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Performance between the Hydraulic Gradient Method and the Perturbation Method for the Analysis of Water Supply Networks

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Abstract: The HGM (Hydraulic Gradient Method), it is used in most of the current commercial software, such as EPANET, WaterCAD, MikeNet, among others, the same that corresponds to an iterative method that depends on initial estimated parameters and programming structures that ensure convergence to obtain results with the highest precision, in addition to this the method makes use of non-linear equation systems. Likewise, the execution time for large extensions of water distribution networks is considerably high. On the other hand, the PM (Perturbation Method), is a new direct solution method, which makes use of principles of quantum mechanics to transform nonlinear equations into simpler linear systems. Obtaining a simple and robust optimization method that only requires simple and direct mathematical processes. Using the MathCad and Python programming languages as a verification tool, multiple tests were carried out, the results for the hydraulic parameters showing that the flow rates and pressures obtained by the HGM and the PM are extremely similar, in the same way the execution time (time run) have been 77.09% favorable to the PM. In other words, the PM presents efficiency to estimate the hydraulic characteristics such as the pressures at the nodes and the velocities in the pipes of the drinking water distribution networks.

Key words: Perturbation method, quantum mechanics, hydraulic gradient.

1. Introduction

Currently, the accelerated advances in computer technologies and the development of new mathematical methods to solve problems that previously seemed impossible or difficult to solve manually, are becoming increasingly easy and obtaining more accurate results, giving us reliability when using these results for the design and construction of structures.

Today in the field of engineering specifically in the field of Hydraulics and Hydrology drinking water systems are analyzed using commercial software such as WaterGEMS, Epanet, WaterCAD among others, all based on the HGM which is an iterative method from solution. The new method called PM, has been developed and implemented using programming routines, written in the Python programming language.

The present investigation aims to compare and evaluate which of the methods has more efficiency when it comes to analyzing drinking water systems.

According to Basha y Kassab [1] applying the PM to the head loss equation transforms it into a series of infinite equations (perturbation expansion), demonstrating that even the third order provides sufficiently precise solutions.

According to Bender, Milton, Pinsky and Simmons [2], the delta perturbation expansion method was developed specifically to solve problems of quantum field theory, we have realized that it can be a powerful tool in the analysis of any nonlinear problem.

According to Fujiwara and Khang [3], the solution methods for the optimal design of water distribution

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networks available in the literature end after finding a local optimal solution or its approximate one.

Sánchez [4], the perturbation method is classified as a numerical method for approximate solution of partial differential equations (it is equivalent to the finite element method, finite difference, etc.); the approximate solution of the governing equations of water flow in drinking water supply systems is obtained directly, it does not require iterative processes, hence the difference with the other numerical methods.

In the same way, in the HGM research, we have as background:

Todini and Pilati [5] demonstrated that to find the solutions of the system of equations in linear parts and in non-linear parts that describe the problem of network flow, the application of the Newton-Raphson technique in the space of unknown pipe flows, unknown pipeline flows and nodal pressures, where the existence and uniqueness of the solution can be tested and leads to an extremely convergent scheme.

Rodas [6], the methodologist currently used for the design of drinking water networks does not provide optimal results, since only permanent flow modeling is performed, creating an uncertainty in the design of the pipe diameter that does not meet the hydraulic parameters (flow, speed and pressure) in transient flow that necessarily occurs in a drinking water distribution system, but it is important for the preselection of the classes of pipes.

Zapata [7], the greater the total number of pipes that a drinking water distribution system has, the greater the number of possible diameter combinations in the different pipes; in such a way that to arrive at an arrangement of diameters such that it satisfies the conditions that a network must meet in order to be efficient, a selection is carried out iteratively based on identifying the pipes of greatest importance for the conduction and on these, propose changes in diameters to meet the objective of lower cost, satisfying the minimum and maximum pressure load conditions in knots and verifying velocities in pipes.

Flores [8], the HGM is the appropriate mathematical method to perform the hydraulic analysis of water distribution networks, its effectiveness has been proven not only by obtaining the set of suitable diameters, 100% acceptance, for each pipe, but also in complying with the limits. speed and pressure in each of them.

2. Material and Methods

The analysis of the pipe network involves the determination of the flow rates and pressure head of the pipe that satisfy the continuity and energy conservation equations. These can be declared as follows:

Continuity: the algebraic sum of the flows that enter are equal to the algebraic sum of the flows that go out through the pipes.

Conservation of energy: the algebraic sum of the pressure drops in the pipes, in any closed circuit formed by pipes is zero [9].

2.1 HGM (Hydraulic Gradient Method)

It is based on the fact that, in a permanent flow, in addition to the conservation of mass equations in each node and the conservation of energy in each pipe in the network, the gradient method requires the following vector and matrix definitions:

NT: Number of pipes in the distribution network.

NN: Number of knots with unknown pressure.

[A12]: “Connectivity matrix” associated with each node. Its dimension is NT × NN:

-1 corresponding to the initial node of the pipe;

+1 corresponding to the end node of the pipe;

0 somewhere else;

NS: number of nodes with known pressures.

[A10]: Topological matrix: pipe to node for NS fixed height nodes. Its dimension is NT × NS with a value of −1 in the rows corresponding to the pipes connected to fixed heads.

[A11]: Diagonal matrix of dimension NT × NT, defined from 1 to NT:

$$\alpha_{NT} Q_{NT}{}^{(n_{NT}-1)} + \beta_{NT} + \frac{\gamma_{\mathrm{NT}}}{Q_{\mathrm{NT}}}$$

$[Q]$: Vector of assumed flows with dimension NT × 1;

$[H]$: Unknown pressure vector with dimension NN × 1;

[Ho]: Known pressure vector with dimension NS × 1.

Fig. 1 shows the general process of the HGM. According to Fig. 1, the Reynolds number [10] and the coefficient of friction of the pipes [11] are calculated.

An initial flow is assumed and then calculate the head loss with the following equation [11-19]:

$$[H_{i+1}] = -[([A21][N]^{-1}[A11]^{-1}[A12])^{-1}][[A21][N]^{-1}([Q_i] + [A11]^{-1}[A10][H_0]) + [q] - [A21][Q_i]]$$

Once the head loss has been calculated, the new flow rates of the pipes are determined with the following matrix equation [11-19]:

$$[Q_{i+1}] = ([I] - [N]^{-1})[Q_i] - [N]^{-1}[A11]^{-1}([A12][H_{i+1}] + [A10][H_0])$$

Once we have the assumed flows and the new flows, we subtract both flows to obtain the error, this difference must be zero, when it is different from zero the procedure is repeated several times as shown in Fig. 1, in this way it is verified the fulfillment of this condition to determine the flow rates and pressure drops in the analyzed distribution network.

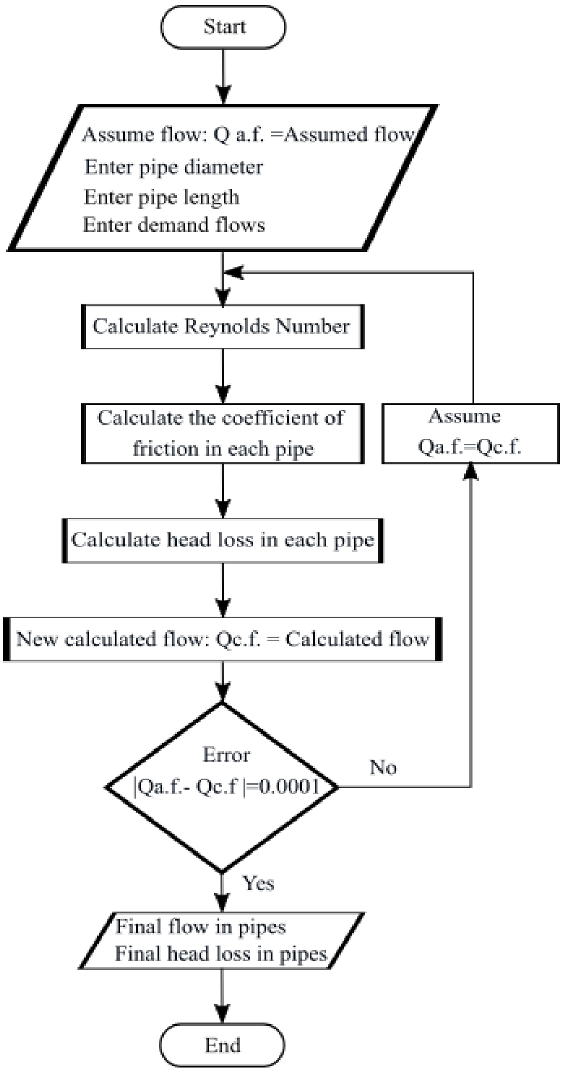

Fig. 1 Flowchart implemented in the Python programming language for HGM.

2.2 PM (Perturbation Method)

The PM seeks approximate analytical solutions to complex equations that are difficult to solve [21]. PM reduces a complex problem to a simpler one by using the Taylor series by generating a series of easy to calculate equations.

The flow can be expressed in the following way in order to apply the disturbance method:

$$Q = \alpha h^x \tag{1}$$

Where the constant α and x depends on the flow equation used thus we have for:

Darcy—Weisbach

$$\alpha = \pi \sqrt{\frac{gD^5}{8fL}}, x = \frac{1}{2} \tag{2}$$

Donde:

Q: Flow flowing through the pipe.

h: Piezometric pressure loss.

x: Constant that varies from 0.5 to 0.54 depending on the formula used.

For our analysis the Darcy Weisbach equation is used.

To find the solution of this equation, a perturbation technique called delta expansion δ was used, which consists of making a change of variable in the following way:

$$x = \delta + 1 \tag{3}$$

Where: δ is a disturbance parameter.

We convert the original problem into a disturbed problem by entering a small parameter delta δ. To obtain this we substitute Eq. (3) in Eq. (1) and we obtain:

$$Q = \alpha h^{1+\delta} \tag{4}$$

The perturbation theory expresses the solution of a problem "x" in a formal power series for some small parameter δ, the main term in this power series is the solution of the problem that can be solved exactly, while the other terms describe the deviation or variation in the solution, due to the deviation from the initial problem. Formally, we have for the approximation to the complete solution a series with the small parameter δ, like the following:

$$x(\delta) = c_0 + c_1\delta + c_2\delta^2 + c_3\delta^3 + \cdots \tag{5}$$

In this example, c_0 would be the known solution to the initial problem with exact solution, and $c_1, c_2, \ldots$ represent the higher-order terms that can be found iteratively by some systematic procedure.

We express the solution for "h" as a function of the small parameter "δ"

$$h = h_0 + h_1\delta + h_2\delta^2 + h_3\delta^3 + \cdots \tag{6}$$

Where:

$h_{0,1,2,3,..}$: Pressure drops for 0, 1, 2, 3 power state.

Properties of logarithms and the Maclaurin series that will be used to calculate the equations:

$$z = e^{\ln z} \tag{7}$$

$$\log z^a = a \log z \tag{8}$$

$$\log(a * b) = \log a + \log b \tag{9}$$

$$e^z = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \frac{z^4}{4!} + \cdots \tag{10}$$

$$\ln(1+z) = z - \frac{z^2}{2} + \frac{z^3}{3} - \frac{z^4}{4} \pm \cdots \quad |z| > 0 \tag{11}$$

Before replacing this expression, we must linearize the expression "$h^{1+\delta}$".

Using the properties of logarithms and Maclaurin series:

$$h^{1+\delta} = h * h^{\delta} = h * e^{\ln(h^{\delta})} = h * e^{\delta \ln h}$$

$$= h * (1 + \ln h\delta + \frac{\ln h^2}{2}\delta^2 + \frac{\ln h^3}{6}\delta^3 + \ldots) \tag{12}$$

We replace the series (6) in Eq. (12) term by term.

Solving each term within the parentheses of Eq. (12) we have:

$$\begin{aligned} h^{1+\delta} &= h_0 + (h_1 + h_0\ln(h_0))\delta \\ &+ \left(h_2 + h_1 + h_1\ln(h_0) + \frac{h_0}{2}\ln(h_0)^2\right)\delta^2 \\ &+ \left(h_3 + h_2 + \frac{{h_1}^2}{2h_0} + (h_1 + h_2)\ln(h_0) + \frac{h_1}{2}\ln(h_0)^2 + \frac{h_0}{6}\ln(h_0)^3 + \right)\delta^3 \\ &+ \left(h_3 + \frac{2h_1h_2}{h_0} - \frac{{h_1}^3}{2{h_0}^2} + \frac{{h_1}^2}{h_0}\ln(h_0) + \frac{h_1}{6}\ln(h_0)^3 + \frac{h_2}{2}\ln(h_0)^2 + h_3\ln(h_0)\right)\delta^4 \\ &+ \left(\frac{2h_1h_3}{h_0} + \frac{{h_2}^2}{h_0} - \frac{{h_1}^2h_2}{2{h_0}^2} + \frac{h_1h_2}{h_0}\ln(h_0) + \frac{h_2}{6}\ln(h_0)^3 + \frac{h_3\ln(h_0)^2}{2}\right)\delta^5 + \cdots \end{aligned} \tag{13}$$

Finally, from Eq. (13) we obtain the power series for the head losses for the flow equation, thus leaving the following formula:

$$Q = \alpha h^x = \alpha h^{1+\delta} = \alpha h_0 + \alpha(h_1 + h_0 \ln h_0)\delta + \alpha[h_2 + h_1 + h_1 \ln h_0 + \frac{h_0}{2}(\ln h_0)^2]\delta^2 + \alpha[h_3 + h_2 + \frac{{h_1}^2}{2h_0} + (h_1 + h_2)\ln h_0 + \frac{h_1}{2}(\ln h_0)^2 + \frac{h_0}{6}(\ln h_0)^3]\delta^3 + \ldots \tag{14}$$

In this way, the disturbed flow equation corresponding to Eq. (14) is obtained.

Zero order:

$$\alpha h_0 \tag{15}$$

Order one:

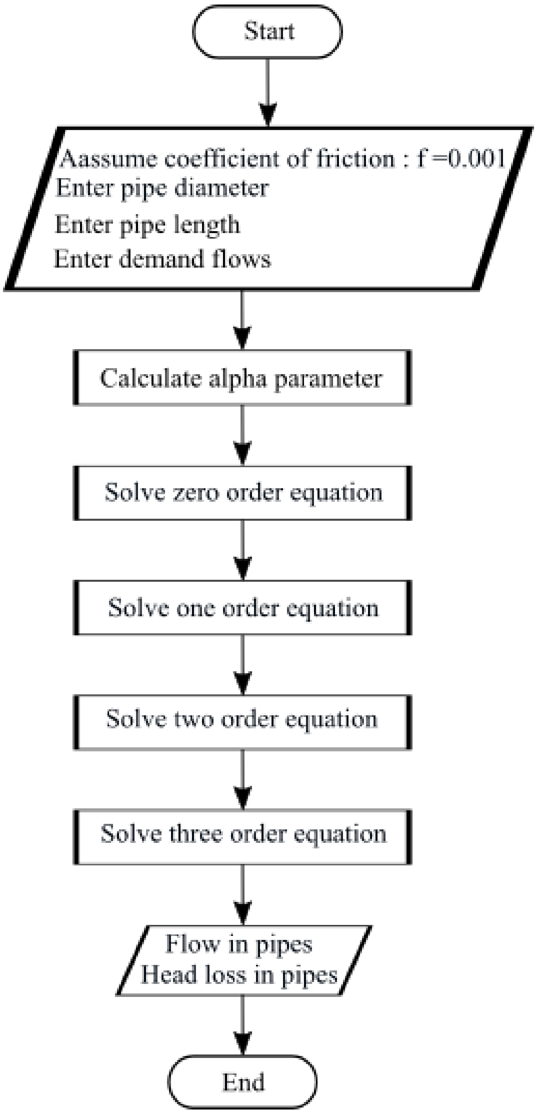

Fig. 2 Flowchart of the routine programmed in the Python programming language for the PM.

$$\alpha(h_1 + h_0 \ln h_0)\delta \quad (16)$$

Order two:

$$\alpha[h_2 + h_1 + h_1 \ln h_0 + \frac{h_0}{2}(\ln h_0)^2]\delta^2 \quad (17)$$

Order three:

$$\alpha[h_3 + h_2 + \frac{{h_1}^2}{2h_0} + (h_1 + h_2)\ln h_0 + \frac{h_1}{2}(\ln h_0)^2 + \frac{h_0}{6}(\ln h_0)^3]\delta^3 \quad (18)$$

These last expressions (15, 16, 17 and 18) represent the disturbed degrees, which were implemented in the Python programming language, which offers us an intensive and effective handling of the required matrix calculation, through the NumPy package that provides and sets our arrange all the power of environments like Matlab or Octave.

3. Applications

3.1. Hanoi Network

According to Castro and Saldarriaga [11], the Hanoi network has been treated by several authors and is included for the purpose of comparing the results with those obtained in other investigations.

Table 1 shows the hydraulic properties of the Hanoi network consisting of 31 nodes and 34 pipes.

The first step is to divide the system into a series of finite elements as shown in Fig. 3, identifying its end points as “knots”, a pipe must be fully identified in the network by its initial and final node, implicitly establishing the direction of the flow flow in the pipeline.

The network has 01 reservoir (node 1) located at a height of 100 m and the network nodes will be on the same level 0, that is, plane with zero elevation, in addition to this the demands on nodes 2 to 32, are described in Table 1.

Table 2 shows the physical properties necessary to find the number of Reynods.

With all the data entered, we proceed to execute the routine implemented in the Python programming language, obtaining the flow rates and pressures with both solution methods (HGM and PM).

Table 3 shows the results obtained by the HGM and the PM after the simulation. The flow rates of the pipes and the pressures in each node of the Hanoi network are appreciated.

3.2 Statistical Analysis Flow Variable

From Table 3, we calculate the Pearson correlation coefficient (r) for the flows, obtaining r = 0.999999664, which indicates that there is a perfect positive correlation. The index indicates a total dependence

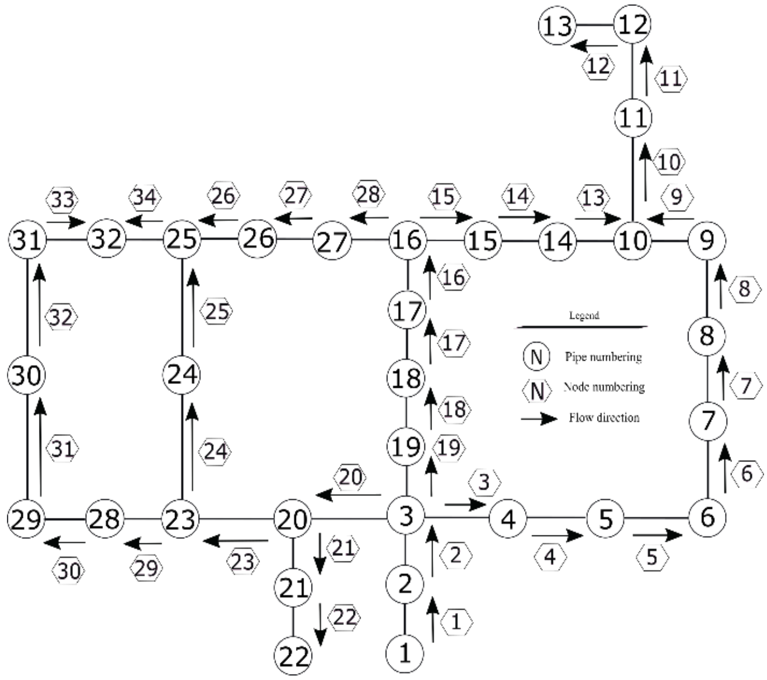

Fig. 3 Representation of the Hanoi network as a series of finite elements.

Table 1 Pipe and node data in the Hanoi network.

Pipeline	Length (m)	Diameter (mm)	Node	Demand (Lt/s)
1	100.00	1016.00	1	0.0
2	1350.00	1016.00	2	247.2
3	900.00	1016.00	3	236.1
4	1150.00	1016.00	4	36.1
5	1450.00	1016.00	5	201.4
6	450.00	762.00	6	279.2
7	850.00	609.60	7	375.0
8	850.00	609.60	8	152.8
9	800.00	609.60	9	145.8
10	950.00	762.00	10	145.8334
11	1200.00	609.60	11	138.889
12	3500.00	508.00	12	155.5557
13	800.00	609.60	13	261.1113
14	500.00	508.00	14	170.8335
15	550.00	609.60	15	77.77784
16	2730.00	762.00	16	86.11118
17	1750.00	1016.00	17	240.2780
18	800.00	1016.00	18	373.6114
19	400.00	1016.00	19	16.66668
20	2200.00	1016.00	20	354.1670
21	1500.00	508.00	21	258.3335
22	500.00	508.00	22	134.7223
23	2650.00	609.60	23	290.2780
24	1230.00	406.00	24	227.7780
25	1300.00	304.80	25	47.22226
26	850.00	304.80	26	250.0002
27	300.00	304.80	27	102.7779

Table 1 to be continued

28	750.00	762.00	28	80.55562
29	1500.00	508.00	29	100.0000
30	2000.00	406.40	30	100.0000
31	1600.00	406.40	31	29.16669
32	150.00	406.40	32	223.6113
33	860.00	406.40		
34	950.00	609.60		

Table 2 General properties of the Hanoi fluid network.

Description	Value
Gravity	$9.807\ \mathrm{m/s^2}$
Kinematic viscosity	$1.14 \times 10^{-6}\ \mathrm{m^2/s}$
Absolute Roughness of the Pipe	1.5×10^{-6} m

Table 3 Flows and pressures in the Hanoi network calculated by the HGM and the PM.

Pipeline N°	HGM Flow (m^3/s)	PM Flow (m^3/s)	Node	HGM Pressure (mH_2O)	PM Pressure (mH_2O)
1	5.53888	5.53888	1	100.000	100.000
2	5.29168	5.29168	2	97.8468	97.9318
3	1.67204	1.67424	3	71.8073	72.2843
4	1.63594	1.63814	4	69.7712	70.2545
5	1.43454	1.43674	5	67.2724	67.7629
6	1.15534	1.15754	6	64.8003	65.2956
7	0.78034	0.78254	7	62.7197	63.2234
8	0.62754	0.62974	8	57.0895	57.6155
9	0.48174	0.48394	9	53.3236	53.8572
10	0.55556	0.55556	10	51.1449	51.6781
11	0.41667	0.41667	11	50.0043	50.5365
12	0.26111	0.26111	12	47.5002	48.0316
13	-0.2196	-0.2174	13	40.0021	40.5241
14	-0.3904	-0.3882	14	51.6629	52.1972
15	-0.4683	-0.4661	15	53.9041	54.4329
16	-1.0792	-1.0740	16	55.3263	55.8532
17	-1.3194	-1.3143	17	66.4548	66.9439
18	-1.6930	-1.6879	18	69.0120	69.4979
19	-1.7097	-1.7046	19	70.8643	71.3443
20	1.67383	1.67677	20	66.8201	67.3163
21	0.39306	0.39306	21	60.0161	60.5226
22	0.13472	0.13472	22	59.6955	60.2005
23	0.92660	0.92955	23	42.6997	43.3650
24	0.32411	0.32545	24	31.1461	31.8148
25	0.09633	0.09767	25	25.8180	26.5117
26	-0.1720	-0.1691	26	35.8772	36.5967
27	-0.4220	-0.4191	27	54.5152	55.0398
28	-0.5248	-0.5218	28	38.2413	38.9108
29	0.31221	0.31382	29	28.1050	28.7950
30	0.23166	0.23327	30	25.2176	25.9071
31	0.13166	0.13327	31	25.1969	25.8866
32	0.03166	0.03327	32	25.1955	25.8859
33	0.00249	0.00410			
34	-0.2211	-0.2195			

between the two variables called the direct relationship: when one of them increases, the other also does so in a constant proportion.

We performed the Student’s t-test for independent samples, giving us the following results and tables:

There is a confidence interval for the difference between the means at 95% that estimates the degree of relationship between the two methods: [-0.735; 0.731]

The approach is as follows: the null hypothesis is taken that the mean of the flows obtained with both methods is the same, so we call the mean of the flows found: $H_0: \mu_{CaudalxPM} = \mu_{CaudalxHGM}$

(Both flows have the same value)

$H_1: \mu_{CaudalxPM} \neq \mu_{CaudalxMGH}$

(Both flows have different values)

From Table 4, we have a value t = -0.006. Since there are 68 individuals in total and two groups are compared, our parameter "t" has 66 degrees of freedom (D.F. = 68-2 = 66).

The value of the parameter “t” is not significant, since the tabulated value for an error α=0.05 ($t_{12,\alpha/2} = 1.997$) is higher than found t=-0.006.

Therefore, the null hypothesis is not rejected and it is concluded that there are no significant differences between the flows obtained with both methods.

3.3 Statistical Analysis Variable Pressure

From Table 3, we calculate the Pearson correlation coefficient (r) for the Hanoi network pressures, obtaining r = 0.999994301, which indicates that there is a perfect positive correlation. The index indicates a total dependence between the two variables called the direct relationship: when one of them increases, the other also does so in a constant proportion.

We performed the Student t test for independent samples giving us the following results and tables:

There is a confidence interval for the difference between the means at 95% that estimates the degree of relationship between the two methods: [-10.049; 8.988].

The approach is as follows: it is taken as a null hypothesis that the mean of the pressures obtained with both methods is the same, if we call the mean of the pressures found as μ:

$$H_0: \mu_{PresiónxPM} = \mu_{PresiónxHGM}$$

(Both pressures have the same value).

$$H_1: \mu_{PresiónxPM} \neq \mu_{PresiónxHGM}$$

(Both pressures have different values).

From Table 5, we have a value t = -0.111 As there are 64 individuals in total and two groups are compared, our parameter “t” has 62 degrees of freedom (D.F. = 64-2 = 62).

The value of the parameter “t” is not significant, since the tabulated value for an error α = 0.05 ($t_{12,\alpha/2} = 1.999$) is higher than found t = -0.018.

Therefore, the null hypothesis is not rejected and it is concluded that there are no significant differences between the flows obtained with both methods.

3.4 Statistical Analysis Variable Calculation Time

After multiple simulations for the HGM and the PM, Table 6 was prepared, where the execution time taken by each method to calculate the flow rates and pressures in a drinking water supply network was recorded.

The calculation times were obtained after 22 simulations, 11 for the HGM under the same conditions, another 11 simulations for the PM, obtaining the results in Table 4.

Table 4 t-test for two independent samples—Hanoi Red Flow Rates.

Difference	-0.002
t (Observed value)	-0.006
|t| (Critical value)	1.997
D.F.	66
value-p (bilateral)	0.996
alpha	0.05

Table 5 t-test for two independent samples - Hanoi Red pressures.

Difference	-0.530
t (Observed value)	-0.111
|t| (Critical value)	1.999
D.F.	62
value -p (bilateral)	0.912
alpha	0.05

Table 6 Calculation Time in the Supply Network-Hanoi Network.

HGM	PM	Difference		
Time1 (Seg.)	Time 2 (Seg.)	Time1 - Time2 (Seg.)	Percentage variation as a function of Time 1	Conclusion
0.34209	0.30814	0.0340	9.92%	Time reduction
0.06678	0.01297	0.0538	80.58%	Time reduction
0.08776	0.01293	0.0748	85.27%	Time reduction
0.08577	0.02352	0.0622	72.57%	Time reduction
0.10014	0.01396	0.0862	86.06%	Time reduction
0.13315	0.02100	0.1122	84.23%	Time reduction
0.07881	0.01397	0.0648	82.28%	Time reduction
0.11070	0.01296	0.0977	88.29%	Time reduction
0.10673	0.01399	0.0927	86.90%	Time reduction
0.10572	0.01596	0.0898	84.91%	Time reduction
0.10672	0.01395	0.0928	86.93%	Time reduction
		Average	77.09%	Time reduction

From Table 6, an average reduction of 77.09% in the simulation time of the water supply network can be seen, showing that the PM is more efficient in 77.09% compared to the HGM.

4. Conclusions

Two numerical techniques called HGM and PM have been used to determine the efficiency and execution time between both methods, making use of the Hanoi network.

The results in the pressures and flows between both methods are very favorable, providing a perfect and positive correlation coefficient.

The execution time between both methods and subject to similar computing circumstances, has obtained a 77.09% reduction in time using the PM. Thus, proving that PM is more efficient than HGM.

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