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A Brief Discussion on the Social Philosophical Significance of Gödel's Incompleteness Theorems

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This paper applies Gödel's Incompleteness Theorems to the evolution and development of human social systems. Although Gödel's Incompleteness Theorems originated in the field of mathematics, their influence has long extended beyond mathematics, making an impact on philosophy, systems science, and the humanities and social sciences. The paper analyzes the autonomy and completeness of human social systems, arguing that evolving human societies are generally self-consistent. However, if the completeness of a human social system is compromised, the system either maintains self-consistency, ceases to evolve forward, enters a death spiral, and eventually decays and disintegrates. Or the system addresses the completeness issue, enters a state of non-self-consistency, introduces new axioms, becomes self-governing again, and enters a new form. From the sociological perspective, this is articulated as social revolution—the system continues to evolve forward; the absence of social revolution—the system does not evolve forward (Jin, 1988).

Keywords: Gödel's Incompleteness Theorems, human society, logical system, self-consistency, completeness

In the 1930s, Austrian mathematician Kurt Gödel proved a theorem that shook the mathematical world, known as Gödel's Incompleteness Theorem. This theorem has multiple forms of expression.

Gödel's Incompleteness Theorems (Summarized in one sentence):

Any arithmetic formal system that includes the axioms of natural numbers cannot simultaneously satisfy consistency and completeness.

More detailed, there are Gödel's First and Second Incompleteness Theorems:

Gödel's First Incompleteness Theorem: If a formal system containing the axioms of natural numbers is consistent, then it is incomplete. There must exist a proposition within this system that cannot be either proven or disproven.

Gödel's Second Incompleteness Theorem: The consistency of any formal system containing the arithmetic axioms of natural numbers cannot be proven within the system if it is consistent.

Jin Guantao said: "The discovery of Gödel's theorem is an epoch-making advance in the mathematics of this century. It represents a profound revolution in mathematical thinking, and also a revolution in philosophical thought" (1988, p. 218). In 1988, he further stated:

> On the other hand, sometimes a philosophical revolution occurs after a scientific revolution. The scientific philosophy's view that logic as a tool of cognition must have absolute certainty is based on mathematicians' belief in the completeness

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> and non-contradiction of logical systems. Now the foundation has collapsed, but it will take some time for the impact to reach the edifice of philosophy. (p. 218)

Jin Guantao made these remarks in 1988, which were published in his newly published book *My Philosophical Exploration*. Thirty years have passed since then, and what new understandings, extensions, or applications of Gödel's Incompleteness Theorem have systemic philosophy brought forth? Below, we attempt to explicate Gödel's Incompleteness Theorem in a more popular form.

We know that general logical systems have the following properties:[1]

1. Validity—If all propositions in a logical system are true, then the conclusions derived from them must also be true;

2. Reliability—All theorems in the system are true;

3. Consistency—No axiom within a logical system can contradict any other axioms or theorems (lack of contradiction);

4. Completeness—There are no valid propositions in the system that cannot be proven or disproven.

This is discussing general logic systems in the sense of mathematics or logic. However, in our daily lives, more general systems, such as natural systems, human social systems, etc., in fact, also have their own strong logical chains and their own cause-and-effect relationships. For instance, the concept of nation in human society, starting from scattered primitive human groups, has undergone evolution through tribes, alliances, slavery systems, feudal systems, monarchy, constitutional monarchy, multiparty systems, two-party systems, and various forms of state management attached to them, all the way to the present day. Does it also possess the four properties of the aforementioned general logical system? I would like to consider this from a holistic perspective, from the angle of system exchange theory. Human societies formed in different historical periods and stages are, first and foremost, complex systems; secondly, they are logical systems, and also possess the above four properties.[2]

Firstly, there is the validity. Here, it is mentioned that in a logical system, if a proposition is true, then the conclusions derived from it must also be true. We can extend this understanding to the context of social systems. If the fundamental political, cultural, and economic institutional arrangements are effective and true, then the management outcomes under their governance will also be effective and true.

Furthermore, on the topic of reliability, as mentioned above, if the fundamental social systems are true, then all laws, regulations, rules, and even local customs and agreements that are based on them are also true. The moral systems and contractual systems constructed around them are true as well.

The third aspect is self-consistency. Regarding self-consistency, the management of social systems is more self-consistent. Within the system, from the constitution at the top to the local rules and agreements at the bottom, there should be no contradictions between them; they should not conflict with each other, restrain each other, or hinder each other's progress. Generally speaking, the early stages of the establishment of various dynasties are the most self-consistent phase.

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[1] Here, the four basic properties of logical systems are listed. In fact, logical systems have many more properties, such as simplicity, decidability, independence, and extensibility.

[2] The concept of system mentioned in this article is more limited to the concept of system exchange. The system exchange concept posits that a logical system is essentially also a system exchange system. The logical chain or exchange chain of this system is a dynamic, generative, and evolvable system. This system originates from a few basic assumptions at the beginning of the system and ends with the breakdown of system completeness.
For the concept of system exchange, please refer to the author's book "*The Essence of Systems: An Initial Exploration of System Exchange Theory*." The book was published by Wuhan University of Technology Press in 2021.

Finally, there's completeness. We can understand this as the absence of unsolvable problems within the social system, especially in the early stages of a dynasty's establishment. Within a certain space and time, a dynasty is capable of solving numerous social issues and defusing various social contradictions, typically experiencing a flourishing period during its early stages. However, the extent of this space and time is subject to a degree of limitation; exceeding it may lead to the emergence of a different set of problems.

Indeed, within a certain period and space, a newly established social system can continuously solve numerous social problems and resolve various social contradictions. We can consider the social system to be complete during this time. However, as the social system evolves and progresses, after a period of time, people will discover that there are always some real or imagined social problems that persist over time and cannot be resolved. The duration of this situation varies, sometimes spanning decades, at most a hundred years, during which people identify the problems and find solutions. In some cases, the period can be very long, lasting thousands of years, affecting vast spaces, and no solution is found, which may be what we refer to as the incompleteness of the system.

Actually, the so-called completeness or incompleteness is relative to the self-consistent scope. Being incomplete in one self-consistent scope does not mean it is incomplete in another self-consistent scope. If we expand the self-consistent scope (in time, space, or conceptually), the original incompleteness may become complete. However, as the system becomes complete, it may enter a state of self-inconsistency, or it may discover new incompleteness phenomena. Generally, the state of system self-inconsistency is not allowed; otherwise the system will decline and perish. Therefore, the system can only enter an incomplete state again, but this system is not the previous one. By this time, the system has already evolved into a new form, a higher-level and larger-scale self-consistent state.

Jin Guantao also pointed out in his book *My Philosophical Exploration*: "In any theoretical system, no matter how perfect it is, no matter how well it conforms to reality, it cannot be self-contained. There will always be new problems that the old theory poses but cannot solve" (1988, p. 219). For example, China's authoritarian society that lasted for over two thousand years, this is a typical example of an old theory presenting new problems that it cannot solve within a social system's existence.

At this point in the discussion, we realize that there exists incompleteness within social systems, which can only be resolved through transformative changes to the self-consistency of society. However, as self-consistency improves, the scale of the social system undoubtedly expands, and the depth, breadth, and even dimension of its self-consistency cannot be compared to that of the previous state. It's just that, under the influence of new self-consistency, the system operates to a certain stage, and again, various incomplete events will occur. We can solve some, but there will always be some (at least one) that cannot be resolved under your governance of self-consistency. At this time, the same transformative changes to social self-consistency will occur again…

Thus, we witness the actual existence of the evolution of social systems (especially evolutionary leaps), which is the result of the alternating influence of the concepts of self-consistency and completeness. This evolution proceeds by alternating between completeness and incompleteness. Each cycle of self-consistency and completeness concepts essentially represents a natural iteration of the social system. Observing human society through this pattern, we seem to have found a new perspective on observing the development and progress of human society. It appears that we can uncover some of the deep-seated logical reasons behind the development and progress of human society.

In the field of political economy, there is an expression regarding the relationship between the mode of production and productive forces, as well as the superstructure and economic base. It states that the mode of production must adapt to the productive forces, and the superstructure must adapt to the economic base. If it is an advanced adaptation, then the production relations or superstructure will not be accepted by society. If it is a lagging adaptation, society will inevitably fall into turmoil. I believe that the theoretical basis for this statement should be Gödel's Incompleteness Theorems, or rather, a specific application of Gödel's Incompleteness Theorems.

In 1453, the Arabians captured Constantinople, posing a significant threat to the survival of Europeans. This meant that the incomplete space of European society had encountered a problem. They were forced to embark on the Great Navigation, which lasted for more than three hundred years, in an attempt to overcome or resolve the issue of this incomplete space. The Europeans succeeded, and they initiated the Industrial Revolution, which continues to this day.

By 221 BC, during the Qin Dynasty in China, the standardization of chariot tracks, the unification of writing, the establishment of commanderies and counties represented new axioms introduced to address the historical problems of incompleteness. These new axioms enabled Chinese society to achieve self-consistency in its historical development phase and completely transitioned Chinese society from a hunting civilization to an agricultural one.[3]

References

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[3] The way Chinese people overcome the incompleteness problem is by completely rebuilding the entire society, transforming the incompleteness problem into a completeness problem. This indeed is a method to solve Godel's incompleteness problem, and it is both useful and effective. This has formed the super-stable system structure of Chinese society. However, the issue is that the Chinese repeatedly rebuild society, with their economic foundation being agrarian civilization. But what about after agrarian civilization? For instance, in the current industrial civilization, or even the intelligent civilization, how should the Chinese respond? Can the Chinese break out of this vicious cycle?