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A Fractal Orifice-Throat Model for Seepage Characteristics of Multiscale Porous Media

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Abstract: The seepage characteristics of multiscale porous media is of considerable significance in many scientific and engineering fields. The Darcy permeability is one of the key macroscopic physical properties to characterize the seepage capacity of porous media. Therefore, based on the statistically fractal scaling law of porous media, fractal geometry is applied to model the multiscale pore structures. And a two-dimensional fractal orifice-throat model with multiscale and tortuous characteristics is proposed for the seepage flow through porous media. The analytical expression for Darcy permeability of porous media is derived, which is validated by comparing with available experimental data. The results show that the Darcy permeability is significantly influenced by porosity, orifice-throat fractal dimension, minimum to maximum diameter ratio, orifice-throat ratio and tortuosity fractal dimension. The present results are helpful for understanding the seepage mechanism of multiscale porous media, and may provide theoretical basis for unconventional oil and gas exploration and development, porous phase transition energy storage composites, CO_2 geological sequestration, environmental protection and nuclear waste treatment, etc.

Key words: Multiscale porous media, fractal geometry, Darcy's law, permeability, orifice-throat model.

1. Introduction

The seepage flow through porous media plays an important role in many fields, such as oil and gas engineering, underground water utilization, environmental protection, energy storage, earthquake prediction and biomedical science etc. As the theoretical foundation for seepage flow through porous media, the linear Darcy's law was firstly proposed by Darcy [1] in 1856 through an experiment. Dupuit [2] has subsequently improved Darcy's law and proposed Dolby's formula for gradient seepage in 1857. Then various permeability models have been presented to characterize the seepage capacity of porous media. Lei et al. [3] obtained an empirical formula for the permeability based on the seepage experiment on particle beds. Scholz et al. [4] measured the permeability of randomly placed elliptical particles, and argued that the permeability depends on the probability of overlapping particles rather than the particle shape. Jiang et al. [5] obtained the empirical formula for the permeability of power-law fluid through porous media. Adler et al. [6] measured the permeability of glass particles and proposed an empirical permeability model. Zhang et al. [7] analyzed the flow resistance of the orifice-throat model by numerical simulation. Wang et al. [8] studied the relationship between permeability and pressure gradient by numerical simulation, and analyzed the pressure response characteristics of power-law fluid through porous media. Lee et al. [9] derived a general model for the permeability of porous fiber. Ahmadi et al. [10] calculated the permeability of single-size spherical arrays using the volume averaging method.

However, it is difficult to characterize the microstructure quantitatively and accurately by using Euclidean methods. Fortunately, many studies show that the pore structures in a lot of natural porous media indicate fractal scaling laws [11]. Therefore, fractal geometry has been proposed and applied to study the seepage

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flow characteristics of porous media. For example, Yu et al. [12] proposed a fractal model for the permeability of the randomly disordered porous media. Wu et al. [13] developed a new fractal model for the periodic pore morphology and derived the Darcy permeability expression. Xu et al. [14] used the fractal capillary bundle model to deduce the analytical expression for the relative permeability of unsaturated porous media. Wang et al. [15] investigated the seepage characteristics of power-law fluid through porous media by applying arbitrary cross-sectional capillaries based on fractal theory. Cai et al. [16] used the orifice-throat model to investigate the influence of flow curvature and pore connectivity on the permeability of porous media. Sumantri [17] combined the Newton viscosity law and fractal geometry to derive the permeability of sandstone. Zhao [18] established a mathematical model for the coupling seepage-consolidation of rock based on a fractal fracture network model.

It can be found that both the pore size distribution and orifice-throat structure show important effect on the seepage flow through multiscale porous media. Therefore, a two-dimensional fractal orifice-throat model is developed to study the seepage flow through multiscale porous media. The orifice-throat structure is used as the basic unit for seepage flow, and pore fractal dimension and tortuosity fractal dimension are introduced to characterize the pore size distribution and tortuousness of flow paths, respectively. And the analytical expression for the Darcy permeability is derived. The influence mechanisms of pore structure on seepage characteristics are analyzed in detail.

2. Fractal Orifice-Throat Model

According to the statistical fractal scaling law of porous media, the multi-scale pore structure and curved flow channel of porous media are characterized by a two-dimensional fractal orifice-throat model shown in Fig. 1, where the flow path of fluid in porous media is represented by a bundle of orifice-throat chains. The length of the concatenated orifice-throat is L_0. And each chain has n square orifice-throat structural units, where the diameter of the throat and orifice are λ and $\beta\lambda$, respectively. The orifice-throat ratio is β. Based on the orifice-throat structural unit shown in Fig. 1c, the porosity can be expressed by:

$$\phi = 1 - \left[(\beta - 1) / \beta \right]^2 \tag{1}$$

When the porosity is $\phi=1$, the orifice-throat ratio is $\beta=1$, however, the orifice-throat ratio approaches to infinity as the porosity $\phi=0$. It is assumed that the tortuous length of the orifice-throat chain follows the following fractal scaling law [19]:

$$l_t(\lambda) = \lambda^{1-D_T} \left[(\beta - 1)\lambda \right]^{D_T} = (\beta - 1)^{D_T} \lambda \tag{2a}$$

$$l_o(\beta\lambda) = (\beta\lambda)^{1-D_T} \lambda^{D_T} = \beta^{1-D_T} \lambda \tag{2b}$$

where l_o and l_t represent the actual length of fluid flow path in a orifice and throat, respectively. The area of a single orifice-throat structural unit is:

$$s = \lambda l_t(\lambda) + \beta\lambda l_o(\beta\lambda) = \lambda^2 \left[(\beta - 1)^{D_T} + \beta^{2-D_T} \right] \tag{3}$$

Since the number of orifice-throat unit in a chain is $n = L_0 / (\beta\lambda)$, the area of a single concatenated orifice-throat chain is:

$$s_{ot}(\lambda) = ns = \beta^{-1} \lambda L_0 \left[(\beta - 1)^{D_T} + \beta^{2-D_T} \right] \tag{4}$$

The size of the orifice-throat satisfies the self-similar fractal scaling law [19]:

$$f(\lambda) = D_f \lambda_{\min}^{D_f} \lambda^{-(D_f+1)} d\lambda \tag{5}$$

where D_f is the orifice-throat fractal dimension. The above formula is the probability density distribution function of the orifice-throat scale λ. There is a quantitative relationship between the orifice-throat fractal dimension D_f and porosity ϕ:

$$\phi = (\lambda_{\min} / \lambda_{\max})^{D_E - D_f} \tag{6}$$

where $\lambda_{\min}$ and $\lambda_{\max}$ denote the minimum and maximum orifice-throat diameter. The Euclidean dimension D_E takes 2 and 3 in 2D and 3D space, respectively. If the

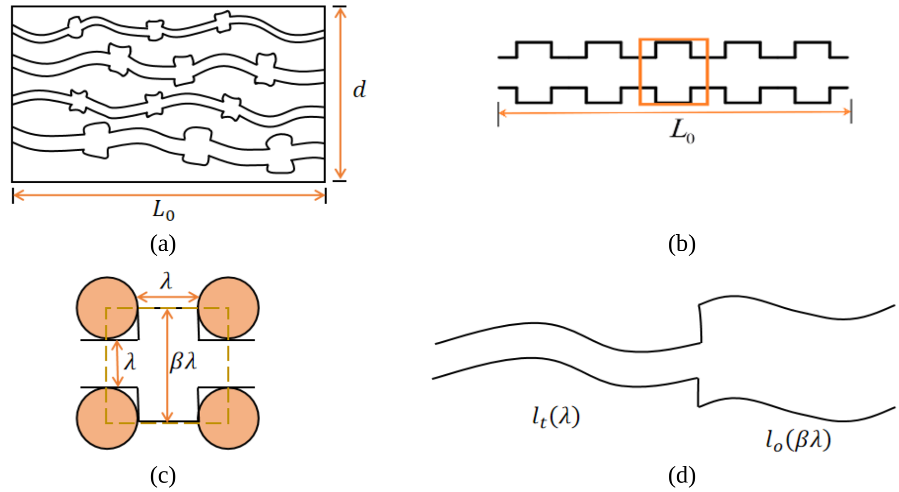

Fig. 1 Schematic diagram of fractal orifice-throat model for multiscale porous media: (a) representative elementary area, (b) concatenated orifice-throat chain, (c) orifice-throat structural unit, (d) equivalent orifice-throat.

fractal probability density distribution function satisfies the normalization condition, then $(\lambda_{\min}/\lambda_{\max})^{D_f}=0$. The number of concatenated orifice-throat chain with the size distribution interval in λ and $\lambda+d\lambda$ is:

$$dN = D_f \lambda_{\max}^{D_f} \lambda^{-(D_f+1)} d\lambda \tag{7}$$

The total area of the orifice-throat chains can be gotten by combing Eqs. (4) and (7):

$$S_{ot} = \int_{\lambda_{\min}}^{\lambda_{\max}} s_{ot}(\lambda)\,dN = \frac{D_f}{1-D_f}\frac{\lambda_{\max} L_0}{\beta}\left[(\beta-1)^{D_T}+\beta^{2-D_T}\right]\left[1-\phi^{(1-D_f)/(2-D_f)}\right] \tag{8}$$

The cross-sectional area of porous media can be written as:

$$S = S_{ot}/\phi = \frac{D_f}{1-D_f}\frac{\lambda_{\max} L_0}{\phi\beta}\left[(\beta-1)^{D_T}+\beta^{2-D_T}\right]\left[1-\phi^{(1-D_f)/(2-D_f)}\right] \tag{9}$$

Thus, the side length of the porous media sample is:

$$d = \frac{D_f}{1-D_f}\frac{\lambda_{\max}}{\phi\beta}\left[(\beta-1)^{D_T}+\beta^{2-D_T}\right]\left[1-\phi^{(1-D_f)/(2-D_f)}\right] \tag{10}$$

3. Darcy Permeability Model

The steady-state laminar flow of incompressible Newtonian fluids through porous media is studied, assuming that the inner wall of the orifice-throat channel is smooth without slip interface. Based on mass conservation and Hagen-Poiseulle equation [11], the flow rate through an orifice-throat channel can be expressed as:

$$q = \frac{1}{12}\frac{\lambda^3}{\mu}\frac{\Delta p_t}{l_t(\lambda)} = \frac{1}{12}\frac{(\beta\lambda)^3}{\mu}\frac{\Delta p_o}{l_o(\beta\lambda)} \tag{11}$$

where Δp_o and Δp_t represent the pressure drop along the orifice and throat, respectively, μ is the fluid viscosity. The pressure drop along an orifice-throat unit is $\Delta p = \Delta p_t + \Delta p_o$. Thus:

$$\Delta p_t = \frac{\beta^{2+D_T}(\beta-1)^{D_T}}{1+\beta^{2+D_T}(\beta-1)^{D_T}}\Delta p \tag{12a}$$

$$\Delta p_o = \frac{1}{1+\beta^{2+D_T}(\beta-1)^{D_T}}\Delta p \tag{12b}$$

Therefore, the pressure drop along the concatenated orifice-throat chain is expressed as:

$$\Delta P = n\Delta p \tag{13}$$

Inserting Eqs. (12) and (13) into (11) results in:

$$q=\frac{1}{12}\frac{\lambda^{3}}{\mu}\frac{\beta^{3+D_T}}{1+\beta^{2+D_T}\left(\beta-1\right)^{D_T}}\frac{\Delta P}{L_0} \tag{14}$$

The total flow rate through the porous media can be obtained by using the fractal scaling law of orifice-throat:

$$Q=\int_{\lambda_{\min}}^{\lambda_{\max}}qdN=\frac{\lambda_{\max}^{3}}{12\mu}\frac{D_f}{3-D_f}\frac{\beta^{3+D_T}}{1+\beta^{2+D_T}\left(\beta-1\right)^{D_T}}\frac{\Delta P}{L_0} \tag{15}$$

The apparent velocity through porous media can be expressed as:

$$v_s=\frac{Q}{d}=\frac{1}{12\mu}\frac{1-D_f}{3-D_f}\frac{\beta^{4+D_T}}{1+\beta^{2+D_T}\left(\beta-1\right)^{D_T}}\frac{1}{\left(\beta-1\right)^{D_T}+\beta^{2-D_T}}\frac{\phi}{1-\phi^{\left(1-D_f\right)/\left(2-D_f\right)}}\lambda_{\max}^{2}\frac{\Delta P}{L_0} \tag{16}$$

The fluid flow through porous media satisfies the linear Darcy's law:

$$v_s=\frac{K}{\mu}\frac{\Delta P}{L_0} \tag{17}$$

Therefore, the effective permeability can be obtained by using Eqs. (16) and (17):

$$K^{+}=\frac{1}{12}\frac{1-D_f}{3-D_f}\frac{\phi\Big/\left[1-\phi^{\left(1-D_f\right)/\left(2-D_f\right)}\right]}{\left(1-\varepsilon^{1/2}\right)^{4+D_T}\left[\varepsilon^{D_T/2}\Big/\left(1-\varepsilon^{1/2}\right)^{2+2D_T}+1\right]\left[\varepsilon^{D_T/2}\Big/\left(1-\varepsilon^{1/2}\right)^{D_T}+1\Big/\left(1-\varepsilon^{1/2}\right)^{2+D_T}\right]} \tag{18}$$

where $\varepsilon=1-\phi$, and the dimensionless effective permeability is defined as $K^{+}=K\big/\lambda_{\max}^{2}$.

4. Results and Discussion

In order to validate the present fractal model, the predicted permeability was compared with two sets of experimental data [20, 21]. The fractal dimension of tortuosity is calculated by the formula $D_T=1+\frac{\ln\tau_{ave}}{\ln\left(L_0/\lambda_{ave}\right)}$, where τ_{ave} is the average tortuosity, λ_{ave} is the average aperture. The average tortuosity can be calculated by $\tau_{ave}=\frac{1}{2}\left[1+\frac{\sqrt{1-\phi}}{2}+\frac{\sqrt{1-\phi}\sqrt{\left(1/\sqrt{1-\phi}-1\right)^{2}+1/4}}{\left(1-\sqrt{1-\phi}\right)}\right]$ [22]. As shown in Fig. 2, the predicted permeability agrees well with the experimental data ($\lambda_{\min}/\lambda_{\max}=0.06$, $L_0=12cm$, $\lambda_{ave}=0.1cm$). The value of permeability increases with the increase of porosity.

The macroscopic permeability is greatly influenced by the microstructure of saturated porous media, including porosity, orifice-throat and tortuosity fractal dimension, orifice-throat ratio, minimum and maximum orifice-throat size. The effect of the ratio of minimum to maximum orifice-throat size and porosity on dimensionless Darcy permeability is depicted in Fig. 3. It can be clearly seen that the ratio of minimum to maximum orifice-throat size takes important influence on the effective permeability even when the porosity is same. Under same porosity, the effective permeability decreases with the increase of orifice-throat fractal dimension. It can be attributed to the increment of orifice-throat structure and portion of small orifice-throat unit.

Fig. 4 shows the relationship between the effective permeability and tortuosity fractal dimension as well as orifice-throat ratio. The increase of tortuosity fractal dimension means the orifice-throat chain is more tortuous, which can enhance the flow resistance. Thus, the effective permeability decreases as the tortuosity fractal dimension increases. At the same time, the dimensionless Darcy permeability increases as pore-throat ratio decreases. It can be attributed to enhanced porosity by reducing orifice-throat ratio.

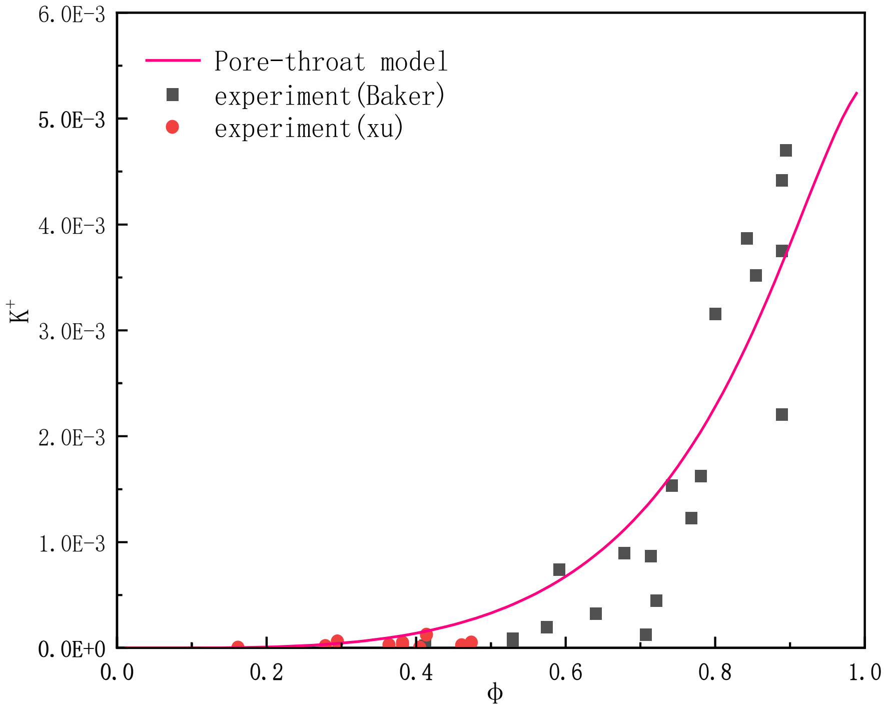

Fig. 2 A comparison of effective permeability between fractal pore-throat model and experimental data.

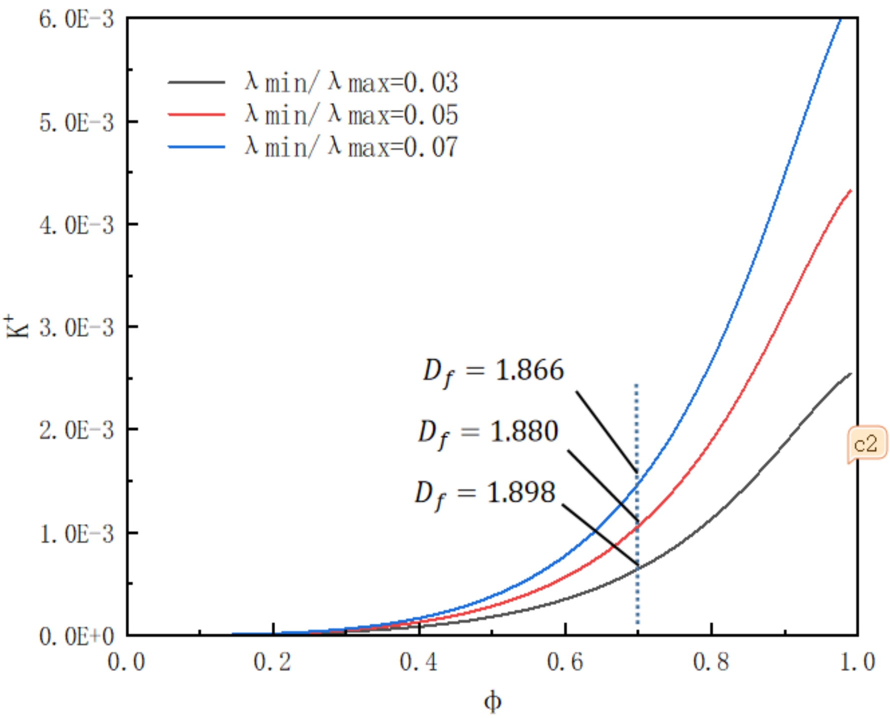

Fig. 3 Influence of porosity and minimum-maximum diameter ratio on dimensionless Darcy permeability.

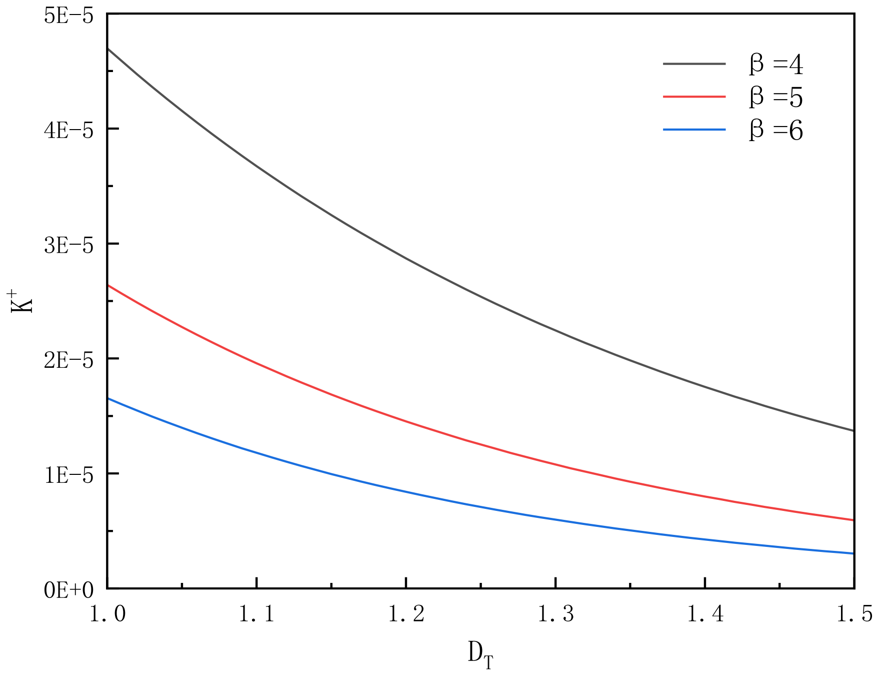

Fig. 4 Influence of orifice-throat ratio and tortuosity fractal dimension on dimensionless Darcy permeability.

5. Conclusion

In this paper, a two-dimensional fractal orifice-throat model for Darcy seepage flow has been developed based on the statistically fractal scaling laws of the multiscale porous media. The analytical expression for the Darcy permeability was presented, which has been validated by comparing with experimental data. The results demonstrate that the permeability of porous media is significantly influenced by the microstructure parameters of multiscale porous media, including porosity, the ratio of minimum to maximum orifice-throat size, orifice-throat ratio, orifice-throat and tortuosity fractal dimensions. The effective permeability increases with the increase of porosity, however, it can be reduced by increasing tortuosity fractal dimension and orifice-throat ratio. When the porosity is certain, the increase of orifice-throat fractal dimension can further lower the effective permeability. The proposed fractal orifice-throat model may help understand the seepage mechanisms in multiscale porous media, and provide theoretical basis for oil-gas and geothermal development, Carbon Capture, Utilization and Storage, stored energy and environmental protection etc.

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