

Some Classical Terzaghi Primary Consolidation Differential Equation Mathematical Added Variabilities in Order to Get Better Field Monitoring Data Match Results

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Abstract: In this paper is illustrated a mathematical added variability of the classical Terzaghi primary consolidation equation (1923) considering independently as variables the Consolidation Coefficient C_v and as well the Height H_i of the consolidating Laboratory Consolidation Test soil sample in order to finally grasp the low permeability layer time behaviour. It is easy to show that, when the C_v variation is positive, each of these two added variabilities differentiations has as maximum a factor 2 related to the laboratory evaluated coefficient of consolidation, for a certain incremental load of reference in a Laboratory Consolidation Test. At this scope, it is analysed the overall behaviour of a typical clayey material, from the mineralogical point of view, namely especially either composed by lean clay with main kaolinite mineralogical content or fat namely with Illite mineralogical content or even very dilatant namely principally constituted by Montmorillonite. The Montmorillonite variability with C_v is negative, and consequently the differentiation enhancement factor can become naught. As it is known so far, in normal conditions of a soft clay, a difference in Construction Values of the Coefficient of Consolidation is up to 23 times greater than laboratory evaluated results, and this according to the author's experience, may be also mainly explained not starting from Laboratory Consolidation Test Data, but through a more general macroscopic behaviour of the soil underneath the newly loaded area, putting aside the case of temperature-induced changes. In conclusion, it is suggested how to model the analytical problem of the so modified Terzaghi Primary Consolidation differential equation in order to better manage the construction unknowns of the phenomenon.

Key words: Coefficient of consolidation, embankment construction, primary consolidation, soil permeability, settlements, vertical settlements, void ratio.

1. Introduction

1.1 The Classical Terzaghi Primary Consolidation Differential Equation

In Ref. [1], it is resumed the most noticeable work in geomechanics done by Terzaghi since the very beginning of the modern approach in soil mechanics, namely the Theory of Consolidation and its classical three-dimensional differential equation:

$$\frac{\partial u}{\partial t} = C_v \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

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where u is the excess pore pressure, t the time and x,y,z the spatial variability, while C_v is described afterward.

1.2 Taylor & Merchant (1940) Combination of Primary Consolidation and Creep

In Ref. [2], also it is developed the full differential equation in terms of the void Ratio e , namely the ratio between the void volume and the solid volume, to take into consideration both the primary consolidation phenomenon and the viscous secondary consolidation phenomenon and namely, after integration:

$$\Delta e = \int_0^{t_p} \left[\left(\frac{\partial e}{\partial \sigma'_v} \right)_t \frac{d\sigma'_v}{dt} + \left(\frac{\partial e}{\partial t} \right)_{\sigma'_v} \right] dt + \int_{t_p}^t \left(\frac{\partial e}{\partial t} \right)_{\sigma'_v} dt$$

where σ'_v is the effective stress.

1.3 The Double Layer Theory and the Clay Platelets Hydrogen Bonds

Moreover from the chemical point of view, we know that between Clays platelets there is an equilibrium between thermically provoked repulsive forces and attractive Van der Waals Forces that act in the so-called Freunlich's diagram as indicated in Ref. [3], and especially it is noticeable that the clay platelets sheets are hold together by the presence of Hydrogen Bonds.

2. The Coefficient of Consolidation and Height of the Consolidating Layer Variability with Time

2.1 (t/T_v) Variability with C_v

Also, from Refs. [1, 4, 5] we have that the time t for the primary consolidation to be fully achieved is given by:

$$\log_{10} \left(\frac{T_v}{t} \right) = \log_{10} \left(\frac{C_v}{H^2} \right) = \text{const}$$

being T_v , the time factor and H the semi-height of the soil specimen, namely:

$$\frac{t}{T_v} = \frac{H^2}{C_v}$$

Where, being k is the soil permeability and γ_w the water unit weight, C_v is:

$$C_v = \frac{k}{m_v \gamma_w}$$

And in terms of oedometric modulus, initial void ratio and the coefficient of compressibility a_v ,

$$\frac{1}{E_{oed}} = m_v = \frac{a_v}{(1 + e_0)}$$

But actually, during a laboratory evaluated Consolidation Test Load Increment, both C_v and H have indeed to be thought as variable, and so, if we isolate this aspect, differentiate the ratio (t/T_v) with respect to the independent variable C_v , we get:

$$\frac{d \left(\frac{t}{T_v} \right)}{d C_v} = - \frac{H^2}{(C_v)^2}$$

Hence integrating, we have:

$$\begin{aligned} \left(\frac{t}{T_v} \right)_f &= \left(\frac{t}{T_v} \right)_i - H^2 \int_{(C_v)_i}^{(C_v)_f} \frac{d C_v}{(C_v)^2} \\ &= + \frac{H^2}{C_{v_i}} + H^2 \left(- \frac{1}{C_{v_f}} + \frac{1}{C_{v_i}} \right) \\ &= H^2 \left(- \frac{1}{C_{v_f}} + \frac{2}{C_{v_i}} \right) \end{aligned}$$

where the indexes f and i mean final and initial.

We also have that:

$$k = D_s^2 \frac{\gamma}{\mu (1 + e)} c$$

where μ is the water dynamic viscosity, c is a soil shape constant related to the soil – water system and D_s is the inverse of the surface to volume ratio for the soil particles.

This means that in the normal variability of the Void Ratio e , for Kaolinite and Illite Clay Minerals, and for a consolidating layer underneath an Highway or Railway track, in oedometric condition, could have a very limited maximum of the ratio $\Delta[e^3/(1 + e)]$, that along with a normal trend of E_{oed} of 20% or more, would entail an augmentation of the coefficient of consolidation C_v during the consolidation process.

Moreover, we also know that the compression index C_c is:

$$C_c = 1,35 \frac{PI}{100}$$

where the plasticity index PI is greater for Illite Clay Minerals and this is thought to be due to a greater presence of the hydroxyl radical OH in the mineral for the kaolinite, likely in the way indicated by Low [6], that renders the water breakage action on the typical hydrogen bonds, easier in producing the soil liquid state, namely, it does produce a lower liquid limit and so a lower plastic limit.

On the contrary for Montmorillonite Clay, C_v normally tends to diminish first all because of a different value of the clay overall geo chemo mechanical compressibility and therefore the actual C_v

value must be considered as by far greater, for this case, than the usual results of the Laboratory Consolidation Tests.

So focusing on Kaolinite and Illite Clay Minerals, we get, for example for $C_{vf} = 4/3 C_{vi}$;

$$\left(\frac{t}{T_v}\right)_f = \frac{5H^2}{4 C_{vi}}$$

And also for $C_{vf} = 3/2 C_{vi}$:

$$\left(\frac{t}{T_v}\right)_f = \frac{4H^2}{3 C_{vi}}$$

So, one is authorized to search the limit:

$$\lim_{\frac{C_{vf}}{C_{vi}} \rightarrow \infty} \left(\frac{t}{T_v}\right)_f = \frac{2H^2}{C_{vi}}$$

and on the contrary, for a Montmorillonite Clay:

$$\lim_{\frac{C_{vf}}{C_{vi}} \rightarrow \frac{1}{2}} \left(\frac{t}{T_v}\right)_f = 0$$

2.2 (t/Tv) Variability with Hi

Likewise, if we differentiate the ratio (t/Tv) with respect to the independent variable H of the soil specimen, we get:

$$\frac{d\left(\frac{t}{T_v}\right)}{dH} = - \frac{2H}{C_{vi}}$$

Hence integrating, we get:

$$\begin{aligned} \left(\frac{t}{T_v}\right)_f &= \left(\frac{t}{T_v}\right)_i - \int_{H_i}^{H_f} \frac{2HdH}{C_{vi}} \\ &= + \frac{H_i^2}{C_{vi}} + \left(-\frac{H_f^2}{C_{vi}} + \frac{H_i^2}{C_{vi}}\right) \\ &= \frac{2H_i^2}{C_{vi}} - \frac{H_f^2}{C_{vi}} \end{aligned}$$

And we get, for example for $H_f = 2/3 H_i$;

$$\left(\frac{t}{T_v}\right)_f = \frac{14 H_i^2}{9 C_{vi}}$$

And go to the limit:

$$\lim_{\frac{H_f}{H_i} \rightarrow 0} \left(\frac{t}{T_v}\right)_f = \frac{2H^2}{C_{vi}}$$

3. Discussion

3.1 The Actual Cv Value

Therefore the actual C_v from Laboratory Consolidation Test for a certain incremental load for kaolinite and Illite clays would totally be up to a maximum of 4 times the ordinary deducted value either if calculated with the Taylor or the Casagrande Technique or even if calculated as the average value between the two procedures.

This evaluation does not consider other *in situ* discrepancies and variabilities from the Laboratory traditionally evaluated Coefficient of Consolidation. Indeed, these variabilities are mainly due to the soil volume large heterogeneities under the Constructed Highway Embankment as discussed in a Companion Paper [7].

All these variabilities could be cumulatively included in some indications given after 7 Italian soft clay deposits comprehensive investigation values, where were attained up to 23 times the laboratory C_v values [8].

4. Conclusions

The classical fundamental primary consolidation Terzaghi differential equation has been revisited with some added variabilities to take into account the fact that Coefficient of Consolidation C_v is indeed not constant as supposed, but changes with the soil specimen geo chemo mechanical compressibility, and by the height reduction of the sample itself.

The analyzed added variabilities differential equations indicate that, putting aside the traditional discrepancies in the C_v value due to the two procedures namely the Casagrande and the Taylor one, a factor of 4, for clays with either Kaolinite or Illite mineralogical content, is the maximum difference with the traditional Laboratory Consolidation Tests

evaluations.

Contrarily in the case of Montmorillonite Clay C_v diminishes with the compressibility and therefore the actual value to be considered will be by far greater and correspondingly the actual primary consolidation phenomenon will be by far much shorter in time.

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