Economics World, Apr.-Jun. 2020, Vol.8, No.2, 51-63
doi: 10.17265/2328-7144/2020.02.002

Portfolio Selection Using Double GAs Searching for Cardinality and Integer Multiplied Optimal Weights

Gumsong Jo, Duson Kim, Namung Ri, Hoyong Kim
Kim Il Sung University, Pyongyang, Democratic People's Republic of Korea

This paper studied cardinality constrained portfolio with integer weight. We suggested two optimization models and used two genetic algorithms to solve them. In this paper, after finding well matching stocks, according to investor's target by using first genetic algorithm, we gave optimal integer weight of portfolio with well matching stocks by using second genetic algorithm. Through numerical comparisons with other feasible portfolios, we verified advantages of designed portfolio with two genetic algorithms. For a numerical comparison, we used a prepared data consisted of 18 stocks listed in S & P 500 and numerical example strongly supported the designed portfolio in this paper. Also, we made all comparisons visible through all feasible efficient frontiers.

Keywords: portfolio tselection, cardinality, efficient frontier, geneic algorithm (GA), Sharpe ratio

Introduction

Stock investment goes with the risk caused by changes of prices that is one of the characteristics of capital market. Portfolio optimization deals with the problem of allocating one's capital to different assets to reduce risk while increasing returns. Many studies on portfolio optimization have been done so far. Modern portfolio theory is representative of them that is composed of Markowitz' portfolio theory published in 1952 and Capital Asset Pricing Model introduced by Sharpe (Amenc & Le Sourd, 2003). Markowitz (1952) introduced return and variance (risk) as estimates of portfolio and suggested return and risk calculation formula. In the context of a portfolio, variance is the volatility of an asset or group of assets. Larger variance value indicates greater volatility (Amenc & Le Sourd, 2003). Covariance is used for representing how two assets are related, i.e., how closely returns of two assets move together (Reilly, 1989).

In 1958, James Tobin showed the "Efficient Frontier" and "Capital Market Line" based on Markowitz's works (Reilly & Brown, 1997). All portfolios on the efficient frontier are optimal compared to other feasible portfolios. The efficient frontier can be described by the curve in the risk-return space with the highest expected rates of return for each level of risk. Sharpe ratio was introduced to compare a portfolio with others. Sharpe

---

Gumsong Jo, Dr., Faculty of Finance, Kim Il Sung University, Pyongyang, Democratic People's Republic of Korea.
Duson Kim, PhD., Department of International Finance, Faculty of Finance, Kim Il Sung University, Pyongyang, Democratic People's Republic of Korea.
Namung Ri, Dr., Department of International Finance, Faculty of Finance, Kim Il Sung University, Pyongyang, Democratic People's Republic of Korea.
Hoyong Kim, Dr., Department of International Finance, Faculty of Finance, Kim Il Sung University, Pyongyang, Democratic People's Republic of Korea.
Correspondence concerning this article should be addressed to Gumsong Jo, Faculty of Finance, Kim Il Sung University, Ryongnam-Dong, TaesongDistrict, Pyongyang, Democratic People's Republic of Korea.

ratio is a tangent line with efficient frontier of portfolio and tangent point is optimal portfolio. The higher the ratio is, the better its risk adjusted performance (Pedersen, 2014).

Recent decades' mean-variance models were expanded by adding reasonable constraints, such as cardinality, quantity, pre-assignment, round lot, class constraint, and transaction cost. Some works require strictly K assets to be included in a portfolio (Chang, Meade, Beasley, & Sharaiha, 2000; Fernandez & Gomez, 2007; Jin, Qu, & Atkin, 2014; Woodside-Oriakhi, Lucas, & Beasley, 2011; Xu, Zhang, Liu, & Huang, 2010), while some others use relaxed version (Ruiz-Torrubiano & Suarez, 2010; Schaerf, 2002). Some scholars included the round-lot constraint into portfolio optimization (PO) problems, which makes it more difficult to find a feasible solution. Some of these were measured in units of money (Bonami & Lejeune, 2009; Golmakani & Fazel, 2011; Kellerer, Mansini, & Speranza, 2000; Lin & Liu, 2008; Mansini & Speranza, 1999; Speranza, 1996), while others imposed that the continuous weight variables should be an integer multiple of a given fraction (Golmakani & Fazel, 2011; Skolpadungket, Dahal, & Harnpornchai, 2007; Streichert, Ulmer, & Zell, 2004). Some of them included transaction cost at the same time. In Mansini and Speranza (1999), it showed that a PO problem with minimum lot and without any fixed transaction cost is NP-complete.

When all practical constraints have been included in the mathematical model, PO problem has been rendered to be complex for direct solving by traditional numerical approaches (Chang et al., 2000; Hajinezhad, Eatiy, & Ghanbariy, 2013; Fernandez & Gomez, 2007; Jansen & van Dijk, 2002; Lin & Liu, 2008; Thein, 2015; Skolpadungket, 2013; Ruiz-Torrubiano & Suarez, 2010; Schaerf, 2002; Skolpadungket et al., 2007; Speranza, 1996; Woodside-Oriakhi et al., 2011; Li, 2015). Many scholars had proposed heuristics algorithms, such as genetic algorithm (GA) (Chang et al., 2000; Lin & Liu, 2008; Skolpadungket et al., 2007; Li, 2015), tabu search (TS) (Chang et al., 2000; Schaerf, 2002), simulated annealing (SA) (Chang et al., 2000; Gilli & Këllezi, 2000; Kellerer & Maringer, 2001), and neural network (NN) (Fernandez & Gomez, 2007) to give a solution to PO problems with practical constraints. These methods generally fall under the class of adaptive optimization algorithms, which include genetic algorithm, tabu search, and simulated annealing, and have been used extensively to solve global optimization problems with arbitrary objective function and constraints (Murray, & Shek, 2012). Loraschi, Tettamanzi, Tomassini, and Verda (1995) studied for application of genetic algorithm in portfolio optimization problems.

In this paper, we focus on the problem of giving a solution to general mean-variance portfolio with cardinality and integer weight constraints by using two genetic algorithms separately. Here, we imposed two practical constraints (cardinality and integer weight) together and improved an efficiency of portfolio by using two genetic algorithms. Also, we made all comparisons visible through efficient frontiers.

Following this instruction, in Section 2, we present two model formulations for portfolio selection with cardinality and integer weight constraints. Section 3 describes two genetic algorithms that should be used in model optimizations. In Section 4, we gave numerical result to support our models and concluded this paper in Section 5.

Portfolio Model With Cardinality and Integer Weight Constraints

It is necessary to have a clear understanding about the general mean-variance model that was introduced in (Amenc & Le Sourd, 2003). General mean-variance model assumes that the rate of return of the stock follows normal distribution. In this sense, it is difficult to forecast the future rate of return because the rate of return is a random variable. There are expected rate of return and rate of risk (standard deviation) representing the efficiency of portfolio investment. The Optimized Portfolio Model is simply said to maximize Sharpe ratio of return and variance (risk) and is defined as

$$\max\{\frac{E_{rp}}{\sigma_{rp}} | \sum \omega_i = 1\} \tag{2-1}$$

where

$$E_{rp} = \sum_{i=1}^{n} \omega_i E(r_i),\, n = 1,2,...N \tag{2-2}$$

$$\sigma_{rp} = \sum_{i=1}^{n} \sum_{j=1}^{n} \omega_i \omega_j \operatorname{cov}(r_{i,j}) \tag{2-3}$$

In reality, however, there are alsoother factors which make investors to have several considerations to invest. Investors could face restrictions on investments in specific fields of the economy and limitations on number of stocks in investment. This needs to make it more practical.

The only constraint of the Markowitz's model (Amenc & Le Sourd, 2003) is that the total sum of the weight of portfolio investment must be 1.This insufficient condition raises some problems in reality. One of the problems is that it is difficult to convert weighted capital into an integer number of each asset against its price. In stock market, stocks are generally traded in integer number not percent.

The other problem is transaction cost. Highly diversified portfolio causes high transaction cost because many numbers of stocks should be included in portfolio. In order to reduce transaction cost investor wants to limit number of stocks in portfolio, it is called as cardinality constraint.

Cardinality and integer weight constraints are noted as

$$\frac{x_i p_i}{\sum x_i p_i} = \omega_i \tag{2-4}$$

$$\varepsilon_i z_i \le x_i \le \delta_i z_i,\ x_i \in N \tag{2-5}$$

$$\varepsilon_i \le \delta_i,\ \varepsilon_i, \delta_i \in N \tag{2-6}$$

$$\sum z_i = K \tag{2-7}$$

$$z_i = \{0,1\} \tag{2-8}$$

Where ε_i and δ_i are lower and upper bound, z_i is cardinality constraint, x_i is number of i^{th} security, p_i is price of i^{th} security (Chang et al., 2000; Fernandez & Gomez, 2007). Here it is very important to find *k*-number of well-matching stocks in portfolio. Well-matching stocks means that stocks in portfolio are effectively correlated each other to minimize a variance of the portfolio for a given return on portfolio. It does not mean a group of *k*-number of the best stocks with highest return and lowest variance, but a group of *k*-number of effectively correlated stocks so that investors could achieve his own objective. In order to solve this problem, we designed two models. First model is for well-matching stocks and second model is for integer multiplied optimal weight of portfolio with well matching stocks. So, we designed two models against previous studies.

The first model is

$$\min\{\sum\sum \omega_{c_i}\omega_{c_j}\operatorname{cov}(r_{c_i}, r_{c_j}) \Big| \sum \omega_{c_i} E(r_{c_i}) = R,\ \sum \omega_{c_i} = 1,\ 0 \le \omega_{c_i} \le 1,\ 1 \le i \le k\} \tag{2-9}$$

where R is desired expected return and c_i stands for c_i th stock and the solution of first model is $C = \{c_1, c_2, .. c_k\}$.

The second model is

$$\min\left\{ \frac{\sum \omega_{c_i}\omega_{c_j}\operatorname{cov}(r_{c_i}, r_{c_j})}{\sum E(r_{c_i})} \middle| \omega_{c_i} = \frac{l_{c_i} p_{c_i}}{\sum l_{c_i} p_{c_i}},\ 0 \le l_{c_i} \le 100 \right\} \tag{2-10}$$

where l_{c_i} is optimal integer weight of c_i th stock and p_{c_i} is the price of c_i th stock.

This model suggests double optimization problem with the mixed quadratic and integer programing which typical numerical method could not give a solution.

Portfolio Optimization Using Two Genetic Algorithms

Portfolio selection model with cardinality and integer weight constraints is unable to be solved mathematically. In this section, optimal number of stocks will be calculated using genetic algorithm, afterwards, by using GA again optimal integer weight of portfolio will be found.

GA enables to solve calculation-intensive mathematical problems in relatively short time period.

GA is a type of optimization algorithm, meaning it is used to find the optimal solution to a given computational problem that maximizes or minimizes a particular function. GA represents one branch of the field of study called evolutionary computation, in that it imitates the biological processes of reproduction and natural selection to solve for the “fittest” solutions.

GA includes the following fundamental construction factors:

- Fitness function for optimization;
- Set of chromosomes (populations);
- Selection of chromosomes for regeneration;
- Cross-over for the generation of new chromosomes;
- Mutation for the generation of new chromosomes.

In order to get the best solution in a short time, this paper generates two genetic algorithms and use one by one.

The first genetic algorithm is for finding well-matching stocks and the second genetic algorithm is for searching integer optimal weights of well-matching stocks in portfolio.

The genetic algorithm used in this paper consists of a bundle of the following seven components.

$$\mathrm{GA} = (F, P_0, M, \Phi, \Gamma, \Psi, T) \tag{3-1}$$

Where F is a fitness function, P_0 is an initial population, M is a population size, Φ is a selection, Γ is a crossover, Ψ is a mutation, and T is the maximum evolution generation number.

Initial Population P_0

In the first GA, a solution $x=(x_1,x_{2,}...x_n)$ is represented by the chromosome $C=(c_1,c_{2,}...c_n)$, where genes are restricted in $1\le c_i\le n$ and $C\in N$. To generate random integer number within 1 to n, let us use equation below.

$$c_i=\mathrm{int}(1+(n-1)*rand) \tag{3-2}$$

where n is number of total stocks.

In the second GA, a solution $x=(x_1,x_{2,}...x_k)$ is represented by chromosome $C=(c_1,c_{2,}...c_k)$, where genes are restricted in $\varepsilon_i\le c_i\le\delta_i$ and $c_i,\varepsilon_i,\delta_i\in N$. To generate random integer number within ε_i and δ_i, let us use equation below.

$$c_i=\mathrm{int}(\varepsilon_i+(\delta_i-1)*rand) \tag{3-3}$$

where k is cardinality constraint.

For all regeneration stages, integer converting function should be applied because all genes should be natural number.

Fitness Function and Determination of Genetic Operators

Fitness function for the first GA is Equation (2-9) that is $\min\{\sum\sum\omega_{c_i}\omega_{c_j}\,\mathrm{cov}(r_{c_i},r_{c_j})\,|\sum\omega_{c_i}E(r_{c_i})=R,\ \sum\omega_{c_i}=1,\ 0\le\omega_{c_i}\le 1,\ 1\le i\le k\}$

where R is desired expected return of portfolio.

For the second GA, the fitness function is (2-10) that is

$$\min\left\{\frac{\sum\omega_{c_i}\omega_{c_j}\,\mathrm{cov}(r_{c_i},r_{c_j})}{\sum E(r_{c_i})}\middle|\,\omega_{c_i}=\frac{l_{c_i}p_{c_i}}{\sum l_{c_i}p_{c_i}},\ \varepsilon_{c_i}\le l_{c_i}\le\delta_{c_i}\right\}$$

where p_{c_i} is the price of c_i th stock.

Three operators including selection, crossover, and mutation were applied in genetic algorithm. The selection operator leaves good members and removes bad members from population, and it helps ensure good members to next generation. In general, probability of each member being selected is the ratio of fitness of the member to total sum of all member fitness values. Hence, the probability of the member which has low fitness value being selected is low and it hardly participates in the next calculation.

Determination of Hyper Parameters

This paper used four hyper parameters M, P_c, P_m, and T were used to control the flow of genetic algorithm.

There is no specific function or way to set these values. These values were determined freely or by having several experiments considering several factors.

Size of population M has an impact on calculation speed, efficiency and the variety of population. If the size is small, the calculation speed can become faster but the variety of population become lower and the algorithm could not get optimal solution. Conversely, if the size is too big, the efficiency of the calculation could be low.

Crossover probability P_c has the decisive influence on capability and speed of generating new members. The bigger the crossover probability is, the more members are generated but the higher the probability of dominant pattern being destroyed.

The mutation probability P_m controls the capability of generating members and T is number of generations that also have an impact on calculation speed.

Numerical Example for Comparison

Let us assume that we make k = 4 cardinality and integer weight constrained portfolio with 18 number of stocks listed in S & P 500. Table 1 shows statistic data about monthly expected return and variance of 18 stocks returns from October 2011 to September 2015. In this example, proportional transaction cost is 0.002 (see Table 5 for original data of 18 stocks).

According to individual Sharpe ratio ranks of 18 stocks in Table 1 when cardinality constraint k = 4, the best stocks are VF, GD, BLL, and CLX that is C = [10, 1, 18, 16]. Now, let us see the result of GA for searching well matching stocks.

In the GA number of variables is 4, so we generate chromosome with four genes and fitness function is

$$\max\{\frac{\sum \omega_{c_i} E(r_{c_i})}{\sum\sum \omega_{c_i}\omega_{c_j}\operatorname{cov}(r_{c_i}, r_{c_j})} | \sum \omega_{c_i} = 1,\ 0 \le \omega_{c_i} \le 1\} \qquad (4\text{-}1)$$

where genes c_i and c_j stand for which stocks are. This GA gave a solution C = [1, 8, 10, 16]. As a result of GA, Portfolios 1 and 2 are almost the same except for 8th and 18th stock.

Table 1

Monthly Expected Return and Variance of 18 Stocks Listed in S & P 500 (2011.10-2015.9)

No.	Name of stocks	Price of stock	Expected return	Variance on return	Sharpe ratio	Rank
1	GD	140.0407	0.019342	0.039244	0.492856	2
2	ORCL	36.71993	0.005367	0.039989	0.134205	13
3	JCI	37.05637	0.011933	0.03399	0.351082	7
4	BBY	37.16841	0.014217	0.101684	0.139812	12
5	BBBY	60.80896	0.001802	0.045993	0.039182	17
6	BAC	15.82756	0.021756	0.068852	0.315987	9
7	ADI	55.8169	0.012513	0.043994	0.284415	10
8	APPL	112.884	0.016802	0.064413	0.260849	11
9	VLO	60.10108	0.029431	0.085223	0.345342	8
10	VF	70.66012	0.017796	0.03592	0.495434	1
11	QCOM	54.42233	0.002577	0.040685	0.063343	15
12	PBI	20.09741	0.002973	0.065893	0.045117	16
13	ESRX	83.19615	0.017648	0.049481	0.356658	6
14	DUK	69.36038	0.003525	0.034116	0.103322	14
15	CVX	77.21582	-0.00309	0.042288	-0.07316	18
16	CLX	112.2899	0.01126	0.027872	0.404	4
17	CAH	81.8494	0.014273	0.038415	0.371543	5
18	BLL	65.12827	0.015885	0.034648	0.458477	3

Table 2 shows Sharpe ratio of three feasible portfolios without transaction cost. Portfolio 1 consists of stocks with high individual Sharpe ratio, Portfolio 2 has stocks searched by GA for maximum Sharpe ratio, and Portfolio 3 is made with all stocks. According to correct calculation for Sharpe ratio of two portfolios, Portfolio 1 is better than Portfolio 2, because GA can find approximate answer rather than correct one. And also, Table 2 shows that Portfolio 3 with all stocks has best maximum Sharpe ratio, but on the contrast, Table 3 and Figure 1 show that Portfolio 3 has worst maximum Sharpe ratio because of transaction cost.

Table 2

Sharpe Ratio of Portfolios 1, 2, and 3 Without Transaction Cost

	Portfolio 1	Portfolio 2	Portfolio 3
Sharpe ratio	0.7477	0.7164	0.7650

Table 3

Sharpe Ratio of Portfolios 1, 2, and 3 With Transaction Cost

	Portfolio 1	Portfolio 2	Portfolio 3
Maximum Sharpe ratio	0.7307	0.6693	0.6491
Return (min.-max.)	0.0133-0.0171	0.0125-0.0173	0.0082-0.0257
Standard deviation	0.0199-0.0286	0.0205-0.0340	0.0178-0.0852

Obviously, Portfolio 1 has the best maximum Sharpe ratio, but it could not make a portfolio within wider range of return and risk than other two portfolios. If investor wants to allocate his capital into four assets for monthly expected return 0.015, above Portfolios 1 and 2 could not give a solution so that he should find another portfolio with four assets by using GA. At that time, fitness function of the GA is defined as

$$\min\left\{\sum w_{c_i c_j} \operatorname{cov}(r_{c_i}, r_{c_j}) \middle| \sum E(r_{c_i}) = 0.015,\ 1 \le c_i, c_j \le 18,\ c_i, c_j \in N\right\} \tag{4-2}$$

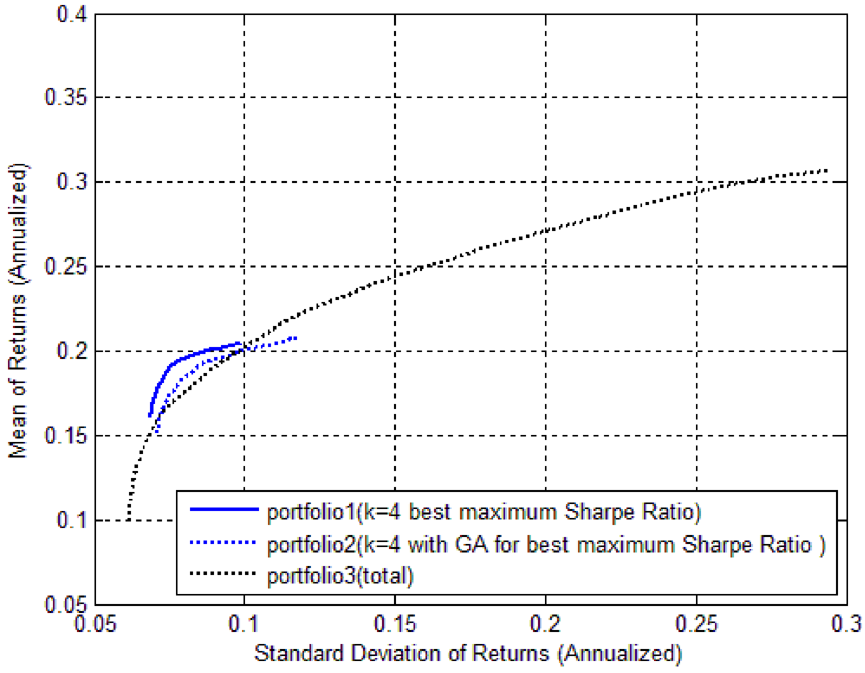

Figure 1. Efficient frontiers without transaction cost.

Where *gens* c_i and c_j stand for which stocks are. This GA gives a solution C = [1, 5, 9, 16], i.e., GD, BBBY, VLO, and CLX.

Figure 2 shows that, at the desired expected return 0.015, Portfolio 3 is better than Portfolio 4.

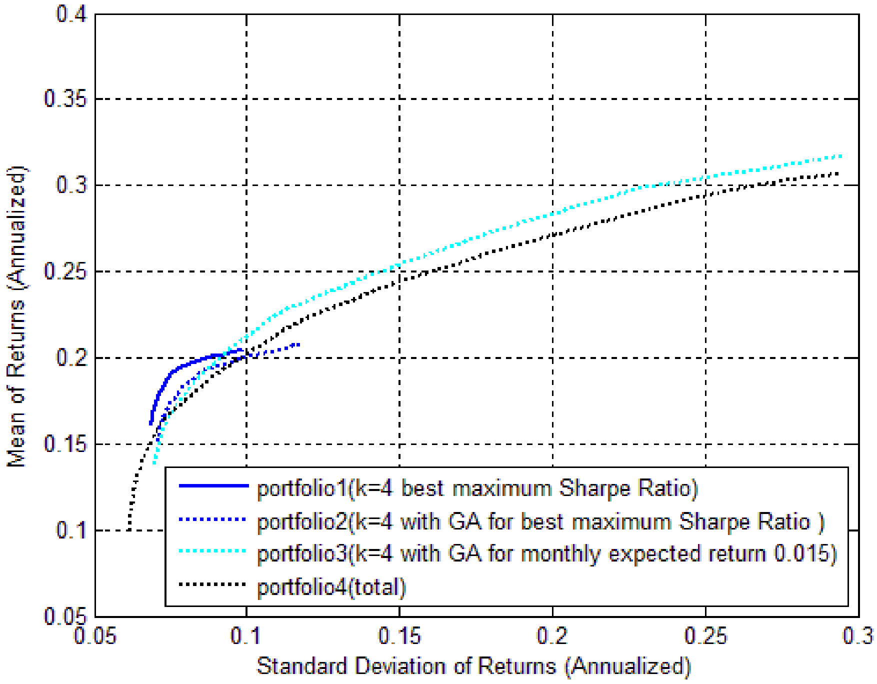

Figure 2. Efficient frontiers of portfolios with transaction cost.

After we find desired assets for portfolio, we have to find optimal integer weight for the portfolio by using second model. Let us find optimal integer weight for the Portfolio 3 with GD, BBBY, VLO, and CLX.

The fitness function of GA is

$$\min\left\{\frac{\sum \omega_{c_i}\omega_{c_j}\operatorname{cov}(r_{c_i},r_{c_j})}{\sum E(r_{c_i})}\middle|\omega_{c_i}=\frac{l_{c_i}p_{c_i}}{\sum l_{c_i}p_{c_i}},\ C=[1,5,9,16],\ 0\le l_{c_i}\le 100\right\} \tag{4-3}$$

where solution l_{c_i} stands for integer weight for c_i th stock.

Table 4 shows optimal integer weight for Portfolio 3 with 0.015 of monthly expected return according to maximum Sharpe ratio.

Table 4

Optimal Integer Weight for Portfolio 3

No(c_i)	Name of stock	Price	Integer weight	Weight (%)
1	GD	140.0407	65	44.84
5	BBBY	60.80896	4	1.20
9	VLO	60.10108	44	13.03
16	CLX	112.2899	74	40.93

Table 5

Original Data on Monthly Returns of 18 Stocks Listed in S & P 500 (2011.10-2015.9)

Month	GD (%)	ORCL (%)	JCI (%)	BBY (%)	BBBY (%)	BAC (%)	ADI (%)	APPL (%)	VLO (%)
2015/9	-5.37	-4.89	-4.84	14.67	-3.40	-7.58	-2.71	-0.46	-7.15
2015/8	1.44	-3.42	-1.57	-2.37	-6.98	-2.28	-6.40	-9.52	-1.25

Table 5 to be continued

2015/7	2.51	-6.74	-5.46	-3.30	-3.51	1.62	-8.43	-2.06	10.61
2015/6	1.85	-2.51	-0.03	-2.75	-1.70	4.60	4.24	-0.59	0.41
2015/5	4.05	1.44	-5.45	-4.59	-2.81	5.33	1.50	1.08	0.04
2015/4	-0.72	0.37	-0.88	-7.55	-2.24	-1.28	7.13	0.90	-2.46
2015/3	-2.49	-0.74	0.87	5.88	-2.33	-2.24	4.39	0.84	4.70
2015/2	0.69	0.13	-0.61	3.22	1.77	0.08	4.67	12.91	19.07
2015/1	-2.61	0.66	-1.48	-2.63	2.54	-7.34	-3.74	-1.56	-0.52
2014/12	-0.76	6.39	1.06	4.09	4.03	1.75	10.34	-0.45	-3.15
2014/11	13.57	5.74	2.44	12.67	8.62	3.09	10.01	11.25	8.28
2014/10	-0.86	-4.81	-5.85	-3.25	0.23	0.07	-7.20	1.05	-6.14
2014/9	5.44	-0.93	0.73	9.65	3.59	7.17	-1.95	2.37	-5.46
2014/8	2.10	0.45	-2.64	-1.72	3.51	-0.17	-4.18	2.75	5.78
2014/7	-1.05	-2.90	0.37	6.10	0.50	0.72	-2.21	4.09	-8.98
2014/6	4.42	0.38	6.33	11.16	-2.11	3.90	5.10	6.79	-3.14
2014/5	5.01	3.56	1.49	1.50	-5.48	-8.42	-2.39	11.45	2.62
2014/4	-0.46	3.55	-2.67	-0.97	-4.43	-5.59	1.88	1.33	3.03
2014/3	3.98	2.90	2.99	4.93	4.78	4.18	3.89	1.34	8.14
2014/2	8.09	-0.15	2.36	-21.04	-6.31	-1.18	1.36	-2.15	-3.73
2014/1	5.47	5.94	5.15	-23.67	-10.82	7.60	-0.27	-4.00	8.44
2013/12	3.30	3.18	4.01	-1.74	1.07	4.85	-0.23	6.93	11.44
2013/11	2.10	4.31	4.67	3.27	0.36	4.49	5.19	3.90	13.34
2013/10	0.12	-0.15	3.27	6.36	3.46	-1.29	-1.26	4.75	5.57
2013/9	2.18	1.88	-0.34	16.47	-1.16	-0.63	-0.98	-0.66	-1.84
2013/8	3.12	2.52	-1.16	11.38	0.14	3.26	1.23	13.04	3.38
2013/7	6.16	-3.54	5.13	7.34	7.81	7.08	4.91	0.48	-7.25
2013/6	1.83	-3.31	-1.28	4.06	0.95	-0.28	-1.04	-4.43	-4.80
2013/5	10.29	4.15	6.56	10.04	4.78	8.56	4.45	6.32	0.90
2013/4	0.27	-5.48	-0.21	14.43	7.83	-0.91	-3.78	-4.99	-9.22
2013/3	3.77	-1.17	1.26	28.07	4.57	3.92	0.43	-3.42	-1.17
2013/2	-5.50	0.29	2.44	15.49	2.62	0.98	5.63	-8.25	23.40
2013/1	3.27	6.64	5.91	16.14	-0.98	7.14	3.45	-6.33	12.16
2012/12	4.41	6.16	4.59	-14.01	-1.13	13.32	3.53	-5.78	9.91
2012/11	-2.36	-1.16	-0.59	-18.18	-4.47	3.03	3.10	-11.17	-0.18
2012/10	1.03	-3.33	0.73	-4.04	-8.84	4.30	-2.99	-6.66	-5.68
2012/9	2.53	2.56	-1.49	-3.84	3.29	13.59	0.48	6.18	9.63
2012/8	0.20	5.60	7.22	-3.65	4.93	4.24	7.24	6.99	16.34
2012/7	1.36	8.42	0.22	-1.12	-9.62	-1.21	0.81	4.48	12.23
2012/6	-3.13	0.78	-3.60	-0.31	-3.41	2.00	0.38	1.61	-0.68
2012/5	-6.02	-5.92	-1.00	-11.54	2.02	-14.98	-4.07	-6.95	-8.11
2012/4	-3.74	-2.53	3.60	-12.36	7.62	-1.91	-2.60	5.36	-9.69
2012/3	2.32	3.14	5.18	1.01	6.68	12.66	-0.66	16.20	8.90
2012/2	1.16	4.25	3.26	1.84	-1.02	18.09	4.52	15.78	12.48
2012/1	7.61	-5.01	5.28	-1.37	0.44	23.01	7.90	9.08	4.34
2011/12	2.28	-7.37	1.51	-7.30	-0.27	-8.40	-0.34	1.94	-9.79
2011/11	2.81	0.46	4.80	6.70	1.93	-6.40	1.04	-2.88	5.50
2011/10	7.17	5.97	5.00	5.07	1.60	1.89	8.69	1.77	21.02
Month	VF	QCOM	PBI	ESRX	DUK	CVX	CLX	CAH	BLL
2015/9	-5.13	-10.13	-2.28	-4.60	-6.81	-5.27	-2.25	-2.40	-4.60

Table 5 to be continued

2015/8	1.87	-4.27	-1.56	-4.00	1.53	-12.89	5.04	-1.37	-2.23
2015/7	4.41	-5.58	-4.05	2.76	0.65	-6.75	3.03	-3.57	-2.56
2015/6	-1.49	-3.58	-4.07	1.23	-4.60	-5.91	-1.94	1.36	-1.24
2015/5	-4.59	1.57	-2.34	1.09	-1.97	-1.94	-1.30	-3.27	0.34
2015/4	-0.22	-2.27	1.09	3.42	2.70	4.00	-0.03	0.90	2.51
2015/3	2.46	0.69	0.87	-0.61	-7.38	-4.10	1.08	3.73	-1.35
2015/2	0.08	-3.24	-5.10	-0.04	-5.32	2.12	1.34	3.90	9.38
2015/1	-1.40	-1.78	-2.30	0.50	4.37	-2.45	5.05	1.39	-4.14
2014/12	3.51	1.80	-0.60	5.98	2.55	-6.23	1.31	2.00	4.90
2014/11	8.50	-3.16	2.90	9.82	3.23	1.50	3.26	5.58	1.06
2014/10	-0.03	-1.77	-8.32	-2.18	5.79	-7.39	6.54	0.50	-0.93
2014/9	5.08	0.83	-1.37	0.88	2.54	-2.47	3.39	4.75	3.23
2014/8	0.61	-5.02	-3.06	7.76	-0.86	-2.95	-2.98	1.50	-0.56
2014/7	-0.74	-0.68	-0.82	-3.45	1.62	2.88	0.60	2.14	3.41
2014/6	0.74	-0.37	4.15	2.01	-0.17	2.60	2.47	4.78	3.80
2014/5	3.25	0.53	3.50	-5.37	-0.85	2.58	-1.08	-3.41	6.51
2014/4	-1.46	2.71	-0.13	-5.66	3.34	4.57	2.34	-4.57	1.28
2014/3	5.32	3.05	2.26	2.55	-1.20	2.54	0.94	3.51	1.59
2014/2	-3.41	1.63	8.25	3.05	3.72	-5.77	-3.25	3.01	5.85
2014/1	0.91	0.67	2.88	6.89	-1.26	-2.07	-4.26	2.45	0.60
2013/12	6.30	3.80	-0.38	4.49	-3.26	1.24	1.80	3.81	1.30
2013/11	10.32	3.98	16.23	3.78	3.22	1.43	7.79	15.62	8.05
2013/10	3.44	-1.00	11.71	-1.93	4.39	-3.36	2.33	5.13	2.12
2013/9	1.04	3.23	0.73	-1.40	-3.09	1.76	-1.51	2.08	0.12
2013/8	-1.38	7.22	19.47	-0.25	-1.73	-2.32	-1.00	3.73	1.81
2013/7	4.96	0.05	-0.06	4.83	3.90	3.08	1.72	4.08	3.53
2013/6	3.10	-3.96	-3.54	0.66	-6.04	-2.74	-2.59	1.24	-5.27
2013/5	6.66	-0.84	1.98	8.17	-2.70	4.69	-2.04	8.33	-3.10
2013/4	4.01	-1.91	1.03	-2.66	4.75	-0.76	3.49	-3.53	2.63
2013/3	6.01	0.47	8.13	5.50	1.55	3.15	4.17	-1.57	1.33
2013/2	3.45	2.36	13.72	1.76	4.11	2.05	7.00	4.27	-2.44
2013/1	-1.16	2.82	8.99	0.75	3.11	5.12	1.50	5.05	3.50
2012/12	-3.81	1.72	-7.34	0.40	3.37	1.87	1.84	3.63	1.08
2012/11	-2.27	2.48	-15.41	-15.10	-4.30	-7.35	0.07	-2.03	3.73
2012/10	2.31	-4.72	-2.27	1.03	0.94	-1.27	2.52	5.73	-0.74
2012/9	5.08	2.31	4.85	4.22	-3.64	3.21	-0.18	-3.36	1.90
2012/8	5.68	9.28	-1.92	6.71	-0.04	4.92	-0.83	-5.97	2.00
2012/7	1.86	-0.22	-4.03	6.78	-2.72	5.81	1.74	2.43	0.28
2012/6	-2.10	-5.97	-2.97	-1.22	5.82	-0.75	4.11	-1.25	1.61
2012/5	-4.52	-8.24	-13.08	-5.94	3.55	-2.12	-1.49	1.04	-6.04
2012/4	0.92	0.51	-5.35	6.49	-0.64	-4.51	2.24	-0.56	5.33
2012/3	4.47	5.33	-2.68	3.34	-0.87	1.87	-0.46	-0.40	1.98
2012/2	6.52	8.45	-3.58	3.39	-0.75	-0.24	0.14	0.36	6.26
2012/1	-0.04	5.37	3.53	11.23	0.96	2.98	4.15	1.80	7.14
2011/12	-0.87	-1.36	-1.68	0.28	3.77	2.10	0.92	-3.85	2.41
2011/11	1.81	5.50	-5.68	11.34	1.61	1.53	-3.27	1.55	1.92
2011/10	5.36	4.08	3.97	6.03	0.03	7.16	0.59	-1.76	6.96

Figure 3 shows how efficient two genetic algorithms work.

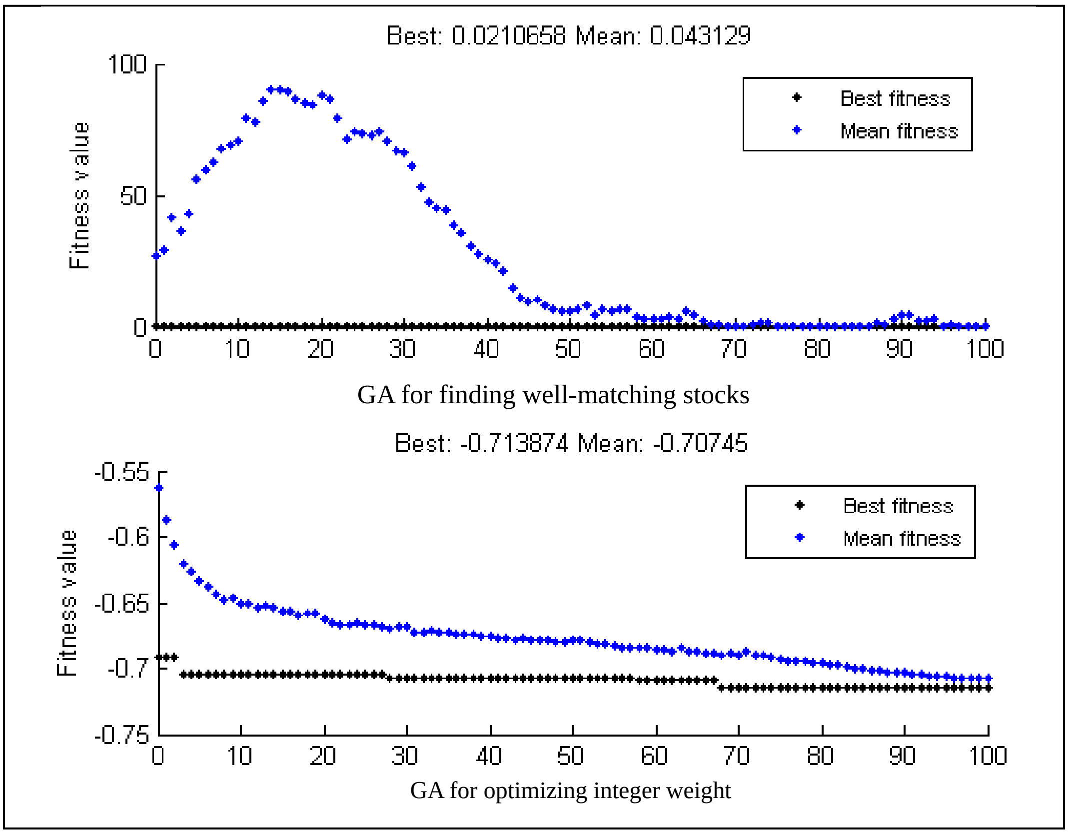

Figure 3. Efficiency of gas.

Conclusion

This paper classified cardinality constrained portfolios as investors' preferences among Sharpe ratio and expected return and gave solutions to each model by using GA. Our research gave all efficient frontiers to all suggested models and compared with each other.

First, we verified that the portfolio with all stocks obviously has more advantage than cardinality constrained portfolio, but in reality, cardinality constrained portfolio is better than total one because of transaction cost. Investors should compose his portfolio with the desired number of stocks to reduce transaction cost.

Secondly, we suggested how to choose which stock should be included in portfolio in terms of investor's target. As shown in this paper, when investor wants the portfolio with the best maximum Sharpe ratio, he simply consists of desired number of stocks with the best individual Sharpe ratio that it has an advantage in calculation time. However, if investor's target is certain return or risk, at that time heuristic algorithms should be used for finding desired stocks. In this paper, simple GA was used and worked well.

Finally, we optimized design portfolios by using another GA. When using GA, we use Sharpe ratio as fitness function to find optimal integer weight of each portfolio. Invested money in each stock is the integer times of stock price. Investors should consider integer weight when optimizing portfolio because stocks are traded in integer number not percent. In order to get a solution of integer weighted portfolio, we used another

GA that fitness function is Sharpe ratio in purpose of simplicity. We examined all genetic algorithms in this paper worked well.

References

Amenc, N., & Le Sourd, V. (2003). *Portfolio theory and performance analysis.* Chichester: John Wiley and Sons.

Black, F., & Litterman, R. (1992). Global portfolio optimization. *Financial Analysts Journal, 48*(5), 28-43.

Black, F., Jensen, M. C., & Scholes, M. (1972). *The capital asset pricing model: Some empirical tests.* New York: Praeger Publishers.

Bonami, P., & Lejeune, M. A. (2009). An exact solution approach for portfolio optimization problems under stochastic and integer constraints. *Operations Research, 57*(3), 650-670.

Chang, T. J., Meade, N., Beasley, J., & Sharaiha, Y. (2000). Heuristics for cardinality constrained portfolio optimisation. *Computers & Operations Research, 27*(13), 1271-1302.

Fernandez, A., & Gomez, S. (2007). Portfolio selection using neural networks. *Computers & Operations Research, 34*(4), 1177-1191.

Gilli, M., & Këllezi, E. (2000). Heuristic approaches for portfolio optimization. *The Sixth International Conference on Computing in Economics and Finance of the Society for Computational Economics*, July 6-8, Barcelona, Spain.

Golmakani, H. R., & Fazel, M. (2011). Constrained portfolio selection using particle swarm optimization. *Expert Systems with Applications, 38*(7), 8327-8335.

Hajinezhad, E., Eatiy, S., & Ghanbariy, R. (2013). Mixed Tabu Machine for portfolio optimization problem. *The 3rd Conference on Financial Mathematics & Applications*, January 30-31, Semnan, Iran.

Jansen, R., & van Dijk, R. (2002). Optimal benchmark tracking with small portfolios. *The Journal of Portfolio Management, 28*(2), 33-39.

Jin, Y., Qu, R., & Atkin, J. (2014). A population-based incremental learning method for constrained portfolio optimisation. *The 16th International Symposium on Symbolic and Numeric Algorithms for Scientific Computing (SYNASC)*, September 22-25, Timisoara, Romania.

Jobst, N., Horniman, M., Lucas, C., & Mitra, G. (2001). Computational aspects of alternative portfolio selection models in the presence of discrete asset choice constraints. *Quantitative Finance, 1*(5), 489-501.

Kellerer, H., & Maringer, D. (2001). Optimization of cardinality constrained portfolios with a hybrid local search algorithm. *The 4th Methaheuristics International Conference (MIC)*, July 16-20, Porto, Portugal.

Kellerer, H., Mansini, R., & Speranza, M. (2000). Selecting portfolios with fixed costs and minimum transaction lots. *Annals of Operations Research, 99*(1-4), 287-304.

Li, Y. B. (2015). Solving cardinality constrained portfolio optimisation problem using genetic algorithms and ant colony optimization (Ph.D. thesis, Brunel University, 2015).

Lin, C. C., & Liu, Y. T. (2008). Genetic algorithms for portfolio selection problems with minimum transaction lots. *European Journal of Operational Research, 185*(1), 393-404.

Loraschi, A., Tettamanzi, A., Tomassini, M., & Verda, P. (1995). Distributed genetic algorithms with an application to portfolio selection. In D. W. Pearson, N. C. Steele, R. F. Albrecht (Eds.), *Artificial neural nets and genetic* (pp. 384-387). Berlin: Springer.

Mangram, M. E. (2013). A simplified perspective of the Markowitz portfolio theory. *Global Journal of Business Research, 7*(1), 1-12.

Mansini, R., & Speranza, M. G. (1999). Heuristic algorithms for the portfolio selection problem with minimum transaction lots. *European Journal of Operational Research, 114*(2), 219-233.

Markowitz, H. (1952). Portfolio selection. *Journal of Finance, 7*(1), 77-91.

Murray, W., & Shek, H. (2012). A local relaxation method for the cardinality constrained portfolio optimization problem. *Springer.* Retrieved from https://link.springer.com/article/10.1007/s10589-012-9471-1

Pedersen, K. G. (2014). Modern portfolio theory—A way to bridge the gap between strategy and risk? (Unpublished Master's thesis, Copenhagen Business School, 2014).

Reilly, F. K., & Brown, K. C. (1997). *Investment analysis and portfolio management.* Chicago: Dryden Press.

Reilly, F. K. (1989). *Investment analysis and portfolio management* (3rd ed.). New York: Harcourt Brace Jovanovish College Publishers.

Ruiz-Torrubiano, R., & Suarez, A. (2010). Hybrid approaches and dimensionality reduction for portfolio selection with cardinality constraints. *Computational Intelligence Magazine, 5*(2), 92-107.

Schaerf, A. (2002). Local search techniques for constrained portfolio selection problems. *Computational Economics, 2*0, 177-190.

Skolpadungket, P. (2013). Portfolio management using computational intelligence approaches (Ph.D. thesis, Department of Computing, University of Bradford).

Skolpadungket, P., Dahal, K., & Harnpornchai, N. (2007). Portfolio optimization using multi-objective Genetic Algorithms. *Congress on Evolutionary Computation (CEC) 2007*, September 25-28, Singapore.

Speranza, M. G. (1996). A heuristic algorithm for a portfolio optimization model applied to the Milan stock market. *Computers & Operations Research, 23*(5), 433-441.

Streichert, F., Ulmer, H., & Zell, A. (2004). Evaluating a hybrid encoding and three crossover operators on the constrained portfolio selection problem. *Congress on Evolutionary Computation (CEC) 2004*, June 19-23, Portland, OR, USA.

Thein, L. K. (2015). Evolutionary approaches to portfolio optimization (Ph.D. thesis, University of Nortingham, 2015).

Tobin, J. (1958). Liquidity preference as behavior toward risk. *Review of Economic Studies, 25*(2), 65-85.

Vercher, E. (2006). Optimization in Finance: Portfolio selection models. *Iberian Conference in Optimization*, November 16-18, Coimbra, Portugal.

Woodside-Oriakhi, M., Lucas, C., & Beasley, J. (2011). Heuristic algorithms for the cardinality constrained efficient frontier. *European Journal of Operational Research, 213*(3), 538-550.

Xu, R. T., Zhang, J., Liu, O., & Huang, R. Z. (2010). An estimation of distribution algorithm based portfolio selection approach. *The 2010 International Conference on Technologies and Applications of Artificial Intelligence (TAAI)*, November 18-20, Hsinchu City, Taiwan.