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Remarks on Value Assignment and Truth

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This article focuses attention on the specification of language expressions in order to express statements having a truth value. Each specified expression (= value expression) introduced can be derived by adding a specification to an expression of the natural language ("Mario Rossi"—"Mario Rossi born in … on …") or it can be associated with it without sharing any part of it ("water"—"H_2O"). We can imagine different languages and different translation procedures from natural language; each procedure imposes a choice among different value expressions. There are two "windows" that the syntax of the specified expressions "opens up to the world", so to speak; these are the following formulas: The value of /x/ = x', where "x", "x'" are names, /x/ is true iffx', where "x", "x'" are sentences.
In the second formula categorical reductions and compositions of the specified expressions must have been performed. In cases where these passages are not available, we would remain with a syntactic expression of the type of sentences, but not, such as to exemplify the traditional definition of truth.

Keywords: meaning, value expressions, truth

Saeptum Scientiarum

According to authoritative words, "the search for the truth is difficult in one respect; it is easy in another respect". I would like to start with some intuitive ("easy") considerations and points that I share about truth. To me, these intuitive considerations are based on common sense, a sense that can be made more common or shared if we want to define an empirical procedure of some kind, like a multiple-choice test. The points can be considered as metaphysical axioms or convictions that no evil genius is able to shake.

Intuitive Considerations

The truth concerns sentences of language. We use the term "true" in many ways but what is relevant here is the attributive use:

We say of a certain sentence that is true or false. We also say that a sentence, like "it rains", is true if and only if it rains. It is difficult not to comply with a statement of the form "'p' is true if p". Can we say that such statements are always acceptable? Obviously not. The fact is that we are not always able to pronounce ourselves on the meaning or on the truth value of a sentence (including those of the quoted form). Many statements of natural language are not suitable for being considered true or false because they are not asserted, contain ambiguous/vague terms or are simply meaningless. Since these statements are the majority of those uttered or written, it follows that truth is a very rare matter.

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The rarity of the truth is not discouraging. Just as for gold, seekers are not discouraged from seeking it, also or mainly in consideration of the value of such a rare object, even those who seek the truth, or that kind of “wisdom” that leads to the place where it is hidden, may perhaps believe by analogy that rarity determines value. In order to sift these statements, the equipment is easily available: It is made up of information that allows us to eliminate vagueness and ambiguity. However, the fact that these devices are readily available does not mean that they are equally effective. In principle, we cannot be sure that they will function in the expected way; all we can do when we buy them is to bet on their effectiveness.

To look for something on the other hand, you need to have an approximate idea of the thing you are looking for and the place where it is hopefully available. My personal starting point is the answer that comes to mind when asking the question: “Why do you think this statement is true?”. A variation could be: “under what conditions do you think it makes sense to predicate the truth of a certain sentence?” My approximate idea, which I hope the following considerations will help to clarify, is that the reason (conditions) depends on a set of elements that have to do with the specification of the terms and the topical ambit of that particular statement. Once we have resolved—to the extent that it is possible to do so—the various incompleteness, vagueness, and ambiguity, to attribute a truth value we need to frame the statement in a certain context, which can be a theory or more weakly a set of assumptions/rules connected with a control procedure.

Some “Metaphysical” Points

To begin with the principles that seem safer: “It is impossible that the same thing at the same time belongs and does not belong to the same thing, according to the same respect” and “it is not even possible that between two contradictors there is an average term, but it is necessary to affirm or deny of the same object, only one of the contradictory predicates”. An additional principle, which can be derived from the same source, is that a thing cannot fail to be identical to itself. I have often asked myself why Aristotle discusses these principles in the *Metaphysics*. After some time spent in historical research and not much in theoretical philosophy, I have convinced myself that if we are interested in truth we must look for it where principles are legal tender, so to speak.

However, since the evidence of the principles we accept is such for us and not for everyone, what Aristotle says must be considered an—albeit plausible—metaphysical hypothesis. Why are the principles plausible? Let us start with that of non-contradiction and identity. The question that needs to be asked is: Can I imagine that a certain thing has and does not have certain characteristics or is not identical to itself? The metaphysical hypothesis that I agree with is that it is not possible. Given this, if we make some assumptions about the meaning of the terms, it cannot happen that the sentences that contain them lead to a contradiction. To put it better: Contradictions are always possible, but when we discover one, we must admit that something is wrong with our way of specifying the terms or assumptions that we have made along the way. As we will see more clearly later, there are different ways of specifying terms, some are equivalent, and some are not. It is a matter of committing oneself to certain specifications, just as the gold diggers invest in a certain technology. If the result obtained is a contradiction—and there are reasons to believe that it derives precisely from a lack of specification—we have failed.

As to the excluded middle, the same question arises: Is it conceivable that something may not prove to be or not to be in a certain way? My metaphysical position is that it is not, but the justification is different from the one just provided and derives from a certain way of “justifying” the principle. Four different ways can be considered. The first consists in considering the principle a general law: In this case, the easily available counterexamples falsify the principle. The second way is to keep the principle and reject the counterexamples as senseless or non-sentences: Some of these counterexamples do not allow themselves to be labeled as “non-sense”. The third rejects the principle and maintains or not the counterexamples based on a different formulation of the logical rules. The fourth (the one I agree with) is to consider the principle in relation to the context in which we evaluate sentences.

If we have specified the meaning of the terms appropriately, we can expect a statement to be true or false. In this case, however, when can we find out that something is wrong? It is a somewhat strange situation, if we drop an apple we cannot observe that it does not fall and does not even stand still in midair. So, when the assumptions we have made do prove suspicious? There may be two different cases. Case a: we look for a certain thing and do not find it, because we have insufficient information: sentences that contain the term not specified are neither true nor false. Case b: It does not happen that something well specified has or does not have a certain property. The first kind of “violation” of the principle of bivalence has to do with the meaning of the terms and is typically found in empirical statements, the second with what happens or does not happen in the world to which we have access by direct or mediated experience, or with what is deductible or non-deductible in a certain theoretical context. The principle seen in perspective a is a rule that requires us to specify the terms; in perspective b, it determines/describes belonging to a certain nomological field.

An “a-type” violation is of a different kind from that affects statements that cannot be decided even in principle, as the deductive system to which they belong may be incomplete. On the other hand, the principle (understood in sense b) applies to sentences of any field, along with the specific regulations of this field. If a sentence is not deductible according to the “regional” legislation of this field, it does not fall under the principle. This leads me to the following definition: A sentence falls under the principle relative to a nomological field if and only if it belongs to this field. On the other hand, in the empirical context, if the terms are under-specified, it may happen that one cannot decide on an attribution of properties precisely because of a defect in the specification. Typically, this is detectable when a device that acquires information relating to the term, in a certain field, is unable to identify the value. Among the many possible cases are the poor efficiency of the fingerprint readers and the recognition of a person based on insufficient information, e.g., a blurry photograph.

It could also be argued, not without some reason that both “violations” of the principle above have a propositional form and therefore are subject to topical rules and principles. If this is true, however, it is necessary to consider the different propositional form and the operating principles. Let me explain with an Aristotelian-like example: Let us take the statements “x is a man” and “every man is mortal”. The first statement has a simple structure which is mirrored by an atomic formula of logic;[1] the second has a molecular structure. The first sentence admits a function from the domain as its interpretation. “Giving” this function simply means giving a set (or a Fregean concept), according to a pre-established procedure. Precisely, this procedure can lead to a violation of the first type of the bivalence law.

The principle of non-contradiction is not subject to all the above interpretations, if only because allowing contradictions or making the principle underivable is less attractive than not having bivalence as a logical law. However, it should be noted that also, in this case, there are perspectives a-b. As we said, the principle of

[1] The simplicity of a nominal or verbal expression must correspond to the simplicity of the semantic-procedural element. The first kind of simplicity is easy to define on the basis of the lexicon, in the absence of theoretical terms; for the second, the matter is more complicated.

bivalence determines/describes belonging to a certain nomological field; for the non-contradiction something more applies, as this principle determines belonging to any field. Bivalence can be seen as a national border (what is not decidable-determinable[2] here can be decidable-determinable elsewhere), non-contradiction as a continental border (in no area can we find two contradictory sentences). The principle of bivalence, moreover, seen in perspective a, is a rule that requires us to specify the terms. Violation of the principle of non-contradiction can also be an indication of a problem with the specification of terms, not in reality with a deficient specification but with a multiple specification. A term may allow and not allow recognition of a certain object based on different (implicit) specifications.

What I have just said poses a series of complicated philosophical problems. Some have to do with a concept of truth—which is not ours—in which the way we come to establish it matters; others concern the time and the alternative organization of events, or controversial aspects of scientific theories, or with the widespread way of describing certain experiments (e.g., the phenomenon of superposition and the alleged dual nature of light). In the space of this article, it is not possible even to summarily address these questions; therefore, I must ask the reader to tolerate a certain dogmatism in my metaphysical background position. To summarize, given a certain specification of the terms in an utterance, discovering a contradiction or failing to evaluate the truth value of the same must make us doubt the adequacy of the specifications. Instead, there is no reason to doubt that what we see clearly is or is not in a certain way.

There is a tendency to consider true what we can verify or what we can achieve with some device at least in principle. The point of view that we propose here is different from some revised form of verificationism: What we can get our hands on following an identification process can give us some certainty of the truth or else make us understand that we have done something wrong, but it has not directly to do with the truth. The specification of expressions is a necessary condition of truth, in other words, the truth of an utterance is inconceivable if the expressions that compose it are not fully specified. Given, however, that an utterance can be true, false or not be true or false for independent reasons. The principles (including the excluded middle) constitute a cage for the sentences that we call "real", a sort of *saeptum scientiarum*: If a statement is real, in some sense, then it respects the principles mentioned above, and vice versa, what respects the principles is real.

Specifications and Gambles

I have talked about the application of principles to a field, and this seems to contrast with their general validity. In the empirical case, I took some liberties by talking about a "cage" for real statements. An expression is specified when—with possible linguistic additions—it can be used on a certain field in an unequivocal way. We shell call *data sheet* the file that contains all the information needed to use an expression unequivocally. If we take an expression of language x as it is, it may be the case that x is F and not F, in violation of a logical law. However, speaking of an expression, taken as it is, is different from considering it as it is normally used. An expression "as it is" or "abstract" from its context of use is, so to speak, written on a blackboard: Either we have no information or we have too much, each piece of which takes us in different directions. If the same expression is considered together with the implicit information (usually available in a certain context of use) and field specifications, the principles are expected to apply to any sentence in which it occurs.

[2] The canonical examples I have in mind are: for decidability, the axiom of choice in the standard axiomatization of set theory; for determinability, the impossibility of determining an astronomical distance in the same way we determine the length of a discrete object such as a table (see for instance the observations in P. Bridgman (1927)).

There follow two simple cases of (non-)violation of principles. First example: Someone says “one has to drive on the left”; someone else says the opposite is true. The second example: Someone says that parallel lines never meet; someone else says that it is not true, that it can happen that two parallel lines meet at infinity. Just as the two pairs of sentences are, they violate the principle of non-contradiction. However, if we integrate the terms appropriately, e.g., saying “one has to drive on the left in England”, or specify the field by saying that we are talking about parallel lines in the context of a certain geometric theory, the violation of principles disappears. We need to look not at the single isolated expression but at its explicit rendering accompanied by contextual information. Nevertheless, why should we always add something to what we say? We are free not to do this, to remain in an a-topical area, but in doing so, we must not pretend to say something true. Nothing wrong, there are many reasons why we may not be interested in telling the truth.

An important point will be the choice of expressions for speaking in a non-contradictory way. Since we have to supply specifications, who ensures that they are good specifications? The language of values is used to translate-disambiguate the expressions of natural language. Each value expression is a specified expression that can be derived from the corresponding natural language expression by adding a data sheet (for “Mario Rossi”, for instance, the sheet should contain the specification “Mario Rossi born in … on ...” or for “water”, “H_2O”). We will indicate an expression of the specified language with the symbolic writing $/x/_{\Delta i}$. A data sheet may not contain all information (e.g., instrumental information) or any information, when a term is used in a certain field in only one way. In the case of empty data sheet, we will write “$/x/_{\Delta = \phi} = /x/$” and, if “x” is a name, “the value of $/x/_{\Delta = \phi} = x$” (for instance, “the value of /3/ = 3”). If the data sheet is not empty, it has a numerical index and the associated expressions of the object language inherit the same index.

One can imagine different languages and different translation procedures from natural language; each procedure imposes a choice—which I would like to call a *gamble*—of one or more expressions of value. What happens to the values expressed is not under our control: It may happen that a value does not exist; this does not conflict with our specification. However, if it happens that, for a certain value x, x is F and x is not F, this goes against our initial gamble of having clearly defined the meaning of “x”. The idea that guides us in specifying the expressions of the language is that once this work is done, we can bet on the bivalence of a certain sentence. An indicator for the quality of the specifications—at least for terms—is the ability to identify the values of expressions by a certain device. Apparently, this is a simple task; however, for many expressions of natural language, it can be complicated or impossible. There are examples everywhere: It is not clear how an adverb, such as “fast” or a qualitative adjective just as “good” might be specified. It seems in fact that the points of the map that we manage to fix with the specifications allow us to isolate a very limited portion of territory surrounded by completely unexplored areas.

We often use language to describe our thoughts, attitudes, desires, etc. By the mere fact that not everything we are talking about is accessible, it is not possible even in principle to provide specifications. We could say that, when we use unspecific expressions, we are not interested in telling the truth. This, however, seems excessive: If I say that I am in a good mood, and I am asked to specify, I would not know what else to say. Those who listen to me seem to understand what I am talking about and as far as I am concerned, that is the way it is, even if I cannot provide objective proof (apart from physical manifestations of some kind). We think we are able to assign a truth-value to “I am in a good mood” because we place that phrase in a more or less shared space. If we bet here that specifications are not necessary, we probably do not run the risk of contradicting ourselves (It is unlikely that we are actually in and not in a good mood), but we (or someone else) may not be able to decide on our mood in certain other circumstances.

In ordinary use, specifications are usually implicit; they depend on contextual or pragmatic factors. This applies both to proper nouns exchanged in a conversation without misunderstanding, to the defined descriptions and predicates in general. One realizes that something is wrong when two people called Andrea are seated at the table or when using descriptions, such as “the man who drinks martinis”. The use of the language I want to focus on is descriptive and non-contextual. In such use, the specifications are intended as the necessary information that enables a certain device to identify one and only one object (in the case of names). The question we must ask ourselves when setting a specification, such as “born in ... on ...” is: “Does this specification actually enable us to identify the object we are talking about? The answer is not definitive; we must give an answer, or rather bet on the answer, but it cannot be taken for granted that it is the right one.

There are things, like trees and animals, but how do we identify them based on the information transmitted by the names “tree” and “animal”? In front of a bush of a certain height, I could remain undecided whether to call it tree or not; likewise for a single-celled creature or one that does not respect the idea—which I assume to be more or less shared—of animal. To be able to use a certain expression in true sentences, it is therefore necessary to add some specification to the expression. Since we are considering different syntactic categories we will decorate the formula introduced above with a categorial index “χ”: ${}^{\chi}/x/_{\Delta\iota}$. I consider a transformation from the ordinary syntactic expression “x” to ${}^{\chi}/x/_{\Delta i}$, which is intended to represent a value expression: The letter χ stands for a syntactic category, Δ_i is a data sheet, containing—as we have just said—missing information. A value expression ${}^{\chi}/x/_{\Delta i}$ expresses a value when the sentence “the value of $/x/_{\Delta i}$ is x_ι” is grammatically well formed: Evidently, this can only be in the case where “x” is a name.

One might wonder if a data sheet could contain terms that require specifications. If our aim is to build an unequivocal language, obviously we should avoid using imprecise expressions, at least in principle. There are in fact cases in which inaccurate expressions assembled make a precise expression, in the sense that whoever receives the information associated for instance with a nominal phrase is able to identify an object. There is not an actual rule, or something that can be a priori wrong. The choice of a specification, however, has consequences: if we bet on the unequivocality of a specified expression, what it expresses cannot have contradictory properties. If this happens, we have lost the bet. Another kind of attention that is required for independent reasons concerns the “quality” of the specifications: by assembling the terms, as happens for the descriptions, the object (value) of the identification can change. *If* you want more stable or absolutely stable specifications, you have to choose them carefully.

When the observation of an object leads to the introduction of a name, the choice of specifications is driven by the possibility of identifying the value based on a set of information. For example, when faced with twins, probably an image cannot serve to recognize one or the other; consequently not even a specification that recalls such a formal element can be useful. The choice of the specification affects the evaluation of sentences that contain one of the specified expressions. It could be the case that one use of these expressions is authorized according to its formal specification and that the phrase in which it occurs is true and at the same time false (whoever is so called has and does not have a scar).

There are particular contexts in which the terms that would need a specification appear as reasonable and sound; this happens quite often in philosophy. One may wonder what a rather clear philosopher, like Hume, means by the term “impression”. How can we verify what he says about it? Moreover, what does “categorical imperative” mean? A student is expected to recite the well-known definition; once this is done, the professor is happy but if we take the expressions one by one in the *definiens* and *definiendum*, there is no way to specify the expressions and find values. Nonetheless, rather surprisingly, one continues to do philosophy and philosophical theses are understood and discussed.

Prima facie the philosophical space is not comparable to the empirical one and to the various theoretical fields. Having said that, being interested in shared truths, two possibilities remain. The first: We just start talking about numbers, scientific laws and, under certain conditions, apples. The second one: We ask ourselves what relationship exists between the areas of application of words, to see if it is worthwhile dealing with philosophy or with our private life without thinking these things are made of the same stuff that dreams are made of. I think we need to focus on what justifies or makes specifications necessary. As we have already said, value expressions belong to a certain field; the field requires certain specifications. When the terms in a certain area are introduced by definition, specifications are superfluous, when there are no definitions, as in the case of natural language, specifications are necessary to make unequivocal use of words.

I would like to make a case of the kind mentioned above: no specifications and no systematic rules, but something sound seems to have been said. My hypothesis is that this may be. How then can it make sense to talk about truth? I do not mean to find out the truth, but to consider the sentence “‘p’ is true if p” sound, with p, a philosophical statement. Incidentally, it should be noted that, in such an environment, there are no empirical or theoretical procedures that allow us to (dis)prove “p”, that is, sometimes they exist but are very weak, such as checking whether it is true that x has a headache by asking x for confirmation. A minimal (perhaps private) field can have modest demands but it is nevertheless a field: We do not take statements on a state of health or philosophical beliefs as literary sentences, there is some reason why someone can say he understands Hegel and someone else judge if this is (not) the case.

The question is: Given the absence of certain rules and procedures, when we speak a philosophical language, or some other more or less private language, do we really run the risk of contradicting ourselves? What could count as evidence to decide that there are no impressions or that the categorical imperative is false or that God does not exist? We observe that bivalence can be respected—for some term expressly introduced—since we support some opinion. Our basic thesis is that the values of the expressions are the nodes of a regional map, or private field. The sentences composed with these expressions have a value in the field (according to a data sheet), except for contradictions or absence of bivalence; if neither of these things is given, then the “theory” has its legitimacy within its narrow confines. Axioms and logical-topical rules are to be considered a sort of legislative element that governs the use of sentences. In the concrete use of language, when we try to establish some truth we must take into account these legislative elements for a specific field. There are no general prohibitions, that is, we can use unspecified or bound terms (as in literature)out of every area (let us say a-topically); we will certainly face violations of the metaphysical principles of which we have spoken, but we could be prepared for this, as when using a map that leads nowhere.

Some Linguistic Cases

It is now appropriate to provide some detail on how expressions of different semantic categories can be specified. The following is a partial and provisional analysis of a fragment of the language of values, or more precisely, we will provide some indications on how this language can be obtained starting from a fragment of natural language. No suggestions are provided regarding expressions that play an important role in building a theoretical lexicon, for example, we do not give indications on how to translate quantifying or connective into a value language. I will not even talk about the characteristics of the specification language: What we would like to do is to form this language from a few basic expressions, but this can almost always not be done; it is also difficult to consider "atomic" specifications, which do not need further specifications. My guess is that there are reasons to bet on certain specifications and these reasons are often "winners", otherwise, there are no such reasons and we must refrain from playing the game of truth.

Individual Expressions

Individual value expressions are intended to have a single value, which can be considered what they stand for, or rather, what they talk about. In the "reality", we are talking about, we suppose things do not have opposite properties at the same time. This constitutes a constraint on the choice of specifications and on the expressions of value themselves, for example. We can specify "Mario Rossi" by adding a certain date of birth; let us say that this information is included in the data sheet Δ_i. In doing so, we could not say that the value of /Mario Rossi/$_{\Delta\iota}$ is Mario Rossi, as we do not know who Mario Rossi is. We will say rather that the value of /Mario Rossi/$_{\Delta\iota}$ is Mario Rossi$_{\iota}$. Obviously, in ordinary use, ambiguous expressions are the norm, but this only means that ordinary expressions are not expressions of value. There are many specifications, for instance: definitions, appositions, empty specifications, complex instrumental descriptions, simple specified expressions that do not embody the term to be specified. How to choose among the different ones available is primarily a conventional and empirical issue; in fact, a certain choice is not without consequences, in the sense that some specifications can lead to inconsistent use.

From a syntactic point of view, a name does not admit adjectival specifications; however, in ordinary discourse, there are clauses that can be added to nouns. For example, I can say "Mario Rossi is an elementary school teacher"; at the same time, I can add the specification in the phrase "Mario Rossi—the person I presented to you yesterday—is an elementary school teacher" or "Mario Rossi—born on January 13, 1984—is an elementary school teacher". If we consider the phrase composed of the proper name and specification, we obtain a sort of definition of Mario Rossi, of the type "Mario Rossi = Mario Rossi born on January 13, 1984". In the expression /Mario Rossi/$_{\Delta\iota}$ used above $\Delta\iota$ can contain a *definiens* of that type. Specifications can lead to the identification of only one value, for instance given /Mario Rossi/$_{\Delta i}$ and /Mario Rossi/$_{\Delta j}$, we can have that Mario Rossi$_i$ = Mario Rossi$_j$. If "Mario Rossi" is specified in this way, it cannot be the case that Mario Rossi is F and not F.[3] In cases of homonymy, Internet providers offer simple specified expressions, perhaps associating a number with the proper noun.

Another case that I think can clarify what we mean by specification is that of expressions, such as "Mario Rossi of which x speaks". Assuming that "x" is adequately specified, one wonders if the clause "of which x speaks" could be of service as a specification of "Mario Rossi". My answer is "No": Mario Rossi mentioned by x can be the Mario Rossi mentioned by y; the fact that someone talks about Mario Rossi tells us nothing about Mario Rossi, i.e., does not provide us with useful information to uniquely identify Mario Rossi. We should ask x or y, to obtain the information that lacks the insufficient specification "of which … speaks". Looking at narrative (taking the example seriously for a while): "Madame Bovary" cannot be identified on the basis of the

[3] However, keep in mind the observation made on the "quality" of a specification. In the case under consideration, the specified expression seems to be of good quality (in the sense intended above).

information provided by Flaubert; among other things, he identifies himself with Madame Bovary on one occasion[4]. These aspects deserve further investigation.

It seems to me a good idea to start from our ordinary experience. If someone asks me to identify Mario Rossi, I have to decline the task, as it is one of the most widely used Italian proper names. I need to know more. One piece of help that might be provided for me is to indicate a causal chain that links the expression to a certain Mario Rossi rather than to another.[5] Knowing nothing about causal chains, I would think that it is not a real help but a joke. At this point, I am provided with information on the place and date of birth. I then try to identify Mario Rossi, perhaps by going to the registry office or in some other way, and I succeed in this task; so, I would say the help I received was enough even if the informer does not give me Mario Rossi's phone number and address. The point I would like to make is that it seems natural to settle for at least that meager indication until you are faced with two Mario Rossis born in the same place on the same day. Similar considerations apply in all those cases where it is a question of identifying an empirical object by using a name. I do not know what snow is from a scientific point of view and probably, if I am not an Eskimo, I do not need a specification of "snow", as I already have all the information I need to identify the snow. I am therefore able to identify snow and reason about its properties based on some system of shared knowledge.

The situation is different if ordinary expressions are understood in a scientific sense. We cannot say that by default we know how to identify a certain chemical substance that has the same smell and taste as something we drink. In these cases, we must distinguish between a common way of identifying objects and a way that presupposes certain special knowledge and a field of reference. In general, we assume that an expression can be disambiguated in two connected ways: by indicating its context of use and by adding specifications. An expression is fully specified when it is associated with a data sheet where all the relevant information is provided. If we change the field (e.g., from empirical common to that of a certain scientific theory), the same expression may need to be specified differently in order not to incur contradictions. For instance, the water of which we have direct experience and an aggregate of H_2O molecules have in common that observation possibly supplemented by laboratory equipment enables us to identify the value of the related expressions. This must be kept in some consideration in identifying the special area to which we assign the expression of value $/\text{water}/_{\Delta i}$. To have experience of the aggregate of molecules we need laboratory equipment and if we do not have information on this aspect, our possible search for the aggregate of molecules will be condemned to failure. It therefore seems reasonable to distinguish between the field of a theory and the empirical "default" field. I use the term "theory" to indicate a certain structured set of definitions and rules, such as a scientific theory, a board game, or a deductive system.

Common Nouns

In the case of proper nouns, the specifications must be—such as to provide necessary identification criteria for that specific value and only for that. For common nouns, things go differently. First of all, we cannot speak of the value of an expression, such as $/\text{man}/_{\Delta i}$, that is, we cannot say, for example, that the value of $^{CN}/\text{man}/_{\Delta i}$ is man_i, simply because the formula is syntactically incorrect. As we understand the values, non-individual expressions, such as common nouns, adjectives and verbs do not have an associated value. Nonetheless, when using these expressions, we may or may not be able to identify values.

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4 Forgetting what Y. Leclerc says about this sentence**:** "on ne la trouve dans aucun de ses texts actuellement connus, ni dans une lettre, ni dans un carnet de notes ni dans le dossier de genèse de *Madame Bovary*".

5 I am referring to the causal-historical theory of reference put forward *inter alios* by Keith Donnellan (1972) and Saul Kripke (1980).

In the biological field, one could choose to adopt a specification that corresponds to the “scientific” definition of what we are talking about. We do not have to do this but it seems a good idea to avoid vagueness or running into inconsistencies. Since we all know how to recognize a man, we can translate “man” into ^{N}/man/$_{\Delta i}$—where “man$_i$” could be the species—and bet on the fact that we will not have identification problems or run into contradictions. The expressions specified in the category of *common* names have no value, as we said. The same goes for plurals: Men are not the value of ^{CN*}/men/$_{\Delta i}$ for the fact that “the value of /men/$_{\Delta i}$ is men$_i$” is ungrammatical. Although common nouns (together with intransitive adjectives and verbs) have no value in themselves, there is a relationship of dependence, so to speak, with proper expressions of value. These latter expressions may appear as appropriate subjects in sentences whose predicate is a common name.

Names of Measurable Quantities

I said that for each field there are principles and logical rules that govern the use of expressions. The principles common to all areas are those of identity, not contradiction and bivalence (as a regional principle). The use of the specified expressions relates to a field that can be extended at least to the point where there are no inconsistencies. A particularly important case is the overlapping of the fields of physics and mathematics. As regards this discussion, I would like to consider some aspects related to expressions of measurement, as a non-secondary part of the problem of the relationship between the mathematical and empirical observational domain. In discussing this, I would also like to suggest a way of (re)considering the discussion in epistemology on the concept of error and “true” measurement of an object.

About meters: It is natural to think that a particular object is or is not x m long. If, by definition, I introduce the expression “x m” in mathematics, it behaves as an element of a literal calculation in the domain for instance of rational numbers. If we can talk about measurable entities, then every expression “*a* is x m long”, with x variable on the domain is true or false. For example, I can say: “given a rectangle 2 m high and whose area is 2 m^2, the base measures 1 m”. It may happen, however, that in measuring physical objects I cannot speak of exact numerical data but have to admit measuring range, for known reasons. In various texts and manuals, the difference between “exact” (“true”) and approximate values is considered. From the point of view that I am proposing, the adjective “exact” means only “mathematical”. Physical quantity names on the other hand are the prototypical example of specified expressions. The “1 m” makes physical sense when a measurement scale, instrument and protocol are established. From our point of view, there is an expression of value /1 meter/$_{\Delta i}$, where Δi contains all the “operational” information. I would like to suggest that the description of the instrument must include that of its sensitivity, which allows accepting measures “around” the one expressed. In practice, if the instrument described in Δ must have a sensitivity of +/- 1 cm, the value expressed by /1 meter/$_{\Delta i}$ will be 1 m plus or minus one cm.

The point is to justify the fact that the specification of mathematical quantities does not coincide with physical measurement. If among the principles we have to accept there is that of non-contradiction, how is it possible that, in a geometric description, one would expect that a body has precisely certain dimensions and in the measurement according to an empirical standard it has a different one or more than one? On the one hand, it must be said that we have measures and not measured things, that is, the mathematical measurement does not correspond to the physical measurement. This is obvious, but does not explain the effectiveness of applying mathematics to the description of the physical world. What really matters is how we relate the specifications, saying, e.g., that the mathematical measurement must fall within the inaccuracy range of the physical measurement. The phrase "such a body is 1 m long" does not contradict the approximate measurement for a certain scale value if it is assumed, for instance, that the former falls within the interval of the latter.

Various elements can lead to instrumental refinement: among them, construction specifications, the physics of the detection system, the sensitivity of the instruments. In order for different instruments not to give conflicting results, it is necessary to set one standard or standards that are more compatible. Just one example about compatibility between instruments, two watches built differently with one-second sensitivity are synchronized at t_0. At the instant t_n, they mark a time differing by one second and a half. Suppose that the measurement interval corresponds to sensitivity, i.e., the time measured experimentally is t +/- 1 second. Since the deviation is greater than the sensitivity, the construction standards must be incompatible. Vice versa, if the second instrument has a sensitivity of one tenth of a second and the measurement interval still corresponds to the sensitivity; decimal measurements do not represent a proof of incompatibility.[6]

Meaning and Truth

We have seen how the value of a nominal value expression $^{N}/x/_{\Delta i}$ is x_i; for example, having provided a certain specification for "Giorgio", e.g., "The individual born in ... on ..." we declare that the value of ^{N}/Giorgio/$_{\Delta i}$ is Giorgio$_i$. It would seem that the declared value corresponds to the individual born in ... on ... and that Giorgio$_i$ is nothing more than the meaning/denotation/reference of the linguistic expression "Giorgio". If so, the introduction of the expressions of value with their decorations could be underestimated from the semantic point of view, at least as regards the names endowed with the ability to refer directly to something. My position is that in most cases the phrase "'Giorgio' refers to Giorgio" (or possible lexical variants) simply does not make sense. I also argue that the apparent meaning of the sentence derives from a set of information available to the speaker in a given circumstance. A grammar of language on the other hand must say something of expressions in a general and a-contextual way, with regard to both the syntactic and semantic side. From my point of view, in order to attribute meaning to "'Giorgio' refers to Giorgio", it is necessary to transform the nominal expression into an expression of value. In the subsequent rewriting of the sentence, the part "refers to" does not appear.

Not all natural language expressions can be specified in the way we have indicated. Consequently, there is no functional correspondence between the set of expressions of the natural language and that of the expressions of the specified language. Our proposal allows two alternatives: the first is to consider natural language expressions as elements of the basic lexicon interpretable on the set of their specifications. The choice of the (partial) interpretation function is what we have labeled as a "gamble". The second way takes as its basic lexicon that of the specified expressions and interprets them using the same specified language. There are no important differences between the two ways of proceeding, however if we consider semantic analysis as a component of a grammar of language, the translation-interpretation in the specified language could be convenient (the syntactic rules manipulate simple signs, while the specifications are sometimes obtained by assembling expressions of different semantic categories). For the translation-interpretation of compound expressions, there is no automatism, even when the components have been specified: one must authorize such translations one by one.

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[6] What we just said obviously does not exhaust the complex subject of measurement. The few considerations above are to be understood as a philosophical glance at a matter that can and must be treated from the point of view of the theory of measurement. T. Eran (2020) provides introductory information on this subject.

The form of value expressions may seem long-winded and therefore unattractive. However, it is necessary to reflect on the fact that when we think of expressing ourselves unequivocally, what we do is share with our interlocutors a series of implicit assumptions relating to an intended use of terms and a reference area. The specifications that are part of the expressions of value simply transcribe the implicit assumptions we make, or more precisely select those that allow the unique identification of an object or consistent use of a certain expression. Moreover, it may happen that expressions of value are bound to a certain (sub) field in which they are used without lengthy additions, such as those considered above, without generating vagueness or ambiguity. This typically occurs in scientific theories.

Some further observations on the semantics of intransitive verbs, adjectives, sentences. I discuss these examples in order to give an idea of how the classic definition of truth can be (re)formulated. I said that only nominal expressions express a value. On the other hand, there are several expressions which, despite not having a value, are related to sets of values. For example, let us consider an intransitive verb like “runs”. In order for this verb to correspond to an expression of value, the added specification must be sufficient to identify what is running without vagueness and ambiguity. Suppose this happens by adding a specification to the data sheet Δi. We will therefore have that the interpretation-translation of “runs” will be ^{IV}/run/$_{\Delta i}$. There is no value corresponding to this expression, as the formula “the value of ^{IV}/run/$_{\Delta i}$ = run$_i$” is not grammatical. However, similarly to what happens for common nouns, intransitive verbs can be related to the values expressed by individual value expressions.

About adjectives: In philosophical literature, adjectives are found almost everywhere; “red” may have the highest frequency occurrence. What need is there to specify “red”? Ultimately, we all know what “red” means, in a conversation at the bar a language-philosopher friend of mine tugged at his sweater and said “Do you think this is purple?” I would put it this way: Suppose one thinks that “red” does not need specifications and therefore the expression of value ^{A}/red/ in the empirical field corresponds to the natural language expression. As you can see without too much effort, the list of colors that we can distinguish, whether or not we are affected by tetrachromacy, is very long: There are several “reds” with which a number is associated, but we want to say that there is a single red. To do this, we rely on our perceptive apparatus, that is, that of the human species not suffering from the aforementioned pathology. However, it is easy to imagine that the “typical” human color recognizer is in difficulty when faced with a particular shade, perhaps at the outer edge of the chromatic range.

What can be said about the statement “this is red”, when an experimental subject has difficulty identifying “this”? The presence of vagueness indicates that the bet we made on the correctness of the specification of “red” has turned out to be a losing one; in fact, the test subject has been unable to say whether the proposed sample is definitely red. Of course, it could be argued that there is the red in itself and therefore the sentence in question has a value regardless of the tool used and its operating protocol, but this is a metaphysical assumption that I do not agree with. What I expect is not to run into deadlock once a detection-measurement system and a protocol is defined. I may not know for sure that I will find myself in such a situation, but this simply means that the bet I made on that certain detection-measurement apparatus is kept “open”, so to speak, I have actually lost, but I may never know.

I come now to the transformation of the rules of composition from the syntax of natural language to that of the language of values. In illustrating the point I use the standard notations of categorical grammar. Syntactical rules allow for something like this:

$$^{x}a \bullet {}^{x\backslash y}b \rightarrow {}^{x\bullet x\backslash y}a \bullet b \rightarrow {}^{y}ab,$$

What we would like to do is establish rules for the language of values that allow for instance:

$$^{x}/a/_{\Delta i} \bullet {}^{x\backslash y}/b/_{\Delta j} \rightarrow {}^{x\bullet x\backslash y}/a \bullet b/_{\Delta i\bullet \Delta i} \rightarrow {}^{y}/ab/_{<\Delta i,\, \Delta i>}.$$

Unfortunately, the expectation of getting an expression of value (from "a", "b", or from /a/, /b/ the value expression /ab/) is often disappointed, due to the fact that the specifications must be provided one by one; it is not stated that there is always a composed *value* expression. A further complication arises from the composition of different data sheets (here we have simply assumed that the product is an ordered tuple). I will leave aside this second issue, which requires a definition of the product of derived expressions, and limit myself to providing some examples of syntactic composition.

Suppose that "man" admits a specification perhaps based on a certain sequence of genetic code and "weighing 70 kg" one based on a comparison with a standard sample; we thus get (neglecting some details that have to do with the measurement interval) the compound expression: $^{N}/\text{man}/_{\Delta i} \bullet {}^{Adj}/\text{heavy 70 kg}/_{\Delta j}$. In the empirical field, we should have that $^{N}/\text{man} \bullet \text{heavy 70 kg}/_{\Delta i \bullet \Delta j} = \dots /_{<\Delta i,\, \Delta j>}$. This "product" is not obtained automatically from the factors, but follows the same procedure that led us to the factors. A specification is constituted by the information necessary to find a certain empirical object or to (re)construct a theoretical one; in the case of complex expressions, we would like to do the same. For /man • heavy 70 kg/, it is quite easy: Once we recognize a man according to his specification, we can weigh him by putting him on the scales. The two operations to which the specifications refer can be carried out "in parallel", therefore, we have reasons for betting on the composed expression. As can be understood, this does not apply in general (take for example "man" and "ideal", where "ideal" is specified by taking into account a hypothetical gas whose molecules occupy negligible space and have no interactions).

The same goes for the composition between individual expressions and intransitive verbs:[7]

$$^{n}a \bullet {}^{n\backslash s}b \rightarrow {}^{n\bullet n\backslash s}a \bullet b \rightarrow {}^{s}ab,$$

In the language of values, we should have:

$$^{n}/a/_{\Delta i} \bullet {}^{n\backslash s}/b/_{\Delta j} \rightarrow {}^{n\bullet n\backslash s}/a \bullet b/_{\Delta i\bullet \Delta i} \rightarrow {}^{s}/ab/_{<\Delta i,\, \Delta i>}.$$

We can define the truth of $^{s}/ab/_{<\Delta i,\, \Delta i>}$ in the following way:

$$^{s}/ab/_{<\Delta i,\, \Delta i>} \text{ is true iff} a_i b_j$$

As we have just said, for the composition of value expressions (in the present case that of a noun and intransitive verbs), it is not certain that, given two categorically compatible value expressions, there is a value expression composed of those two. Betting on a propositional compound is like betting that such expression (say "ab") exemplifies the pattern of truth in the last line above.

The benevolent reader will have realized that our analysis takes place mainly on the syntactic side and where we speak of interpretation of the expressions of the natural language we almost always mean the translation into a specified language. At the same time, there are two "windows" that the syntax of the specified expressions "opens up to the world", so to speak; these are the following formulas:

$$\text{The value of } {}^{N}/x/_{\Delta i,} = x_i$$

$$t/x/_{<\Delta 1 \dots \Delta n>} \text{ is true iff} x_{1\dots n}$$

In the second formula, categorical reductions and compositions of the specified expressions must have been performed. In cases where these passages are not available, we would remain with a syntactic expression

[7] Type decorations are in the style of Lambek categorical grammar (see for instance Lambek 1988).

of the type of sentences, but not, such as to exemplify the traditional definition of truth. This simply means that the statement in question does not belong to the "product" of the respective data sheets of the components and therefore is neither true nor false, even when it has a unitary syntactic sense. If we turn to the ordinary semantic treatment of natural language, or at least of the fragments considered in formal grammars, we may be a little disappointed.

What usually happens in categorical grammar is that all the expressions of the fragment have a certain meaning assigned to them (possibly by means of a translation in a certain "intermediate" logical language). If the syntax has a categorical form, the language of the corresponding semantic category is that of the lambda calculus. The interpretation is defined as a functor from syntax to semantics. Our analysis does not preclude associating lambda-terms with the specified language expressions and establishing that their value is a certain mathematical object. For example, the expression $^{IV}/\mathrm{run}/_{\Delta i}$, can be translated as $^{\langle e,\ t\rangle}/\lambda x.\mathrm{run}_i x/$[8] and this expression has as its value a certain mathematical entity. What cannot be said in general is that the interpretation functor always "remembers" the syntactic structure: In other words, even if specified and categorically compatible expressions are available, their product may not be defined.

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[8] I use here the notation in R. Montague (1973).