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Research on Value Evaluation Method of Investment Project Based on Fuzzy Composite Real Options

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Venture capital investments are characterized by high input, high yield, and high risk. Due to the complexity of the market environment, stage-by-stage investment is becoming increasingly important. Traditional evaluation methods like comparison, proportion, maturity, internal rate of return, scenario analysis, decision trees, and net present value cannot fully consider the uncertainty and stage characteristics of the project. The fuzzy real options method addresses this by combining real option theory, fuzzy number theory, and composite option theory to provide a more accurate and objective evaluation of Public-Private Partnership (PPP) projects. It effectively considers the interaction of options and the ambiguity of project parameters, making it a valuable tool for project evaluation in the context of venture capital investment.

Keywords: real option, fuzzy method, Geske composite option

Introduction

Before the emergence of real option theory, company managers and decision makers always rely on intuition and experience to perceptually measure the interaction between management flexibility and decision-making strategy. Hayes and Abernathy (1980) and Hayes and Garvin (1982) have long recognized that traditional net present value (NPV) methods often underestimate the value of projects, leading to short-term decisions, insufficient investment, and even loss of competitiveness. The reason is that they did not consider it reasonable. In view of this situation, Hertz (1964) and Magee (1964) suggested using simulation and decision tree methods to estimate the value brought by management flexibility, but there are still certain limitations. Then Myers (1987) demonstrated that the widespread application of the NPV method in basic theory is part of the reason for this problem, and proposed that the traditional NPV method has great limitations. What it means is simple because it is based on the assumption that future cash flows occur according to a set budget, and that investments are reversible and irreversible. Most of these assumptions are not met in reality, so he proposed that the real option method is used for major decision-making projects. Since then, more and more research on the theory of real options.

Trigeorgis (1996) divided the real option into eight categories: option to defer, default option, abandon option, expansion option, shrinking option, shutdown and restart options, conversion options, and growth options. Kester (1984) and Trigeorgies (1998) proposed a real-options classification scheme that is motivated by

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similarities and differences with financial options, distinguishes between simple and Composite options, and proprietary and shared options.

Due to the complexity of the financial environment, investors often do not adopt a one-time investment method, but instead use a gradual approach to enter the project to diversify risks. Therefore, venture capital is mostly a staged investment. Then, there are real options in each stage of venture capital, and these options do not exist alone but affect each other. Trigeorgis (1993) specifically studied the mutual effects of real options. He pointed out that the existence of follow-up options can increase the value of pre-emptive options, and the exercise of pre-emptive options will also affect the value of follow-up options. This is because the exercise of a leading option can change the value of the underlying asset itself, thereby affecting the value of subsequent options. Therefore, the value of a set of real options will be significantly different from the sum of individual options, that is, the value of real options is not additivity.

The pricing of real option can be found from the paper of Black and Scholes (1973) and Merton (1973), this is the famous function, B-S model. After that Cox, Ross, and Rubinstein (1997) propose the generalized Cox-Ross-Rubinstein binomial models; it can be used to price discrete time option. Geske (1979) gives composite options (that is, options whose underlying assets are options) evaluation method. In a multi-stage binary tree model, Muzzioli and Torricelli (2004) use standard triangular fuzzy numbers to estimate the probability distribution of stock prices move, and then calculate the option price based on the average of the probabilities obtained. Carlsson and Fuller (2002) use trapezoidal fuzzy numbers to estimate the project cash flow discount value and investment cost, and use the B-S pricing model to calculate the value of delayed real options. Later, the trapezoidal fuzzy numbers were applied to multi-stage venture capital, and multi-step tri-tree model for solving.

Hei and Yang (2006) propose the price of European option under the ternary option model, which is of two different discrete kinds. That is, it is assumed that there are three states of the change in the price of the underlying asset, namely, rising, constant, and falling, and the probability that the price is constant is constant. The author first calculates the value of a single-term tri-tree option, then uses the backward method to derive the value of the option at each node, and finally gives the option price expression by mathematical induction.

Liu (2008) wrote an article about the research and development of option pricing theory in a fuzzy environment from four aspects. One is the study of European option pricing based on fuzzy theory, mainly the introduction of B-S method; the study of American option pricing; the third is the study of binary tree pricing based on fuzzy theory, which mainly introduces the fuzzy European option binary tree pricing and the fuzzy American option binary tree pricing; the fourth is the study of real option pricing based on fuzzy theory. The author suggests the fuzzy binary tree model used in real options calculations. Zhu, Zhang, Chen, and Gao (2008) use normal fuzzy numbers to process the discounted value of the project's net income cash flow, calculate its mean and variance, and insert it into the BS formula (the mean is the value of the underlying asset, and the variance is the volatility of the underlying asset change), and the value of the real option is obtained.

Zmeskal (2010) has proposed the generalized soft binomial American real option pricing model. It is stochastic discrete binomial models and continuous models are usually applied in option, and valuation under fuzzy numbers (T-numbers).

In paper, we will use the Geske composite method to valuate a government project, furthermore, use the fuzzy number to simulate the uncertainty condition, and in the end we will compare the different method and results.

Theory Part

Fuzzy Set and Theory

In 1965, Zadeh first proposed the fuzzy set theory and extended the classic set theory to the membership relationship, describing the fuzziness and uncertainty of the research object. The value of the research object is no longer limited to the integers 0 and 1, but can be in [0, 1] which provides a new processing method for solving the inaccuracy problem with the characteristics of fuzziness and uncertainty, and fuzzy mathematics was born. In the field of value evaluation, the investment value is determined on the assumption that the parameters of the pricing model can be accurately estimated as a fixed value, and the actual situation generally cannot meet the requirements of this assumption. The parameters in the model are due to various uncertain factors. Existence is often difficult to estimate, and accuracy cannot be guaranteed. The fuzzy set theory can solve the uncertainty problem well. Therefore, the theory can be used to deal with the inaccuracy of the parameters in the pricing model, so that the estimated value is more accurate and reasonable, and the practicality of the model is improved.

Definition 1: A fuzzy set is commonly defined by a membership function (μ) as representation from [0,1], $\mu_A(x)$ is the X to collection $\tilde{A}\widetilde{A}$.

Definition 2: If the collection $\tilde{A}$ is in U, $\lambda\epsilon[0,1]$, $\tilde{A}_\lambda = \{\, x|x\epsilon U, \mu_{\tilde{A}}(X) \geq \lambda\}$, we named the $\tilde{A}_\lambda$ the cut.

Definition 3: Trapezoidal fuzzy number:

The regular convex fuzzy set $\tilde{A}$ on the real number field R is called a fuzzy number. Trapezoidal fuzzy number refers to a fuzzy number with a membership function of the form:

$$\mu_{\tilde{A}}(X)\{\frac{x-X^\alpha}{X^L-X^\alpha}, x \in [X^\alpha, X^L]\ 1, x \in [X^L, X^R]\ \frac{X^\beta - x}{X^\beta - X^R}, x \in [X^R, X^\beta]\ 0, x \notin [X^\alpha, X^\beta] \tag{1}$$

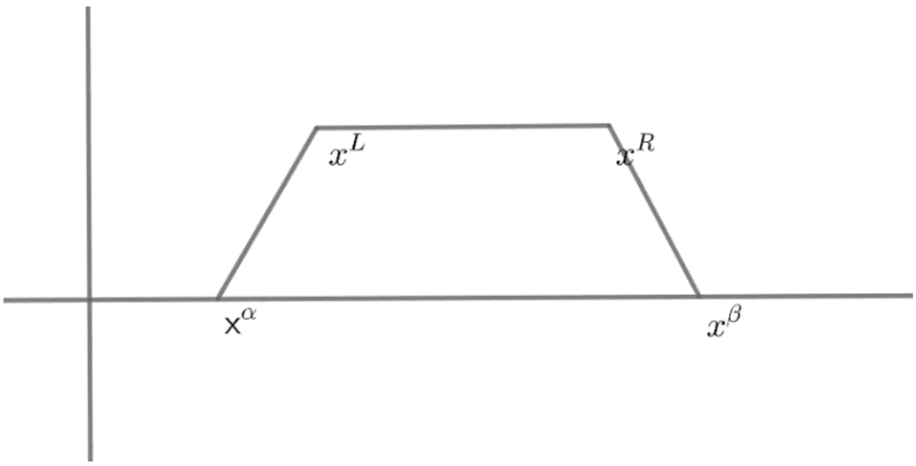

Figure 1. Trapezoidal fuzzy number.

From Figure 1, $X^\alpha, X^L, X^R, X^\beta \in R$ and meeting $X^\alpha \leq X^L \leq X^R \leq X^\beta$, we can sign $\mu_{\tilde{A}} = (X^\alpha, X^L, X^R, X^\beta)$. When $X^L = X^R$, the trapezoidal fuzzy number $\tilde{A}$ degenerates into triangular fuzzy number. When $X^\alpha = X^L = X^R = X^\beta$, $\tilde{A}$ becomes a general real number.

Definition 4: Operation rules of trapezoidal fuzzy numbers:

Similar to general sets, trapezoidal fuzzy numbers can be added and subtracted, and assume the trapezoidal fuzzy number $\tilde{A}_1 = (x_1^\alpha, x_1^L, x_1^R, x_1^\beta)$, $\tilde{A}_2 = (x_2^\alpha, x_2^L, x_2^R, x_2^\beta)$, so we can get:

$$\tilde{A}_1 + \tilde{A}_2 = \left(x_1^\alpha + x_2^\alpha, x_1^L + x_2^L, x_1^R + x_2^R, x_1^\beta + x_2^\beta\right) \tag{2}$$

$$\tilde{A}_1 - \tilde{A}_2 = \left(x_1^\alpha - x_2^L, x_1^L - x_2^\alpha, x_1^R - x_2^\beta, x_1^\beta - x_2^R\right) \tag{3}$$

$$\widetilde{\lambda A}_1 = \left(\lambda x_1^\alpha, \lambda x_1^L, \lambda x_1^R, \lambda x_1^\beta\right) \tag{4}$$

$$E(\tilde{A}) = \frac{X^L + X^R}{2} + \frac{X^\beta - X^\alpha}{6} \tag{5}$$

Geske Composite Real Option Value Evaluation Model

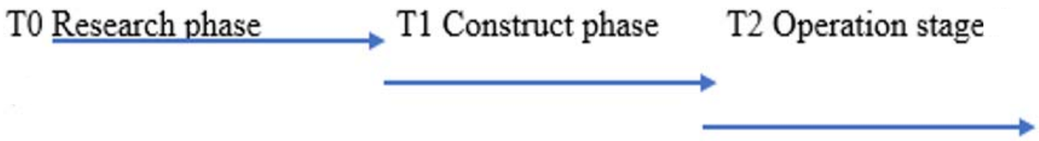

Figure 2. Project investment phase.

According to Geske (1979), we can get the formulation:

$$C = Ve^{-\delta(t_2-t_1)}M(a,b,\rho) - X_2e^{-\gamma(t_2-t_0)}M(a_1,b_1,\rho) - X_1e^{-\gamma(t_1-t_0)}N(a_1) \tag{6}$$

In this formula:

$$a = \frac{ln \ln\frac{V}{V_1} + \left(\gamma - \delta + \frac{\sigma^2}{2}\right)(t_1 - t_0)}{\sigma\sqrt{t_1 - t_0}}, a_1 = a - \sigma\sqrt{t_1 - t_0} \tag{7}$$

$$b = \frac{ln \ln\frac{V}{X_2} + \left(\gamma - \delta + \frac{\sigma^2}{2}\right)(t_2 - t_0)}{\sigma\sqrt{t_2 - t_0}}, b_1 = b - \sigma\sqrt{t_2 - t_0} \tag{8}$$

$$\rho = \sqrt{\frac{t_1 - t_0}{t_2 - t_0}} \tag{9}$$

Here we sign:

$M(a,b,\rho)$ is two-dimensional Gaussian distribution; in this a, b are the upper limit and lower limit, and ρ is the correlation coefficient between variables.

V is the present value of market value.

$N(*)$ is the normal distribution.

t_0 is at the moment when the project is invested in the feasibility study stage, that is, the get time.

t_1 is the decision point for investment in the construction phase, that is also the expiry date of the composite real option C1.

t_2 is the decision point to continue investing in the operation phase; at this moment option C2 expires.

X_1 is the investment expenditure in the construction phase, that is, the strike price of the composite real option C1.

X_2 is the strike price of composite real option C2.

V_1 is the critical value of the project value when the real option C1 is handed over in the first stage, that is also with the critical value of execution; it can be calculated by B-S model.

Fuzzy Geske Model

In this paper, we will make fuzzy process to V and X, and treat them as fuzzy variables. We sign them $\tilde{V} = (V^L, V^R, V^\alpha, V^\beta), \widetilde{X_t} = (X^L, X^R, X^\alpha, X^\beta)$. According to Geske (1979), we can get below:

$$ {}^{-}\tilde{C}^{\lambda} = {}^{-}\tilde{V}^{\lambda} e^{-\delta(t_2-t_1)}\, {}^{-}\widetilde{M}^{\lambda} - {}^{+}\tilde{X}_2^{\lambda} e^{-r(t_2-t_0)}\, {}^{+}\widetilde{M}_1^{\lambda} - {}^{+}\tilde{X}_1^{\lambda} e^{-r(t_1-t_0)}\, {}^{+}N(\widetilde{a_1}) \quad (10) $$

$$ {}^{+}\tilde{C}^{\lambda} = {}^{+}\tilde{V}^{\lambda} e^{-\delta(t_2-t_1)}\, {}^{+}\widetilde{M}^{\lambda} - {}^{-}\tilde{X}_2^{\lambda} e^{-r(t_2-t_0)}\, {}^{-}\widetilde{M}_1^{\lambda} - {}^{-}\tilde{X}_1^{\lambda} e^{-r(t_1-t_0)}\, {}^{-}N(\widetilde{a_1}) \quad (11) $$

Application

Project Introduction

According to the agreement, the Beijing Municipal Government and the franchise company will invest in the Line 4 project at a ratio of 7:3. The project is divided into two parts A and B according to whether the project is profitable. Part A (public welfare part) is 70%, with a total investment of 10.7 billion, including demolition, civil construction, railroads, etc. The infrastructure company is responsible for the project, and the government funds. It is provided to the Public-Private Partnership (PPP) company in two ways: use right investment and lease. Part B (commercial part) is 30%, and the investment of 4.6 billion is handled by the franchise company—MTR Metro Co., Ltd., including the purchase and installation of vehicles, ticket checking systems, communications, elevators, control equipment, and power supply equipment. The capital investment was 1.38 billion yuan, and the remaining 3.22 billion yuan was settled through bank loans. The loan period was 25 years. The loan interest rate was calculated at 5.76% of the five-year annual interest rate in early 2004, and the principal and interest were paid in equal amounts. After the project is completed, MTR Co., Ltd. can franchise for 30 years and pay rent to the government.

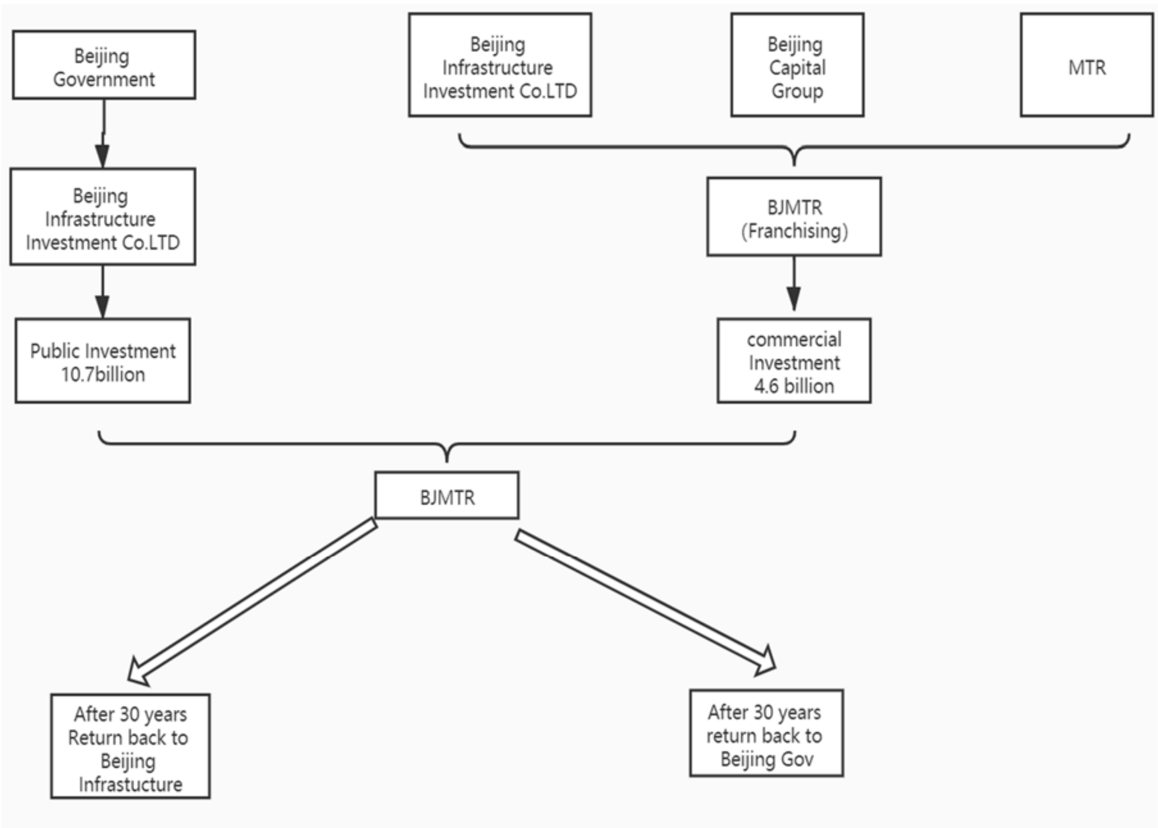

Figure 3. PPP project introduction.

Estimate Parameters

Option strike price. The execution price of the options at each stage is the investment expenditures incurred by investors in order to exercise the options at each stage. The traditional real option method simply divides the investment process of the Line 4 project into two stages, and there is only one decision point after the project is launched (2009). So in the B-S model, the strike price is discount the 30 years operation cost to 2009. However, the composite real option divides the project into three stages; there are two decision point after starting (2005 and 2009). At the point of 2005, the option strike price was equal to investment from 2005 to 2009, and at the point 2009, the option equal to sum investment from 2009 to 2038 discounted to 2009.

Maturity of option. In the Line 4 project, the expiration time of the first option of the traditional B-S model is 2009, and the second is 2038. The Geske composite real option valuation model contains three phased investment options. The first option was obtained in 2004 and expired in 2005. The second option was generated when the previous option expires, that is, it started in 2005 and had an effective period of 2005-2009. The third option is generated immediately after 2009-2038.

Risk free rate. The risk-free interest rates of the three valuation models are all based on the one-year interest rate of my China's first tranche of certificated government bonds issued in 2004, assuming 3%.

The volatility of the price change of the underlying asset. In the B-S model, the volatility of project is assuming equal to the stock market volatility, sign 0.3 and in the fuzzy Geske model, according to Equations (1) to (5) it is calculated 0.18.

Table 1

Line Four Cash Flow (10 Thousand Chinese Yuan)

Year	2004	2005	2006	2007	2008	2009
A_1	92,000	92,000	92,000	92,000	92,000	0
A_2	0	0	0			32,572.45
A_3	0	0	0			77,153.70
A_4	0	0	0			39,000
A_5	-92,000	-92,000	-92,000	-92,000	-92,000	5,581.25
A_6	-92,000	-83,636.36	-76,033.06	-69,120.96	-62,837.24	3,150.47

Year	2010	2011	2012	2013	2014	2015
A_1	0	0	0	0	0	0
A_2	32,572.45	32,572.45	32,572.45	32,572.45	32,572.45	32,572.45
A_3	83,701.80	90,249.90	96,798.00	103,346.10	109,894.2	116,442.30
A_4	39,000	39,000	39,000	39,000	39,000	39,000
A_5	-92,000	-92,000	-92,000	-92,000	-92,000	5,581.25
A_6	6,224.27	8,713.17	10,698.10	12,250.12	13,431.54	14,296.92

Year	2016	2017	2018	2019	2020	2021
A_1	0	0	0	0	0	0
A_2	32,572.45	32,572.45	32,572.45	32,572.45	32,572.45	32,572.45
A_3	116,940.53	117,438.75	117,936.98	118,435.2	118,933.43	119,431.65
A_4	39,000	39,000	39,000	39,000	39,000	39,000
A_5	45,368.08	45,866.3	46,364.53	46,862.75	47,360.98	47,859.20
A_6	13,142.52	12,078.03	11,099.30	10,198.70	9,370.12	8,607.90

Year	2022	2023	2024	2025	2026	2027
A_1	0	0	0	0	0	0
A_2	32,572.45	32,572.45	32,572.45	32,572.45	32,572.45	32,572.45
A_3	119,929.88	120,428.10	120,926.33	121,424.6	121,922.78	122,421.00
A_4	39,000	39,000	39,000	39,000	39,000	39,000
A_5	48,357.43	48,855.65	49,353.88	49,852.10	50,350.33	50,848.55
A_6	7,906.83	7,262.08	6,669.22	6,124.13	5,623.02	5,162.43

Year	2028	2029	2030	2031	2032	2033
A_1	0	0	0	0	0	0
A_2	32,572.45	32,572.45	32,572.45	32,572.45	32,572.45	32,572.45
A_3	122,919.23	123,417.45	123,915.68	124,413.9	124,912.13	125,410.35
A_4	39,000	39,000	39,000	39,000	39,000	39,000
A_5	51,346.78	51,845.00	52,343.23	52,841.45	53,339.68	53,837.90
A_6	4,739.10	4,350.08	3,992.62	3,664.20	3,362.50	3,085.37

Year	2034	2035	2036	2037	2038
A_1	0	0	0	0	0
A_2	0	0	0	0	0
A_3	125,908.58	126,406.80	126,905.03	127,403.3	127,901.48
A_4	39,000	39,000	39,000	39,000	39,000
A_5	86,908.58	87,406.80	87,905.03	88,403.25	88,901.48
A_6	4,527.82	4,139.80	3,784.91	3,460.33	3,163.48

Project Cash Flow

According to the project plan and our calculation, we can get the table below, which contains cash flow from 2004 to 2038.

The data use abbreviation for each item, like construct investment (A_1), loan payment (A_2), operation income (A_3), operation cost (A_4), net income cash flow (A_5), and A_6 means net present value, all data in appendix.

Among these table: $A_5 = A_1 + A_3 - A_2 - A_4$.

Fuzzy Geske Composite Real Option

Under normal circumstances, the fluctuation range of the present value of project income is [-10%, 10%], and when the market environment and other conditions are more favorable to investors, when the present value of income is the largest, it will increase by 30% on the original basis. On the contrary, when the market environment and other conditions are very frequent for investors, when the present value of income is taken to the minimum, it will drop by 15% on the original basis. In the same way, the investment cost under normal circumstances the fluctuation range of is [-15%, 15%]. In the most ideal state, the investment cost will drop by 20%. When the market environment and other factors are the most unfavorable for investors, the investment cost may increase by 25%.

First, we fuzzy our parameters:

$$\alpha = V * 15\% = 628989.55 * 0.15 = 94384$$

$$\beta = V * 30\% = 628989.55 * 0.30 = 188697$$

$$V^L = V * (1 - 10\%) = 628989.55 * 0.9 = 566091$$

$$V^R = V * (1 + 10\%) = 628989.55 * 1.1 = 691889$$

So we get:

$$\tilde{V} = \left(V^L, V^R, V^\alpha, V^\beta\right) = (566091{,}691889{,}94384{,}188697)$$

Then we need fuzzy the parameters step by step:

Step 1	Fuzzy V
Step 2	Fuzzy volatility
Step 3	Fuzzy m_1 and m_2
Step 4	Fuzzy X_0, X_1, X_2
Step 5	Fuzzy normal distribution
Step 6	Fuzzy V_1
Step 7	Fuzzy a, a_1, b, b_1
Step 8	Fuzzy two-dimension Gaussian distribution
Step 9	Fuzzy C

Firstly, we need to fuzzy the volatility; the results show below table:

Table 2

Fuzzy Parameters

Cut		σ		v	M		M1	
λ	-	+	-	+	-	+	-	+
1	0.166068	0.202971	566,091	691,889	0.681285	0.886948	0.507486	0.7993
0.75	0.141068	0.252971	542,504	738,913.3	0.601079	0.877839	0.363873	0.762156
0.5	0.116068	0.302971	518,917	785,937.5	0.536246	0.878229	0.251	0.740157
0.25	0.091068	0.352971	495,330	832,961.8	0.48308	0.885254	0.165143	0.730863
0	0.066068	0.402971	471,743	879,986	0.440204	0.897139	0.102658	0.732956

So

$$\tilde{C} = \tilde{V}e^{-\delta(t_2-t_1)}M(a,b,\rho) - \widetilde{X_2}e^{-r(t_2-t_0)}M(a_1,b_1,\rho) - \widetilde{X_1}e^{-r(t_1-t_0)}N(a_1)$$

$$=\left(V^L, V^R, V^\alpha, V^\beta\right) * e^{-\delta(t_2-t_1)}M(a,b,\rho) - \left(X_2^L, X_2^R, X_2^\alpha, X_2^\beta\right)e^{-r(t_2-t_0)}M(a_1,b_1,\rho) - \left(X_1^L, X_1^R, X_1^\alpha, X_1^\beta\right)e^{-r(t_1-t_0)}N(a_1)$$

$$=\left[V^L e^{-\delta(t_2-t_1)}M(a,b,\rho) - X_2^R e^{-r(t_2-t_0)}M(a_1,b_1,\rho)\ V^R e^{-\delta(t_2-t_1)}M(a,b,\rho) - X_2^L e^{-r(t_2-t_0)}M(a_1,b_1,\rho)\ \alpha e^{-\delta(t_2-t_1)}M(a,b,\rho) - \beta_2 e^{-r(t_2-t_0)}M(a_1,b_1,\rho)\ \beta e^{-\delta(t_2-t_1)}M(a,b,\rho) - \alpha_2 e^{-r(t_2-t_0)}M(a_1,b_1,\rho)\ \right] - \left(X_1^L, X_1^R, X_1^\alpha, X_1^\beta\right)e^{-r(t_1-t_0)}N(a_1)$$

= (111,292.4, 193,195.3576, 223,351.3993, 327,547.3493)

Results Comparison

Starting from the analysis of the characteristics of the compound real option in the Beijing Metro Line 4 project, the fuzzy Geske compound real option value evaluation model is used to estimate its investment value, referring to the actual situation of the Line 4 project with the traditional net present value formula, BS pricing model, and Geske composite real option value evaluation model for comparative analysis of investment decisions. The evaluation results shown in Table 3 can be finally obtained.

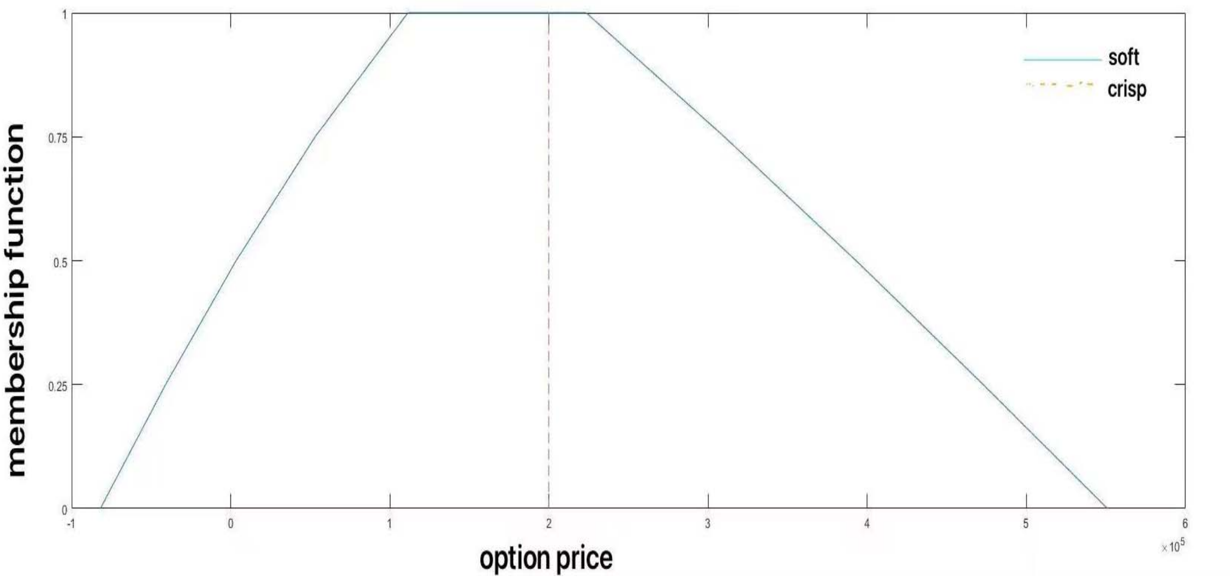

Figure 4. Comparison of crisp and soft results.

Table 3

Projects Value Comparison Under Different Methods

	NPV	B-S	Geske	Fuzzy Geske
Option		175,446.07	199,838.10	189,713.9
Project	-168,989.55	6,456.52	30,848.55	20,725.89

The investment value of the Beijing Metro Line 4 project calculated with the NPV method is -168,989,500 yuan, and the total investment value calculated by the real option B-S model is 64,565,200 yuan. The investment value of the latter is 175,446,700 yuan greater than the former, indicating that the total value of the project can be greatly increased after considering the uncertainty of the project. At the same time, the total investment value calculated by the compound real option Geske model is 308,485,500 yuan, which is higher than the calculation result of the BS model. An increase of 243,920,300 yuan, the increased value is mainly derived from the value appreciation generated by the compounding of a single real option. In addition, the option values calculated by the fuzzy compound real option method are (111,292.4, 223,351.3993, 193,195.3576, and 327,547.3493) yuan at the different member functions. The main reason is the change of the volatility and other parameters, which also shows from the side that the value source of the real option will be affected by other market information and changes in real world.

Conclusions

From the perspective of social capital investors, this paper proposes a PPP project value evaluation model based on the fuzzy compound real option method. Taking the value evaluation of PPP projects as the research object, the relevant concepts and this paper proposes a model for evaluating the value of Public-Private Partnership (PPP) projects from the perspective of social capital investors. The model employs the fuzzy compound real option method. It carefully defines relevant concepts and theoretical foundations and analyzes the existing value of PPP in detail.

The compound real option method is effective in evaluating the value of the project's future uncertainty and management flexibility, making the evaluation result more accurate. However, it has a rigid treatment of parameters that does not conform to actual projects. The value evaluation model based on fuzzy Geske compound real options fully considers the flexibility of investment decisions. It avoids traditional net present value method's ignorance of the uncertainty of PPP projects and makes up for the lack of interaction in the real options method at all stages. As a result, it realizes the staged dynamic investment decision of the project.

The fuzzy Geske composite real option value evaluation model uses the characteristics of fuzzy numbers to accurately express the flexible value of the project. The evaluation obtains the project value that is actually a fuzzy number. The project value is expanded from a specific value to a selection range, which provides investors with more investment decision-making opportunities. Hence, it is more suitable for multi-stage PPP in a complex and uncertain environment project value assessment. Theoretical foundations are carefully sorted out and defined, and the existing value of PPP is analyzed in details.

The compound real option method can effectively evaluate the value of the project's future uncertainty and management flexibility, making the evaluation result more accurate, but the treatment of parameters is more rigid, which does not conform to the actual project. The value evaluation model based on fuzzy Geske compound real options fully considers the flexibility of investment decisions, not only avoids the traditional net present value method ignoring the uncertainty of PPP projects, but also makes up for the ignorance of real options in the real options method at all stages The lack of interaction has realized the staged dynamic investment decision of the project. Fuzzy Geske composite real option value evaluation model uses the characteristics of fuzzy numbers to accurately express the flexible value of the project, and the evaluation obtains the project value of which is actually a fuzzy number. The project value is expanded from a specific value to a selection range, which provides investors with more investment decision-making opportunities, so it is more suitable for multi-stage PPP in a complex and uncertain environment project value assessment.

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