table 3 Node degree distribution frequency and probability of Beijing composite network in 2022.

Interval of node degree	[1,5]	[6,10]	[11,15]	[16,20]	[21,25]	[26,30]	[31,35]
Median	3	8	13	18	23	28	33
Times	8115	1178	245	68	16	4	1
Probability	0.8429	0.1224	0.0254	0.0071	0.0017	0.0004	0.0001

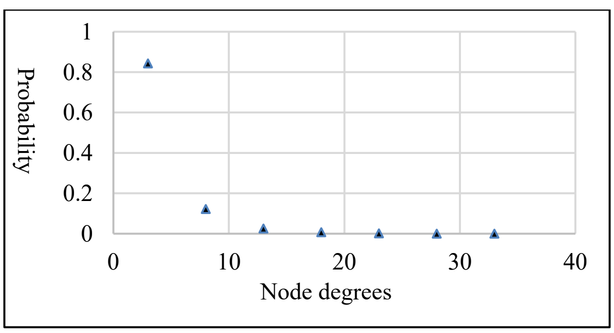

Fig. 9 Probability distribution of node degrees of Beijing composite network in 2022.

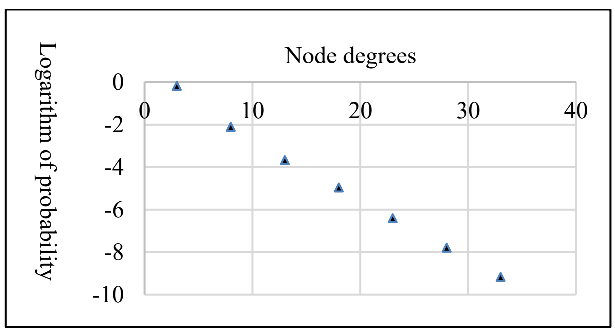

Fig. 10 Relationship between node degree and the logarithm of probability.

Let the node degree be x axis and the probability be y axis in Fig. 9. Let $v = \ln y$ in Fig. 10. The points in Fig. 10 were calculated by the points in Fig. 9.

Here, n = 7. Using the data in Table 3, the value of correlation coefficient *r* was calculated by Eq. (1), r = -0.998. The critical value of $r_{\alpha=1\%}\big|_{f=5}$ was 0.874 found in critical value table at degree of freedom *f* = n - 2 = 5 and level of significant α of 1%. Since $|r| = 0.998 > r_{f=5}\big|_{\alpha=1\%} = 0.874$, the scattered points in Fig. 10 had significant linear correlation. Least square method was used as an approach in Eq. (11) to fit the line with points in Fig. 10.

$$\begin{cases} \hat{\beta}_0 = \overline{v} - \hat{\beta}_1 \overline{x} = 0.423 \\ \hat{\beta}_1 = \dfrac{L_{xv}}{L_{xx}} = -0.294 \end{cases} \tag{11}$$

The linear equation:

$$\hat{v} = 0.423 - 0.294x \tag{12}$$

The fitting line Eq. (12) were drawn with the sample points in one diagram of Fig. 11 with good fitting effect. To take *t* test of Eq. (12), test hypotheses is: H_0:$\beta_1 = 0$.

When the hypotheses were true, there are: Eq. (4), Eq. (5) and Eq. (6).

Statistic *t* was calculated by data in Table 3 and Eq. (4), Eq. (5), Eq. (6): *t* = -38.0.

To check the *t* distribution table, at significant level α of 0.01 and degree of freedom *f* = n - 2 = 5 the value of $t_{\alpha=0.01}\big|_{f=5}$ in table is 3.365. So, $|t| = 38.0 > t_{f=5}\big|_{\alpha=0.01} = 3.365$, null hypotheses H_0 is refused. The linear correlation of equation (12) is significant. The fitting curve Eq. (13) of nonlinear relationship between node degree and probability was deduced from Eq. (12):

$$\hat{y} = 1.527e^{-0.294x} \tag{13}$$

The points of the fitting curve Eq. (13) and the sample points were drawn in Fig. 12 with good fitting effect. It showed that the node degree in Beijing composite network in 2022 had a negative exponential relationship with the probability.

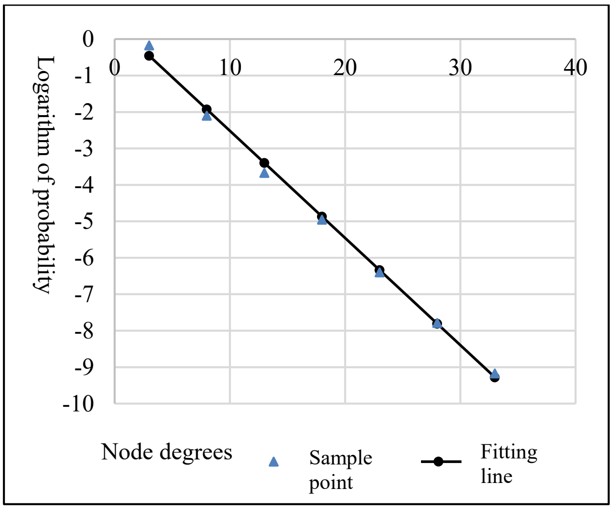

Fig. 11 Fitting linear relationship between node degree and logarithm of probability.

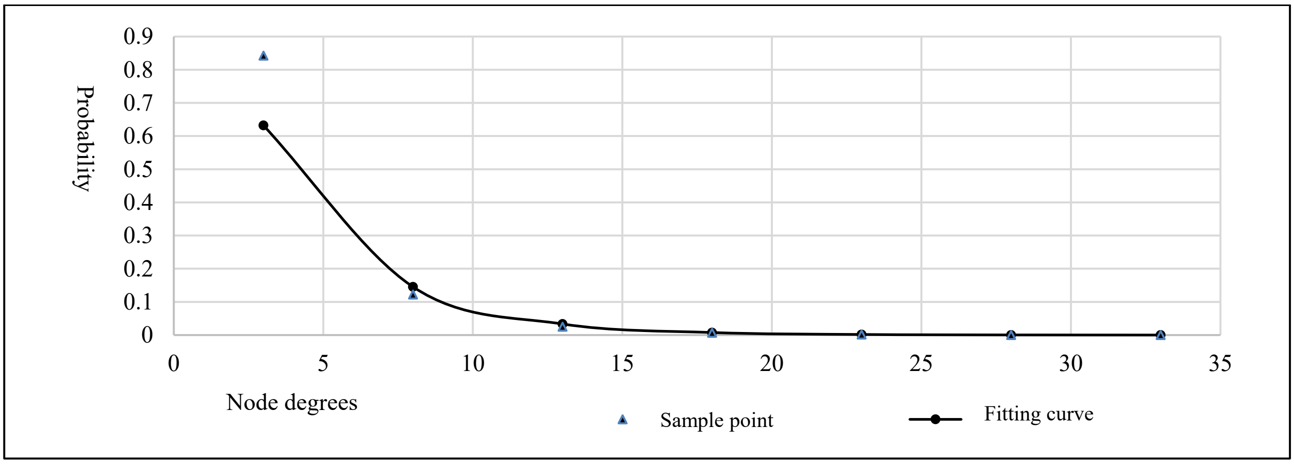

Fig. 12 Fitting nonlinear relationship between node degree and probability.

Table 4 The fitting curve equations of node degree distributions for Beijing subway, bus, and composite network in 2022.

	Subway network	Bus network	Composite network
Fitting curve equation	$\hat{y} = 4.53e^{-1.03x}$	$\hat{y} = 1.709e^{-0.578x}$	$\hat{y} = 1.527e^{-0.294x}$

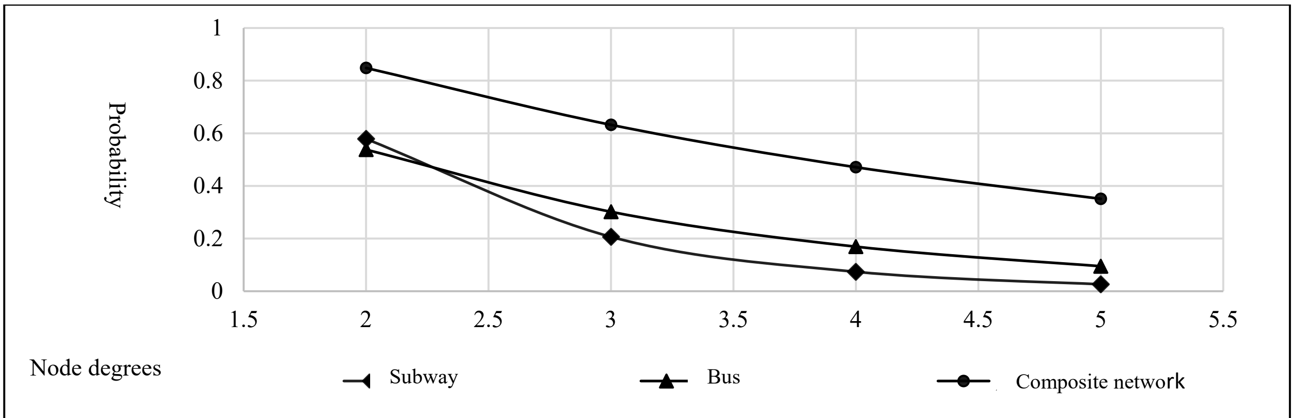

Fig. 13 Interrelationship among the node degree distribution in Beijing subway, bus, and composite networks.

5. Interrelationship Among the Node Degree Distribution in Beijing Subway, Bus, and Composite Network

The fitting curve equations of the node degree probability distributions for Beijing subway, bus, and composite networks in 2022 were listed in Table 4. These three probability distributions are all negative exponential function.

By plotting the curves from Table 4 together in Fig. 13, their relative positions can be observed. The probability distribution curve of the subway is at the bottom, the distribution curve of the bus is in the middle, and the distribution curve of the composite network is at the top. The node degree probability distribution curve of the subway is the steepest, indicating that stations with a node degree of no more than 2 account for a significantly higher proportion compared to other stations. For all three, the occurrence probability of node degrees decreases as the degree increases.

6. Conclusion

After analysis of data, it is found that: the node degree probability distributions for Beijing subway, bus and composite network follow a negative exponential function. The probability distribution curve of the subway network is at the bottom, the distribution curve of the bus network is in the middle, and the distribution curve of the composite network is at the top. The node degree probability distribution curve of the subway is the steepest, indicating that stations with a node degree of no more than 2 account for a significantly higher proportion compared to other stations. For all three, the occurrence probability of node degrees decreases as the degree increases.

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