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Gold Prices as a Mechanism of Control and Equilibrium in Financial Markets

Ewa Drabik
Warsaw University of Technology, Warsaw, Poland

Gold is used as a currencies comparative measure and, because of its properties (it does not rust) and use (in space industry, for example), it has a significant role in balancing both financial markets and economies. During crises, gold seldom loses value. We aim to show that price of gold is a stabilizing factor for the economic balance. We will do so utilizing the chaos theory, which gains more and more popularity in social sciences.

Keywords: gold price, equilibrium, fractal market hypothesis (FMH), attractor, fractals

Introduction

For ages, reductionism was a universal paradigm, assuming complex objects may be reduced to smaller parts. It turned out that some complex systems are irreducible. Theory of nonlinear systems emerged, using differential equations to describe them. Differential equations have their inadequacies, though, and thus unsuitable for analysing stability, as stability is rather an exception than a rule.

In the 1960s M. Hénon, a French astronomer, noticed movement of galaxies is planar in short time intervals, but generally, in longer periods of time, their trajectories are complex and three-dimensional. A few years later it turned out orderly fragments of motion are attractors. N. Z. Lorentz, an MIT meteorologist, presented three differential equations describing changes in atmosphere, and his research led to discovering a strange attractor. In the 1970s, B. Mandelbrot laid foundations of chaos theory. In the 1980s, from observations of economic time series, the fractal market hypothesis (FMH) emerged, which, contrary to the current effective market hypothesis (EMH), does not assume stock market prices are random.

Gold is used as a currencies comparative measure and, because of its properties and use, it has a significant role in balancing both financial markets and economies. During crises, gold seldom loses value. We aim to show, through chaos theory, that price of gold is a stabilizing factor for the economic balance.

Part one deals with gold parity. Part two describes basic terms of chaos theory, like attractor, fractal dimension, phase space reconstruction method, Hurst exponent, or Lyapunov exponent. Illustrations will be given for the described phenomena. Example 4 will illustrate differences between a random time series and deterministic one. The same analysis will be applied to gold prices. The following part will present conclusions.

Ewa Drabik, professor, Faculty of Management, Warsaw University of Technology, Warsaw, Poland.
Correspondence concerning this article should be addressed to Ewa Drabik, Faculty of Management, Warsaw University of Technology, Narbutta 85, Warsaw 02-524, Poland.

Research Design

Research Subject

The paper aims at discussing a specific balance of financial markets in the language of the chaos theory. We will mainly analyse the influence of gold prices on the stability of financial markets. Using chaos theory, it's possible to show gold price changes mechanism has a stabilising effect on the functioning of the financial markets, regardless of the economic situation. We will show the gold prices series, after a phase space reconstruction, constitutes a strange attractor. To this effect, we will utilize the method known as phase space reconstruction.

Theory and Hypotheses

Gold prices are a stabilizing factor for the economic balance. In order to demonstrate this regularity, we will utilise the fractal market hypothesis, which assumes share prices have a cause and effect character. Those prices depend on different attitudes towards risk on the part of various market game participants. Also important are psychophysical characteristics of investors, and the heterogeneous character of the information flow. On top of that, share prices reflect the combination of short- and long-term investments. Investors have different market strategies. Bubbles destabilize the market. In 1929, for example, share prices were rising, investors were buying, until the bubble burst and an unprecedented stock market crash happened, which, nevertheless, did not destroy the system, as it gave way to the next market stage, thanks to gold parity, among other factors.

Gold Parity

Gold has been a crucial means of payment since ancient times. The base of gold currency was established in 1844. The Bank of England was required to buy its banknotes for gold. Gold standard was introduced in 1821, when it was made possible to exchange paper bills for gold. In 1870-1880 the system was adopted by other countries. The outbreak of the Great War in 1914 caused difficulties in maintaining this system. Beginning of the 20th century saw the start of the process of replacing gold with other forms of currency, called the demonetization of gold. The gold standard is an obstacle in buffering competitiveness changes. The gold standard played a part in extending and deepening the Great Depression (1929). Some countries departed from the gold standard and devalued their currencies in the 1930s, and as a result, they enjoyed a higher level of industrial production than others, as well as the ability to pursue an expansive monetary and fiscal policy.

Gold is used as a currencies comparative measure. Various country currencies are precisely related to the value of gold; gold parity ties the currency units to specific amounts of gold. Currency exchange rates are basically ratios of one currency's gold parity to another's. Gold money is substituted with paper money, exchangeable to gold according to the currency's parity. A currency's exchange rate may not exceed the exit gold point, being equal to sum of the gold parity and the cost of transfer (transporting the gold, insuring it, etc.). This is part of the gold price adjustment mechanism.

Fluctuations of gold parity are limited, thus countries using the gold standard may consider the system as very stable. Price changes are automatic, unless surplus is withdrawn from the market. Balance of payments surplus means that growing reserves of gold result in money supply increase and price increase. The opposite process results in export restriction and import increase, thus restoring the balance again. The amount of money in circulation is tied to the amount of gold. The central bank of a country with decreasing gold reserves is obliged to decrease the amount of money in circulation, interest rates are rising, investment activity falls due to more expensive loans, and employment also falls. The decline in household income causes a decrease in consumption and demand, leading in turn to deflation. Money flows to other countries, leading to a positive trade balance and inflation. Central banks lower the interest rates, increasing the economic activity. Employment and production levels increase, as well as consumption and demand. It should be emphasized, however, that some economists gathered around the Austrian school of economics are in favour of the gold currency.

Elements of Chaos Theory and Financial Markets

In 1900 a young French mathematician, L. Bachelier, presented his *Théorie de la spéculation* (“Theory of speculation”) at the University of Paris. His instructor was H. Poincaré, and the main subject of the thesis was finding the formula for European type option pricing. Bachelier’s self-assigned task was ambitious: “to develop a formula for the probability of market fluctuations”. He concentrated on the snapshot of the market in a given moment, assuming the financial instruments price fluctuations over time are virtually unpredictable. Taking this into account, it would seem right to assume the process describing fluctuations is a Wiener process, which has independent increments. Bachelier was virtually unknown in his field, and when he presented his Ph.D. thesis at the Sorbonne, it was graded as *honorable*, less than *très honorable*, which would have let him work at the prestigious university. As things were, he became an assistant professor in Besançon. His innovative considerations, presented in the thesis, were not error-free, an unforgivable offense in the eyes of his fellow scientists. But the main problem was the unusual (at the turn of century) subject matter of the structure of capital markets, an area little frequented by contemporary mathematicians and economists. Unlike them, Bachelier considered it to be a rich data source. His thesis was thus doomed to be forgotten. Over half a century later, it was rediscovered by Savage. In 1980s Mandelbrot became a follower, noting that financial markets should not be treated as linear, if only for their high sensitivity to changes in initial conditions. Mandelbrot was one of the first to state that despite high reverence for the turn of century science, nobody knew how to classify Bachelier’s discoveries.

At the heart of Bachelier’s observations was the following one: “contradicting opinions on the market are so different that buyers believe the prices will grow, and sellers believe they will fall, all at the same time”. He set out to prove the expected value of price fluctuations is zero, thus the best prognosis for tomorrow’s price is today’s price, concluding the process of change in financial instruments prices may be described as the random walk process. He noted the amplitude of changes grows along with growth of the time horizon. Minute by minute changes are small, and with expanding time horizon (to days, weeks, months, years, etc.) the amplitude of changes will grow. He also noted the speed of change is proportional to the square root of time. Indeed, his prediction came true with amazing precision. The empirically verified fact that the speed of price change is proportional to the square root of time, confirmed his hypothesis that fluctuations in share prices can be compared to Brownian motion.

In the 1970s Bachelier’s predictions related to share price changes were not only confirmed, but became the basis for further research. As noted before, Benoit Mandelbrot became a great supporter of Bachelier’s research, who diligently studied his work, *Théorie de la spéculation* (Bachelier, 1964). He took note of some analogies between Bachelier’s “random fluctuations in share prices” and random fractals (one of the basic concepts of a new fields of mathematics, known as chaos theory). A chaotic system is typically a nonlinear system. There are two requirements for a chaotic system: (1) the existence fractal dimension, (2) the dependence on initial conditions (Peters, 1991).

In the 1980s more research appeared supporting the thesis of capital markets *not* behaving in accordance with the theory of random walk. For example, too many changes are observed in the stock market, to ascribe them to noise alone. Economists use a concept of equilibrium in which, assuming no external influences, the system remains “at rest”, mainly when supply is equal to demand. Sometimes an external factor causes disturbance in the system in a way that makes it impossible for it to return to its previous state in a linear way. Free market economy is an evolutionary structure. Balance assumes the absence of emotions, and the state of non-balance is basically one of the conditions for the development of the economy, the characteristic feature of which is that it does not “remember” the past, or, at best, allows some short-term memory. Moreover, such a system is sensitive to changes in initial conditions. Capital markets are very volatile at certain times. Therefore, in the 1980s, the fractal (or “chaotic”) market hypothesis (FMH) was developed, which takes into account the impact of information and the length of the time horizon on the individual investors’ behaviour. The fractal market hypothesis emphasizes the impact of liquidity and investment horizons on the behaviour of investors (Peters, 1991).

The chaos theory was created in the 1970s, for natural sciences. Chaos is a non-linear dynamic system, sensitive to changes in initial conditions. Fractals have certain measurable characteristics, and the property of self-similarity. The self-similarity means that smaller parts of the fractal figure look alike the whole figure itself, just like in the case of Sierpiński triangle (Figure 1). Fractals may have a fractional dimension. Fractal shapes may emerge in a variety of ways. The simplest one is recurrence of the generating rule. The Sierpiński triangle is an example, as well as the Cantor set, and the Koch curve (Peters, 1991). All of them are generated deterministically, and all have a fractal, or fractional, dimension. A primary characteristic of Euclidean geometry is that the dimensions are integers. Lines are one-dimensional; planes are two dimensional. A random time series has a fractal dimension of about 1.5 (Peters, 1991). There are also random fractals (Figure 1), generated using probabilistic rules. It can also happen that a random fractal generation algorithm leads to a figure that can also be generated using a deterministic approach (Example 1).

Example 1. Chaos game. An equilateral triangle ABC is created. Vertex A is assigned the pair of (1, 2), vertex B, of (3, 4), and C, of (5, 6). Starting from a random point of the triangle, you throw a six-sided dice. If 1 or 2 falls, the next point of the figure is placed half way between the original point and vertex A (1, 2). If later 3 or 4 falls, the next point of the figure is placed half way between the original point and vertex B (3, 4). After around 30,000 repetitions, a Sierpiński triangle is drawn (Figure 1).

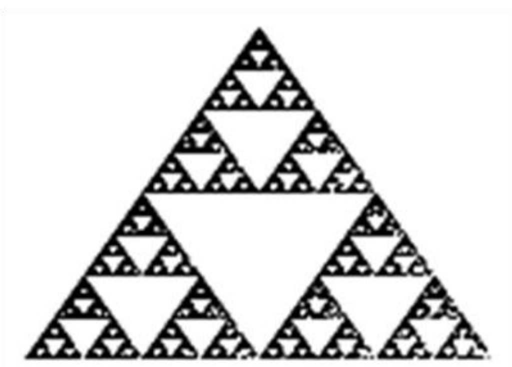


Figure 1. Sierpiński triangle drawn using the algorithm described in Example 1. Source: own research.

A mathematical construct like the Sierpiński triangle is more than a line but less than a plane. Every small triangle of it is geometrically identical to the larger triangle.

In non-linear economic systems, the equivalent of equilibrium is an attractor. An attractor is a subset of the state space, to which, after some time, points traverse (are attracted) from a certain neighbourhood area of this subset. The set of points that are traversing towards the attractor is called its basin of attraction. When more than one attractor exist in a dynamic system (and it is common in the case of stock market), respective basins of attraction are separated by a separatrix. Separatrices are sets of points that do not traverse towards any attractors. When all points in a given basin of attraction traverse to a single point, the attractor is called a point attractor.

Fluctuations of prices and rates of return of financial instruments are usually fractals. They are self-similar in respect to statistical characteristics, like averages, standard deviations, etc. Curves illustrating share prices have fractal dimensions. In 1989, Scheinkman and LeBaron discovered that a series of 5,000 rates of return has a fractal dimension between five and six. It is practically impossible to model such a complex system, and would basically require 10^6 observations (Scheinkman & LeBaron, 1989).

From an investor's viewpoint, who aims to foresee share prices, it is unsatisfactory to say the curves of share price fluctuations are fractals. Contemporary chaos theory research aims to determine the characteristics of the fractal, above all. The goal is to find a general formula for the curve, and self-similar characteristics. In addition, the algorithm for creating the fractal is given (either deterministic, or random), and the fractal dimension of is estimated, which, in turn, determines the number of variables needed to create a model of the dynamic system. The number is not less than the nearest integer number higher than the fractal dimension.

Classic economy tends to see the dynamic economy systems as balanced (point attractors) or oscillating around the state of equilibrium (boundary cycles). Empirically, it turns out economic time series is non-periodic cycles (they have no specific length or time scale). Such cycles are natural to non-linear dynamic systems. They form a family of attractors called chaotic or strange. They are usually depicted in phase space, i.e., all possible states of the system are shown. In the phase space, values of all the variables are shown at the same points in time. If, then, a system is described by three variables, its phase space will be three-dimensional. It is quite simple to construct a phase space described by an equation. When we do not know the variables and are limited to observations only, we can only reconstruct the phase space. An example of a system with known equation is the so-called Hénon attractor, which can be reconstructed by putting the time series describing a given characteristic in *m*-element series, where *m* is the dimension of the state space (Hénon, 1976).

Example 2. Hénon attractor. A set of equations is given:

$$x(t+1) = 1 + y(t) - \mathrm{a} \cdot x^2(t)$$

$$y(t+1) = b \cdot x(t) \tag{1}$$

Figure 2 shows two curves, whose initial conditions differ by 0.01. One can see initially both lines are close, but at some point they diverge abruptly. The Hénon attractor *i* san example of a chaotic system, characterized by high sensitivity of output to small changes in initial conditions. Figure 3 shows the phase space for the Hénon attractor.

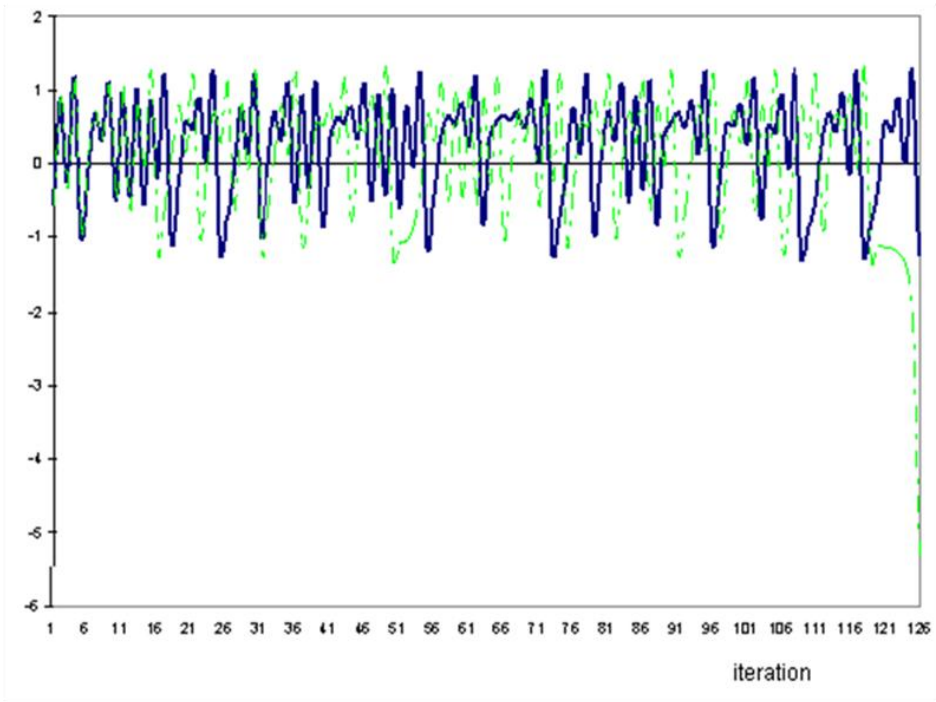


Figure 2. Hénon attractor: sensitivity to small changes in initial conditions. Source: own research.

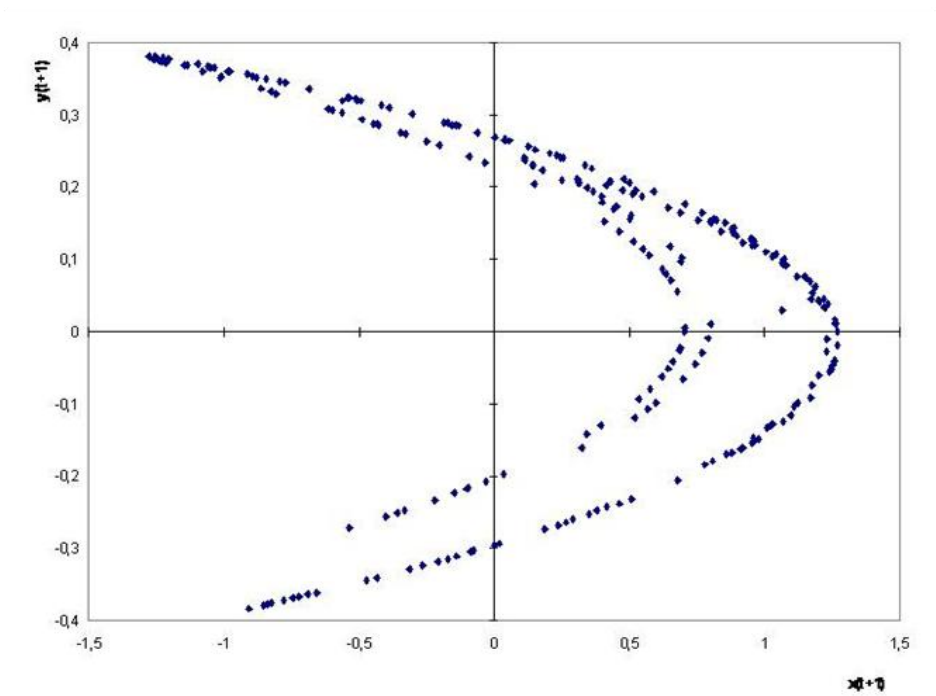


Figure 3. Hénon attractor: phase space; a = -1.4, b = 0.3. Source: own research.

The measures of system sensitivity to changes in initial conditions are numbers called Lyapunov exponents. They describe the speed of divergence of neighbouring orbits describing a given phenomenon in the phase space. The Lyapunov exponent is calculated as follows:

$$L_i = \lim_{t\to\infty} \frac{1}{t} \log_2 \frac{x_i(t)}{x_i(0)} (i = 1,2,...,N) \tag{2}$$

where:

L_i is the Lyapunov exponent for the i^{th} dimension,

$x_i(t)$ is the value of i^{th} variable at time *t*,

$x_i(0)$ is the value of i^{th} variable at time 0,

N is the number of dimensions of the phase space.

It follows from Equation (2) that for each dimension of the phase space there is only one Lyapunov exponent. It allows for attractor classification. For example, a three-dimensional point attractor has three negative Lyapunov exponents (-, -, -). It means all three dimensions converge to a point. A three-dimensional attractor being a boundary cycle has two negative exponents, and the third one is 0 (0, -, -), as boundary cycles converge in two dimensions, and there is no change in the relative position of the points in the third one, resulting in a closed orbit. Three-dimensional "strange" attractors, in turn, have a positive exponent, a negative one, and the third one is 0 (+, 0, -). The positive exponent describes sensibility to change of initial conditions, i.e., to what extent small changes of initial conditions affect the prognosis. The negative exponent makes the diverging points stay in the attractor's basin of attraction. The measure of equilibrium of "strange" attractors is the maximum distance between points that guarantees staying within the respective basin of attraction.

In 1980, Packard, with co-authors, presented David Ruelle's method of phase space reconstruction in a situation, when we do not have the formula describing the system, and we cannot make observations on more than one kind of variables. The method fills other dimensions with one variable's values shifted back in time. Let's say our initial series is A: $U(t+1), U(t), \ldots, U(N)$, where $N \le t$. We create the new series B: $V(t+1) = U(t), V(t) = U(t-1), \ldots, V(N) = U(N-1)$, series C: $Z(t+1) = U(t-1), \ldots, Z(N) = U(N-2)$, etc. The Lyapunov exponent calculated for series constructed this way makes the dynamic system non-linear (Packard, Crunchfield, Farmer, & Shaw, 1980). Applying this approach to a Hénon time series, the resulting diagram is similar to that in Figure 3 (see Figure 4). The amount of delay applied to the series of values does not have to be equal 1. It is calculated as percentage of each dimension's part of the phase space orbit, as follows:

$$q = m \cdot l \tag{3}$$

where:

q is the average orbital period,

l is the time shift,

m is the dimension of phase space.

Delay of the time series *l* is estimated by noting the moment when correlations between observations fade. For example, when we note variables cease to correlate after 10 iterations, and the phase space is two-dimensional, then the delay equals 5. Lack of correlation between variables is determined using a method proposed by Hurst. First you calculate:

$$y_t = \sum_{i=1}^{t} \left(Y_i - \overline{Y}_t\right) \tag{4}$$

where:

Y_i is the logarithmic share price return rate at the moment *i*, i.e.: $Y_i = \ln \frac{X_{i+1}}{X_i}$,

X_i is the logarithmic share price return rate at the moment *i*,

$\overline{Y_t}$ is the average logarithmic share price return rate until the moment *t*.

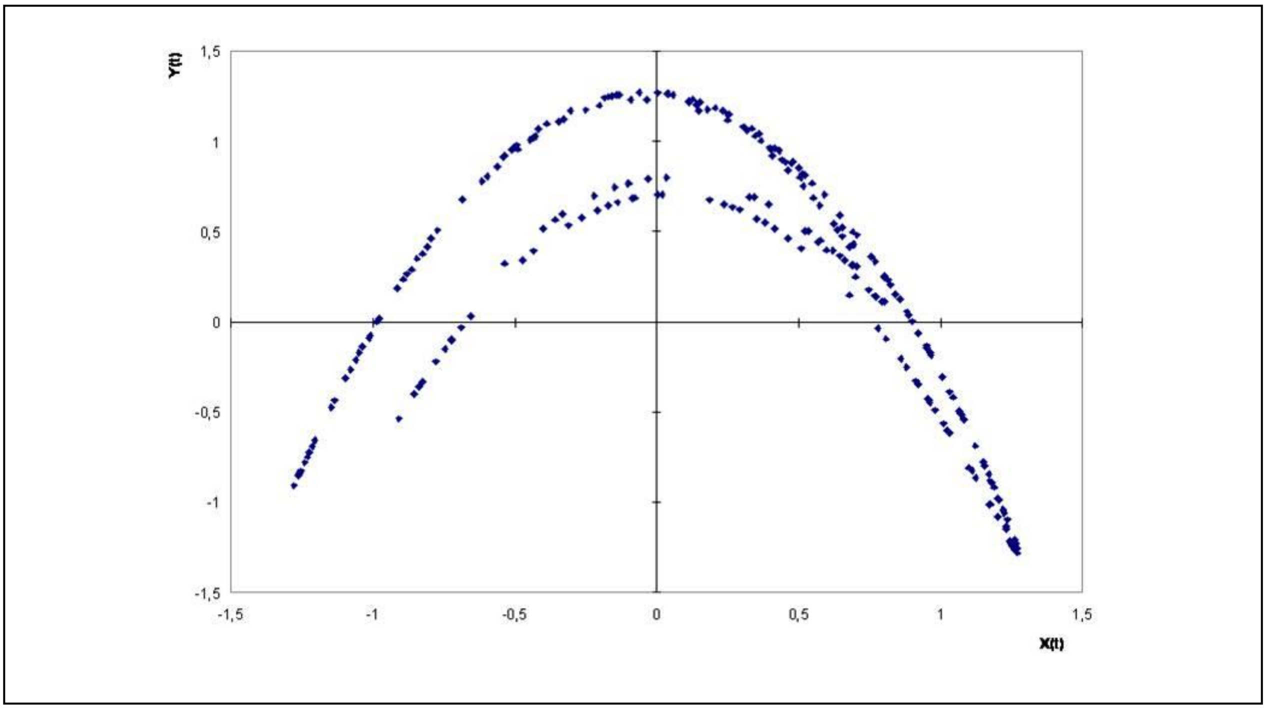


Figure 4. Hénon attractor: phase portrait reconstructed based on *Y* values delayed by 1 iteration. Source: own research.

Next you determine the so-called range of variable y_t:

$$T = \max(y_t) - \min(y_t) \tag{5}$$

One of the better-known characteristics of time series is the so-called Hurst exponent. In order to compare types of time series, Hurst divided the range by standard deviation (*d*) of the original observations. He noticed that thus scaled range should grow in time. He formed the following relationship:

$$\frac{T}{d} = (\mathrm{a} \cdot t)^H \tag{6}$$

where:

$\frac{T}{d}$ is the scaled range,

t is the number of observations,

a is constant,

H is the Hurst exponent.

After logarithmizing both sides of Equation (6), Hurst exponent is calculated:

$$H = \frac{\ln\left(\frac{T}{d}\right)}{\ln(\mathrm{a}t)} \tag{7}$$

It is stated that when $H = 0.5$, the series is not correlated. With $0 \le H < 0.5$, the series is anti-persistent. When $0.5 < H < 1$, the series is trending, or persistent. With $H \to 0.5$, correlations between observations diminish. Hurst exponent also allows for estimating the fractal dimension (*D*) using the following:

$$D = 2 - H \tag{8}$$

We find an interesting illustration of the issues described above in the Kaldor-Kalecki business cycles.

Example 3. Kaldor-Kalecki business cycles. Entrepreneurial activity always trends to balance out savings and investments. By investments we understand the value of planned growth of all kinds of resources, and by savings we understand the amounts of money entrepreneurs plan to save. When investments outgrow savings, the level of economic activity grows. General economic activity is determined as the sum of entrepreneurial and consumer expenditure. Let Y, K, S, I denote, respectively, real income (profit), capital, savings and investments. Investment is a function of profit and capital $I(Y, K)$ and the following conditions are true: $I_Y > 0$, $I_K > 0$, where subscripts denote partial derivatives of respective arguments. The first inequation means the higher production level, the higher capital goods demand. The savings level depends on the profit, and $0 < \mathrm{S_y} < 1$, i.e., propensity to save, which is greater than 0, should be less than 1. It is also true $S_K > 0$. Profit is proportional to surplus of demand on the goods market. The Kaldor-Kalecki model can be described as a system of differential equations:

$$\begin{aligned} \dot{Y} &= \alpha\left[I(Y,K) - S(Y,K)\right] \\ \dot{K} &= I(Y,K) - \delta K \end{aligned} \tag{9}$$

where δ is a constant describing depreciation rate of the capital stock (or, propensity to invest), and $\alpha = const$.

Equation (9) can be chaotic, and then its attractor is “strange”. To illustrate this, let’s assume the investment function is: $I_t(Y_t, K_t) = c \cdot 2^{-1/(dY_t + \varepsilon)} + eY_t + \alpha(f / K_t)^g$, and savings is a linear function of profit: $S_t = sY_t$. In addition, the following parameter values are assumed: $\alpha = 0.20$, $c = 20$, $d = 0.01$, $\varepsilon = 0.00001, e = 0.05$, $f = 280, q = 4.5, s = 0.21$, and the initial conditions are: (a) $\mathrm{Y}_0 = 65$, (b) $Y_0 = 65.1$. In the Kaldor-Kalecki business cycles model, profit behaviour is illustrated in Figure 5. Figures 6, 7, and 8 illustrate the behaviour of Equation (9) in the following phase spaces: (Y_t, K_t), (Y_{t+1}, Y_t), (K_{t+1}, K_t). We can see the system has a strange attractor in each of those spaces.

The Kaldor-Kalecki model is a typical non-linear dynamic model that can, in specific situations, behave chaotically.

Chaos theory tools allow verifying whether the functions of stock prices are stochastic processes, or rather deterministic fractals that may, but do not have to, behave chaotically.

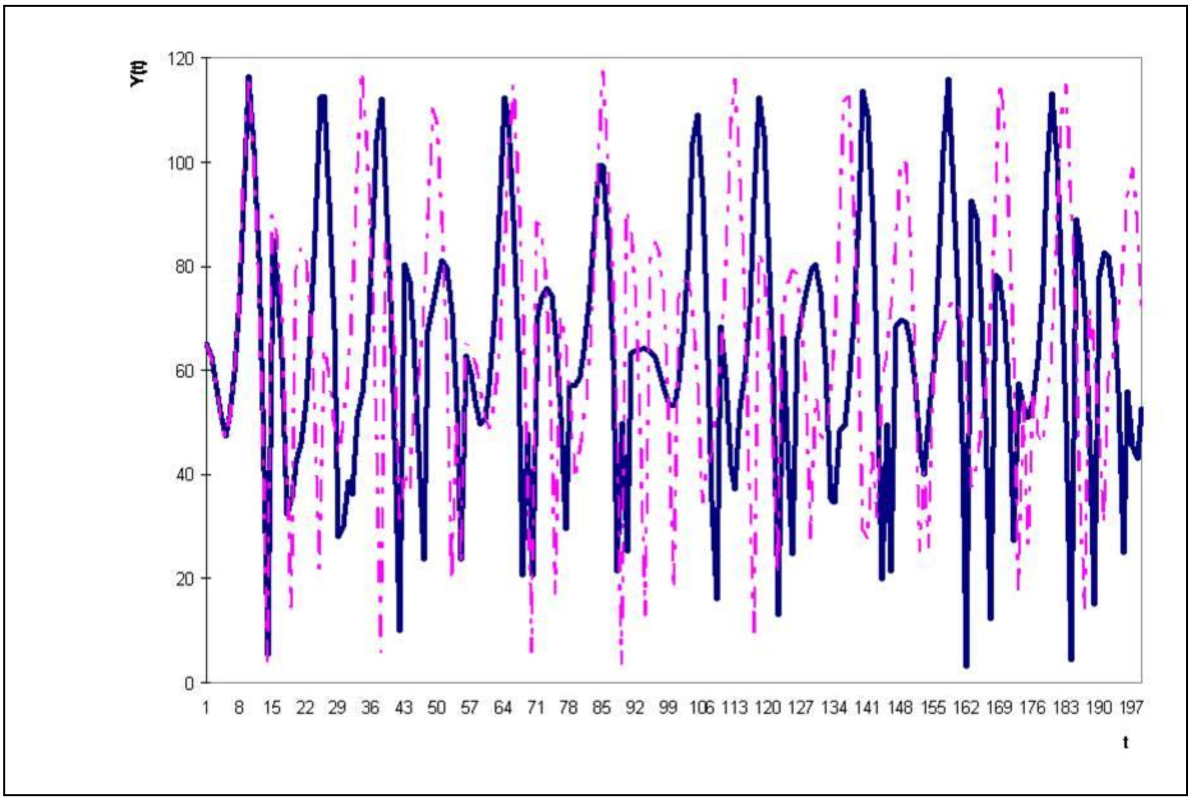


Figure 5. Kaldor-Kalecki business cycles model; profit in time with initial conditions: (a) $Y_0 = 65$ (continuous line), (b) $Y_0 = 65.1$ (dotted line). Source: own research.

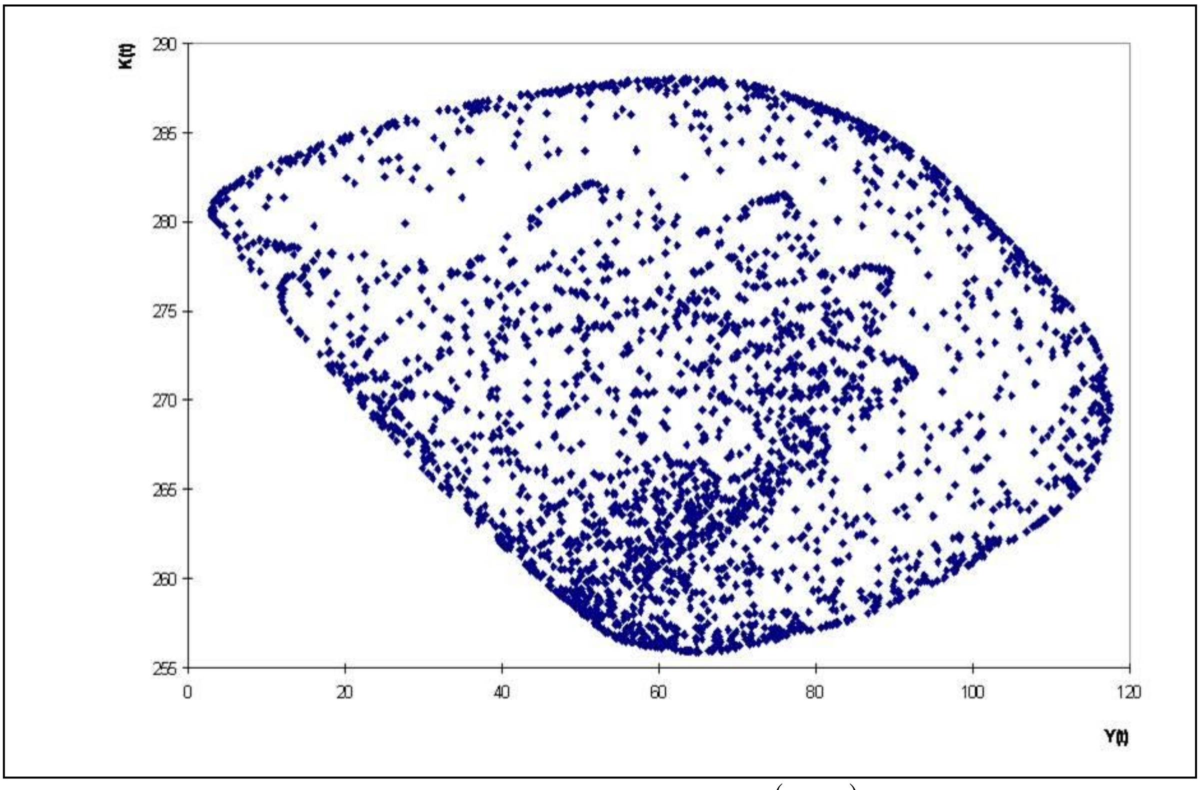


Figure 6. Kaldor-Kalecki business cycles model in the phase space (Y_t, K_t). Source: own research.

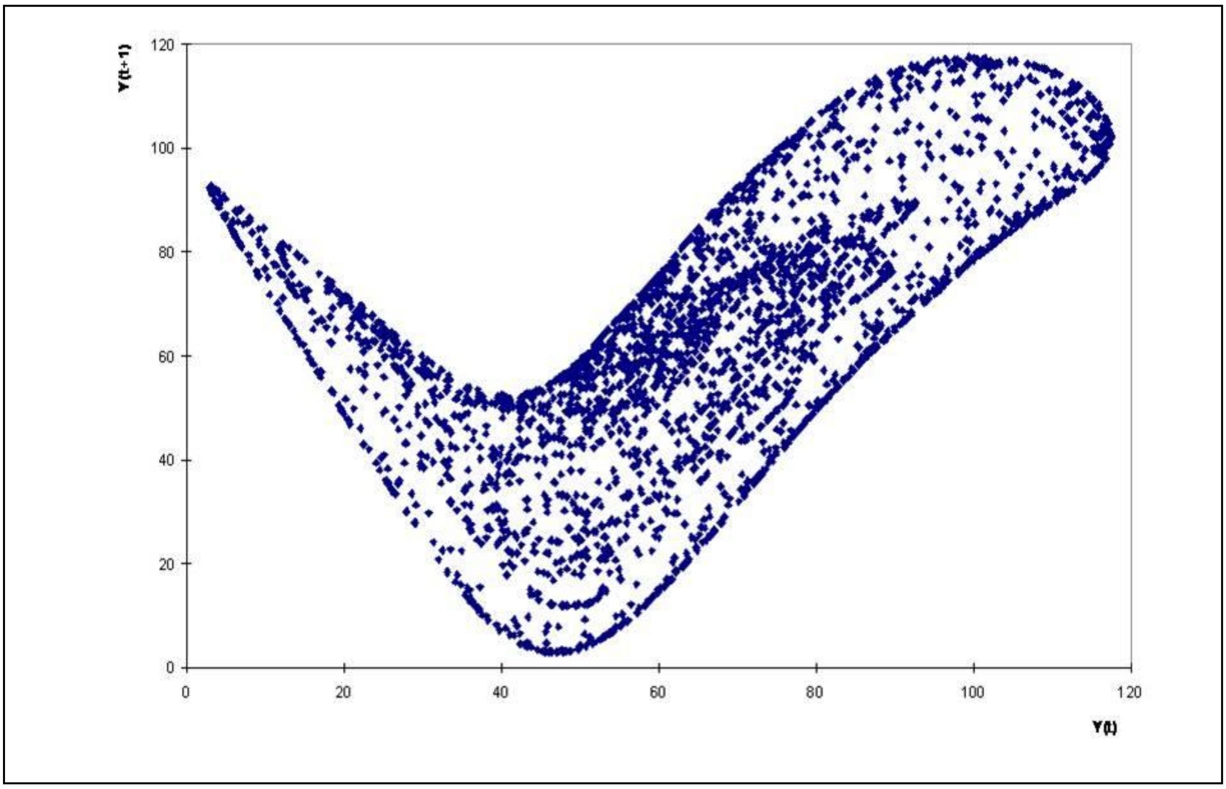


Figure 7. Kaldor-Kalecki business cycles model in the phase space $\left(Y_{t+1}, Y_t\right)$. Source: own research.

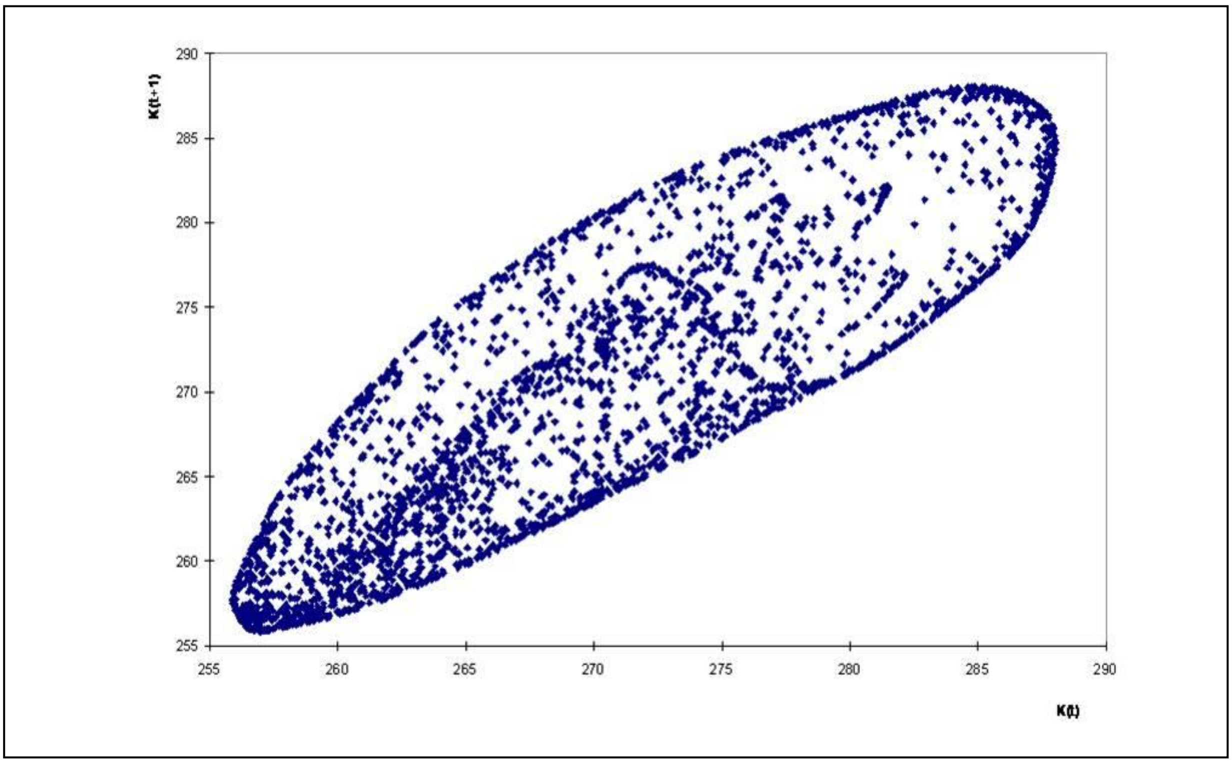
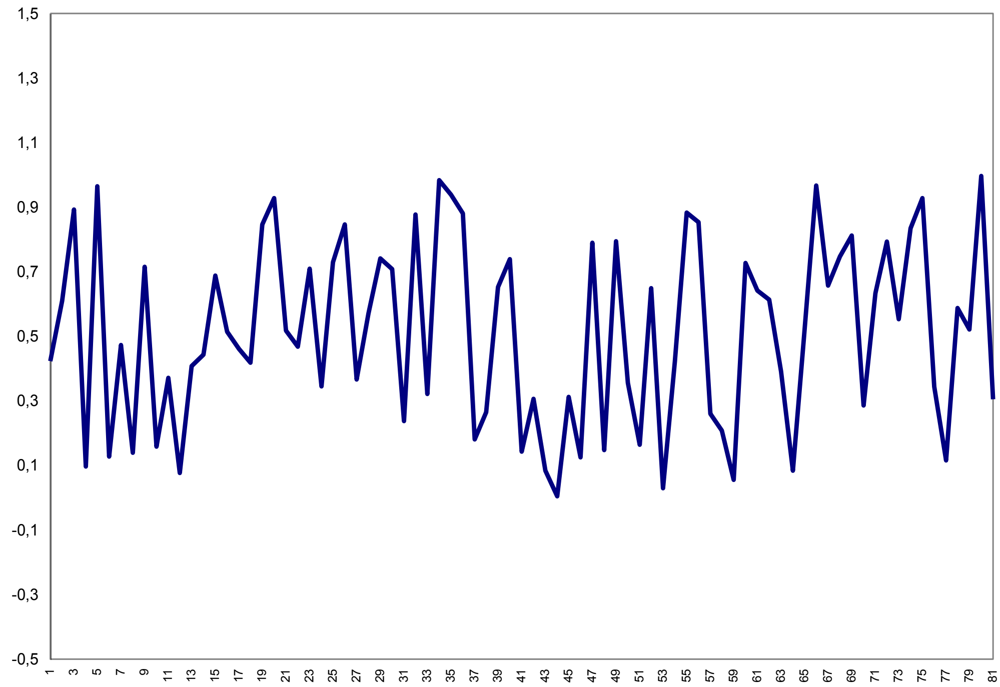
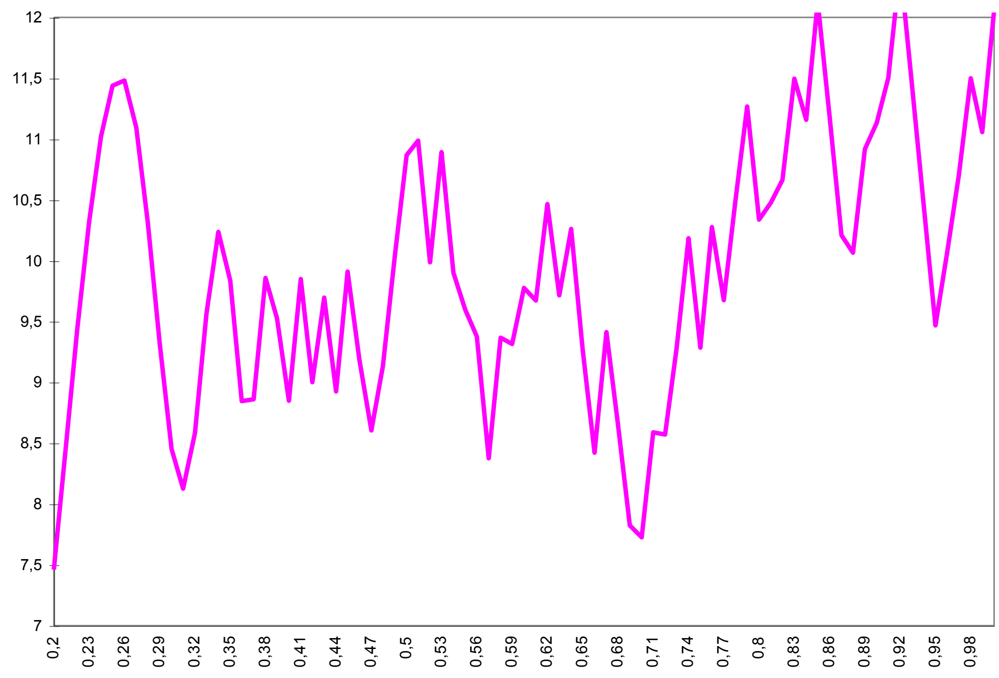


Figure 8. Kaldor-Kalecki business cycles model in the phase space $\left(K_{t+1,}, K_t\right)$. Source: own research.



(a)



(b)

Figure 9. (a) stochastic process, generated by a random number generator; (b) Weierstrass function, where $\varepsilon = 0.1$, $\lambda = 1.1$. Source: own research.

Example 4. Differences between stochastic and deterministic series. Figure 9 shows (a) a 100-element random time series, and (b) a deterministic series illustrating the following function:

$$f(t) = \sum_{i=1}^{\infty} \frac{\sin(\lambda^{i} t)}{\lambda^{\varepsilon_i}} \tag{10}$$

where $\lambda > 1$ and $0 < \varepsilon < 1$. It is a known example of a function that is continuous everywhere but differentiable nowhere, which behaves chaotically for specific values of λ. Figure 10 shows the random series and the Weierstrass function in the phase space.

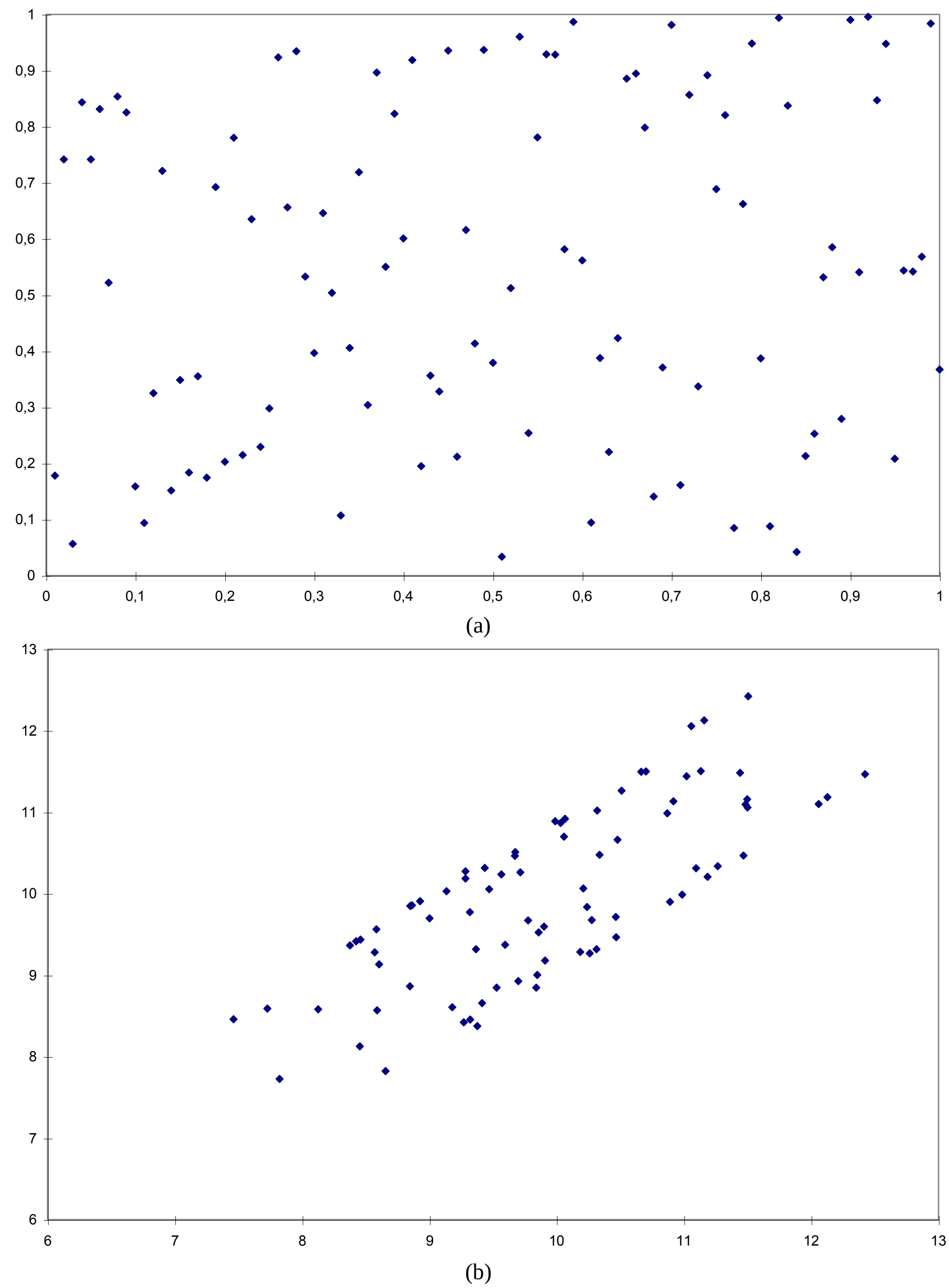


Figure 10. (a) phase space for a 100-element random series; (b) Weierstrass function phase space. Source: own research.

We can see for the random time series (Figure 10a) generated by a random number generator, presented in the phase space, the whole plane is covered with dots. If the number of elements in the series were higher, it would have been impossible to cover the area containing the dots with a finite number of spheres. Regardless of the number of dots, though, the correlational dimension equals 2.

Example 5. Determinism and gold prices. For a time series constructed from gold prices (between 2000 and May 2019), the Hurst exponent calculated by way of Equation (7) is $H = 0.823$ for $a = 1$, $H = 0.58$ for $a = 10$, and H = 0.686 for $a = 3$. In each case it is higher than 0.5, i.e., the gold prices time series is persistent. Also, just like in the case of the Weierstrass function (Example 4), in case of gold prices, the "object" generated in the phase space resembles a strange attractor, whose correlational dimension is between 1 and 2 (to be more precise, D = 1.17 for $a = 1$, $D = 1.32$ for $a = 3$, and $D = 1.42$ for $a = 10$). The object is notably more regular for gold prices (cf. Figures 11 and 12). Both attractors can be covered by a finite number of spheres. This denotes an equilibrium, and that there are deterministic time series, regardless of the market situation. Within the timeframe of the series, there were two financial market crises (2002 and 2008), and they did not cause significant gold prices fluctuations. Gold prices increased, but no major fluctuations were noticed. The conclusion can be drawn that gold prices constitute a good mechanism of financial markets balance control.

Figure 11. Gold price in USD per ounce from 2000 to 2019. Source: www.kitco.com.

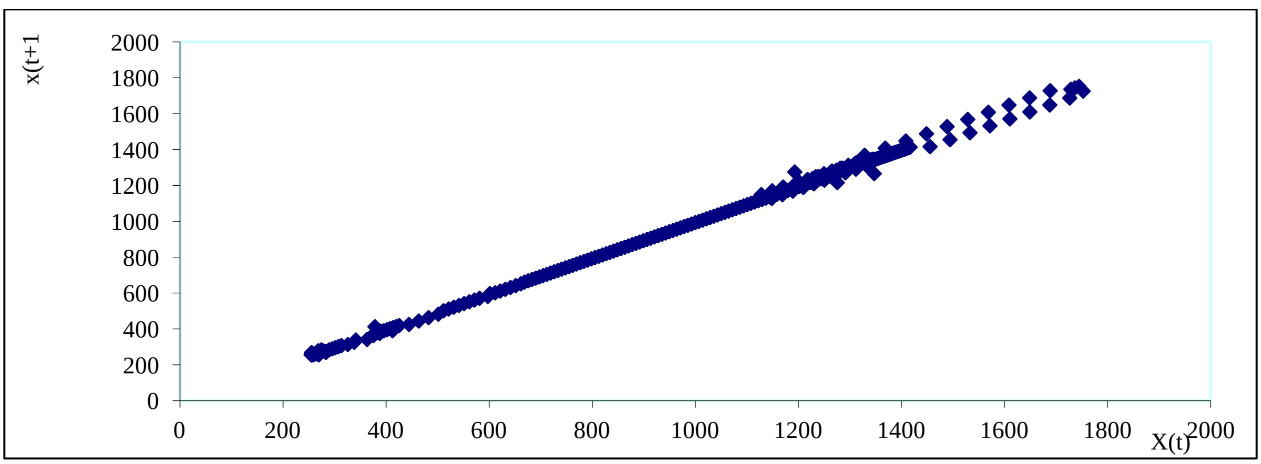


Figure 12. Strange attractor for gold prices presented in the phase space for data from 2000-2019. Source: own research.

Conclusions

Chaotic models have been applied to economic problems, among others, to describing states of equilibrium. The efficient market hypothesis, ascribing the character of Random Walk to share prices, and assuming investors behave rationally and are able to subjectively determine the probabilities of future prices, valid until these days, gave way to new hypotheses, fractal market hypothesis among them, that are more accurate at describing the market reality. Presenting the gold prices from 2000-2019 in the phase space shows they are a mechanism of financial markets control and balance. They often prevent economic catastrophes. Catastrophes are the subject matter of R. Thom's catastrophe theory. A catastrophe is characterised by the fact that a very small change in initial conditions may result in an irreversible change in the results.

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