

The Theoretical Foundations for Solving Typical Geometrical Problems in Engineering

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Abstract: In the present work the analysis of the basic methods for solving typical geometrical engineering problems is made. The basic problem is revealed and the first-level tasks of its solution are defined. A general algorithm for solving typical geometrical problems in the form of standardized logical blocks is offered. The general algorithm for the solution of typical geometrical problems is discussed, taking into account the iterative nature of the proposed methods. The introduced formal general algorithm allows students to master their skills of independent work. Practical engineers can use the developed general approach for solving new difficult real-world problems. Synthesized general algorithm, as the main contribution to geometry, allows, on deductive basis—"from the general to quotient", to teach and study engineering geometry.

Key words: Foundations, method, algorithm, geometry, problem, solution, product.

1. Introduction

1.1 Problem Statement

Spatial imagination and imaginative perception are essential professional skills being developed in the academic course "Engineering Graphics". The basic sections of engineering graphics are positional problems, metric problems, problems of construction of the development of a curvilinear surface, and problems of constructing of axonometric views of a product [1-17]. These four sorts of problems of the course are called as typical geometrical engineering problems.

A number of various methods are used for the solution of typical geometrical engineering problems [18-28]. As a result, comprehending these methods within the limited course time available is virtually impossible.

Hence, there is a contradiction (conflict) between the great variety of known methods for solving typical geometrical engineering problems and the limited course duration. Such a conflict can be partially

resolved through the development of a unifying general approach to the solution of typical geometrical engineering problems.

Thus, the problem can be solved by adding offered changes into the existing methodology of training and engineering practice of the solution of real problems.

The proposed changes will help to solve the following methodological problems:

(1) To offer methods and means for increasing the efficiency of comprehension of the theoretical foundations of engineering graphics and the solution of practical problems.

(2) To train students in the skills of independent work.

(3) To offer graduate students some methods of solving engineering problems related to their specialization in a particular engineering field.

In the conditions of competition, constant increase of requirements to the quality of a product leads to the increase of complexity of real engineering problems under consideration. In such situation known methods of the solution of typical geometrical engineering problems are not always applicable and yield necessary competitive result.

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Hence, there is a contradiction (conflict) between constant increase of complexity of real engineering problems under consideration and absence of adequate methods of their solution.

The solution of this contradiction (problem) can also be partially achieved by working out of the general approach to the solution of typical geometrical engineering problems.

The developed general approach to the solution of typical geometrical problems in certain extent will allow an engineer to generate algorithm of the solution for novel complex real problem.

1.2 Analysis of Researches and Publications

Various methods for solving typical geometrical engineering problems are described in the works of V. E. Mihajlenko, V. V. Vanin, V. Ya Volkov, S. N. Kovalev, V. M. Najdysh, A. N. Podkorytov, I. A. Skidan, N. N. Ryzhov, H. Stachel, G. Weiss, K. Suzuki and other scientists [29-45].

Frolov [32] and Bubennikov [29] described an algorithm for the solution of positional problems in intersections of geometrical images. This algorithm includes three steps. They also developed another algorithm for the solution of metric problems, consisting of five steps [29, 32]. In their works, three methods of construction of the development of curvilinear and many-faced surfaces were proposed: the normal section method, the development of surface on projection plane method, and the method of triangles [29, 32]. However, algorithm of construction of development for generalizing these methods is not offered. The description of various methods of construction of standard axonometric projections of a product is not presented in the guidelines of the general algorithm [29-45].

The knowledge of algorithms facilitates better understanding and comprehension of the essence of methods of the solution of engineering problems correlated with the definition of positional and metric characteristics, constructions of development and

axonometric views of geometrical images.

The problem of synthesis of the general algorithm for the solution of all groups of typical geometrical engineering problems was not the object of the current researches [29-45].

1.3 Aim and Purposes of the Paper

The purpose of the present paper is to develop a general formal algorithm for solving typical engineering graphics geometrical problems.

Such an algorithm should help students to understand in depth the geometrical essence of the studied phenomena, to comprehend the generality of engineering methods and to solve practical problems related to the course of study and those in real life more efficiently. Moreover, the generalized approach to the solution of typical geometrical problems should also help practical engineers to solve new difficult real-life problems.

The paper also aims:

(1) To develop a generalized algorithm for solving typical engineering graphics geometrical problems in the form of standard logic blocks, facilitating students' understanding of the essence of the used method for solving problems of descriptive geometry and allowing engineers to solve new difficult real-world problems.

(2) To describe the developed general algorithm for the solution of typical geometrical problems, considering the iterative nature of used methods.

2. Foundations of the General Approach

2.1 Approach Structure

The developed general algorithm for the solution of typical geometrical problems consists of seven stages, as shown in Fig. 1.

The letters P, M, D, A identifying the types of geometrical problems precede the stage number. Their meaning is as follows: P is for positional problems; M is for metric problems; D is for problems of construction of development; A is for problems of

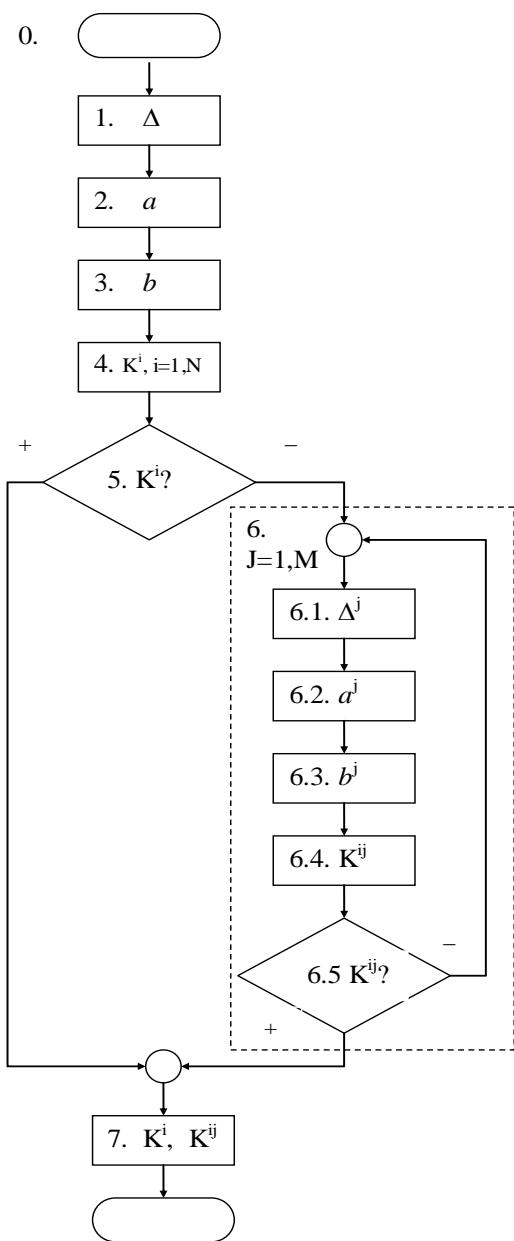


Fig. 1 A flow-chart of the algorithm for solving typical geometrical problems.

construction of an axonometric projection of a product. The letter G placed before the stage number indicates the description is considered a stage of the general algorithm.

The results of the analysis of the basic methods for solving typical geometrical engineering problems in the introduced code are shown in Table 1. The content of the introduced symbolic description is discussed in the following sections.

2.1 Approach Essence

P1. According to the criterion of simplicity of the intersecting lines of the auxiliary geometrical image (intermediary) with the original geometrical images the kind of this auxiliary geometrical image (intermediary) is imaginarily figured out.

On the basis of imaginarily-defined three-dimensional representation of the auxiliary geometrical image, construction of its two-dimensional image is carried out. The result of the first stage is the complex drawing of the intermediary— $\Delta(\Delta_1, \Delta_2)$.

M1. The number of the chosen systems of planes of projections for allocation and a designation of the end result of the solution of a metric problem is defined as $j = M$. Number M belongs to a set of natural numbers.

At the beginning, to the counter of the chosen systems of planes of projections $\Delta^j, j = 1, M$ the first value unit is given— $j = 1$.

The projection of a geometrical image and the axis of coordinates for removal from initial complex drawing $K^i(K_l^i, K_m^i), i = 1, N, l = 1, 4, 5, m = 2, 4, 5$ for a new plane of projections $\Delta^j (\Delta_l^j, l = 4, 5, \Delta_m^j, m = 4, 5, j = 1, M)$ are defined.

The coordinate axis $X_{lm}, l \neq m$ is then constructed with respect to the preserved projection of a geometrical image so that the geometrical image occupied its particular position in relation to the introduced plane of projections— $\Delta^j (\Delta_l^j, l = 4, 5, 6; \Delta_m^j, m = 4, 5, 6; l \neq m)$.

D1. On a curvilinear developed surface between two nearest generating lines a certain region Δ is allocated.

A1. The orthogonal projection Δ (the front view, the top view, the left-side view etc.) containing the maximum information on design features of a product (an aperture, a lug, planes, facets, flutes etc.) is highlighted in the complex projective drawing (or in a prototype photographs).

Three orthogonal projection planes ($j = 0, 1, 2$) are normally used in the axonometry construction.

Table 1 Results of the analysis of the basic methods for solving typical geometrical engineering problems in a symbolic description.

Serial number of a stage of algorithm	Type of an geometrical engineering problem				
	Positional problems	Metric problems	Problems of construction of development	Axonometric problems	General approach new problems
1.	P1	M1	D1	A1	G1
2.	P2	M2	D2	A2	G2
3.	P3	M3	D3	A3	G3
4.	P4	M4	D4	A4	G4
5.	P5	M5	D5	A5	G5
6.	P6	M6	D6	A6	G6
7.	P7	M6	D7	A7	G7

As such, the counter j considered orthogonal planes of projections $\Delta^j, j = 0, M$ gets its first value zero $j = 0$. Number M belongs to a set of natural numbers.

G1. Thus, the auxiliary image Δ is defined at the first stage of the solution of a typical geometrical problem.

P2. The first auxiliary line a of intersecting of the intermediary Δ with the first initial geometrical image Σ is constructed— $a = \Delta \cap \Sigma$. The result of the second stage is the complex drawing of the first auxiliary line $a—a(a_1, a_2)$.

M2. Lines of projective connections, which remain from the saved projection of the geometrical image, are constructed to be perpendicular to the created axis of coordinates— $a \perp X_{lm}, l \neq m (a^n, n = 1, 2, 3, 4)$.

D2. The curvilinear surface located between the selected generating lines is replaced with a flat figure— a .

A2. A geometrical analysis of the orthogonal projection Δ keeping the maximum quantity of images of the constructive elements is carried out. The aim of this analysis is the definition of the characteristic geometrical images (circles, arches, segments of straight lines, points).

In order to construct an axonometry of the characteristic geometrical images their characteristic points are allocated— $K^i(K_1^i, K_2^i, K_3^i), i = 1, N$. Number N belongs to the set of natural numbers. For example, the axonometric projection of a segment of a straight line can be constructed using two

characteristic points $K^i(K_1^i, K_2^i, K_3^i), i = 1, 2$.

The primary a coordinates $x^i, y^i, z^i, i = 1, N$ are defined for each characteristic point $K^i(x^i, y^i, z^i), i = 1, N$ belonging to the analyzed orthogonal projection Δ .

G2. Thus, at the second stage of the solution of a typical geometrical problem for the auxiliary image Δ , the first group of auxiliary actions a is carried out.

P3. The second auxiliary line b of intersection of the same intermediary Δ with the second initial geometrical image Φ is constructed— $b = \Delta \cap \Phi$. The result of the third stage for a positional problem is the complex drawing of the second auxiliary line $b—b(b_1, b_2)$.

M3. For a metric problem, marks b are placed on the constructed lines of projective connections from the created axis of coordinates to a created projection of a geometrical image to visualize the distances accordingly equal to distances from the deleted projection to the deleted axis of coordinates— $b (b^n, n = 1, 2, 3, 4)$.

D3. For a problem of construction of development, the true size of the constructed flat figure is defined— b .

A3. At the third stage of construction of an axonometry of a product by means of the natural coefficients of distortion k_x, k_y, k_z for coordinate axes Ox, Oy, Oz , secondary axonometric coordinates x_K, y_K, z_K of the axonometric projection K into the primary orthogonal coordinates x_K, y_K, z_K points $K^i(x^i, y^i, z^i), i = 1, N$ are defined:

$$\begin{aligned}x_K &= k_x \cdot x_K, \\y_K &= k_y \cdot y_K, \\z_K &= k_z \cdot z_K.\end{aligned}$$

The results b of the third stage for the problem of construction of an axonometry are the secondary axonometric coordinates x_K, y_K, z_K defined for each characteristic point— $(K(x_K, y_K, z_K))^i, i = 1, N$.

G3. Thus, at the third stage of the solution of a typical geometrical problem for an auxiliary image Δ , the second group of auxiliary actions b is carried out.

P4. Points of intersection $K^i, i = 1, N$ of the first auxiliary line a and the second auxiliary line b are defined— $K^i = a \cap b, i = 1, N$. Numbers N belong to the set of natural numbers. The results of the fourth stage are a complex drawing of points of intersecting $K^i = a \cap b, i = 1, N$ the first auxiliary line a and the second auxiliary line b — $K^i(K_1^i, K_2^i), i = 1, N$.

M4. The projection of the geometrical image is constructed by connecting characteristic points $K^i, i = 1, N$ and the constructed projection of a geometrical image is designated (indicated)— $K^i(K_l^i, K_m^i), i = 1, N, l = 1, 4, 5, m = 2, 4, 5, 1 \neq m$.

The discussed algorithm can be used many times for the same engineering metric problem if the following two conditions are met.

(1) Conditions of repeated application of a projections planes change method.

(2) One of the two planes of projections of the replaced system should be a component of the introduced system of planes of projections.

The entered plane of projections should be perpendicular to the saved plane of projection.

D4. The flat figure of the true size is constructed on the plane (drawing)— $K^i, i = 1, N$. Number N is the number of identical flat figures of the true size corresponding to the allocated part Δ of a curvilinear developed surface.

A4. Axonometric projections of the characteristic points are constructed— $(K(x_K, y_K, z_K))^i, i = 1, N$.

Connecting axonometric projections $(K(x_K, y_K, z_K))^i, i = 1, N$ of the characteristic points $K^i(x^i, y^i, z^i)$,

$i = 1, N$ by a thin line, the constructed projections of the geometrical images belonging to an orthogonal projection Δ of a product are marked.

The result of the fourth stage includes the axonometric projections $(K(x_K, y_K, z_K))^i, i = 1, N$ of the characteristic points $K^i(x^i, y^i, z^i), i = 1, N$ and the axonometric projections of the geometrical images belonging to the orthogonal projection Δ of a product.

G4. Thus, at the fourth stage of the solution of a typical geometrical problem for an auxiliary image Δ , the necessary projection of the geometrical image $K^i, i = 1, N$ is constructed.

P5. Sufficiency of the obtained number of points of intersecting $K^i = a \cap b, i = 1, N$ of the first auxiliary line a and the second auxiliary line b for allocation and designation of the end result of solution of a positional problem is verified.

If the number of points of intersecting $K^i = a \cap b, i = 1, N$ of the first auxiliary line a and the second auxiliary line b for obtaining the end result is sufficient, then the last seventh stage of the given algorithm is carried out.

If the number of points of intersecting $K^i = a \cap b, i = 1, N$ of the first auxiliary line a and the second auxiliary line b for obtaining the end result is insufficient, then the following sixth stage of the developed algorithm is carried out.

M5. Sufficiency of the obtained number of the new introduced systems of planes of projections for the allocation and designation of the end result of the solution of the problem (Fig. 1) is verified.

In doing so, the new constructed projection using the characteristic points $K^i, i = 1, N$ is checked out for the purpose of possible definition of the required metric characteristic of the initial geometrical image.

If the number of the new entered systems of planes of projections is sufficient, then the last seventh stage of the developed algorithm is carried out. Otherwise, the sixth stage of the developed algorithm is carried out.

D5. The verification of the completion of

construction of the development of the surface is carried out.

If all identical flat figures of the true sizes corresponding to the allocated fragment Δ of the curvilinear developed surface coincide with a plane and other fragments ($\Delta^j, j = 1, M$) are not allocated, the seventh stage is carried out. Otherwise, the sixth stage of the developed algorithm is carried out.

A5. Sufficiency of the obtained number of axonometric projections $(K'(x_K, y_K, z_K))^i, i = 1, N$ of characteristic points $K^i(x^i, y^i, z^i), i = 1, N$ for the allocation and designation of the end result of the solution of an axonometric problem is verified.

If the number of axonometric projections $(K'(x_K, y_K, z_K))^i, i = 1, N$ of characteristic points $K^i(x^i, y^i, z^i), i = 1, N$ is sufficient, then the last seventh stage of the developed algorithm is carried out. For example, for a flat plate its axonometric projection can be allocated by a thick line. Otherwise, the sixth stage of the developed algorithm is carried out.

G5. Thus, at the fifth stage of the solution of a typical geometrical problem for an auxiliary image Δ sufficiency of the obtained result is verified.

P6. Additional points $K^j, i = 1, N, j = 1, M$ for unequivocal allocation and a designation of the end result of the solution of a positional problem are defined.

The counter j of the additional intermediaries $\Delta^j, j = 1, M$ gave the first unit value $j = 1$.

P6.1. The complex drawing of a new intermediary $\Delta^j, j = 1, M$ is constructed— $\Delta^j(\Delta^j_1, \Delta^j_2), j = 1, M$.

For this new intermediary $\Delta^j, j = 1, M$ steps P6.2, P6.3, P6.4 of the developed algorithm are carried out in full analogy with steps P2, P3, P4.

P6.2. The complex drawing of a new auxiliary line $a^j, j = 1, M$ for the new intermediary $\Delta^j, j = 1, M$ is constructed— $a^j(a^j_1, a^j_2)$.

P6.3. The complex drawing of a new auxiliary line $b^j, j = 1, M$ for the new intermediary $\Delta^j, j = 1, M$ is constructed— $b^j(b^j_1, b^j_2)$.

P6.4. Complex drawings of new points of

intersection $K^{ij} = a^j \cap b^j, i = 1, N, j = 1, M$ for auxiliary lines $a^j, b^j, j = 1, M$ are constructed— $K^{ij}(K^{ij}_1, K^{ij}_2), i = 1, N, j = 1, M$.

P6.5. A decision on the unambiguity of the construction of the end result of the solution of a problem is made.

If the number of points $K^{ij}, i = 1, N, j = 1, M$, belonging to simultaneously two initial geometrical images and various intermediaries $\Delta^j, j = 1, M$ is sufficient for the unequivocal allocation and designation of the result of the solution of a positional problem, the seventh stage of the developed algorithm is carried out.

Otherwise, the value of the counter j of additional intermediaries $\Delta^j, j = 1, M$ is increased by unit ($j = j + 1$). A complex drawing of a new intermediary $\Delta^j, j = 1, M$ is constructed and new points of intersection $K^{ij} = a^j \cap b^j, i = 1, N, j = 1, M$ of new auxiliary lines $a^j, b^j, j = 1, M$ are defined. In other words, the sixth stage of the developed algorithm is repeated.

M6. Additional points $K^j, i = 1, N, j = 1, M$ of the new geometrical image in the new system of planes of projections for unequivocal allocation and designation of the end result of the solution of a metric problem are defined.

The counter j of new entered planes of projections $\Delta^j, j = 1, M$ is assigned with a new value as $j = j + 1$.

M6.1. The complex drawing of a new system of planes of projections for a new entered plane of projections $\Delta^j, j = 1, M$ is constructed— $\Delta^j(\Delta^j_1, \Delta^j_m), j = 1, M$.

The projection of a geometrical image and the axis of coordinates deleted from the initial complex drawing for the introduced plane of projection are defined— $\Delta^j(\Delta^j_1, \Delta^j_m), j = 1, M$.

The coordinate axis $X_{lm}^j, 1 \neq m, j = 1, M$ is constructed relative to the kept projection of the geometrical image so that the geometrical image occupies a particular position in relation to the introduced plane of projections— $\Delta^j(\Delta^j_1, \Delta^j_m), j = 1, M$.

For the introduced plane of projections $\Delta^j, j = 1, M$

steps M6.2, M6.3, M6.4 of the given algorithm are carried out in full analogy with steps M2, M3, M4 points of the developed algorithm.

M6.2. New lines $a^j, j = 1, M$ of projective connections from the saved projection of the geometrical image are constructed to be perpendicular to the created axis of coordinates for the newly introduced plane of projections $\Delta^j, j = 1, M - a^j (a^{jn}_1, a^{jn}_m, n = 1, 2,); a^j \perp X_{lm}^j, l \neq m, j = 1, M$.

M6.3. The distances accordingly equal to distances from the deleted projection to the deleted axis of coordinates $b^j, j = 1, M$ for the new entered plane of projections $\Delta^j, j = 1, M$ are laid down by notches on the constructed lines of projective connections from the introduced axis of coordinates to the constructed projection of the geometrical image— $b^j (b^{jn}_1, b^{jn}_m, n = 1, 2, \dots)$.

M6.4. A newly constructed projection of the geometrical image of the new entered plane of projections $\Delta^j, j = 1, M$ is allocated through connecting the characteristic points $K^{ij}, i = 1, N, j = 1, M$ and designated— $K^{ij}(K^{ij}_1, K^{ij}_m), i = 1, N, j = 1, M$.

M6.5. A decision about the unambiguity of the construction of an end result of the solution of the problem is made.

If the number of points $K^{ij}, i = 1, N, j = 1, M$ of newly constructed projection of the geometrical image for the newly introduced plane of projections $\Delta^j, j = 1, M$ is sufficient for unequivocal allocation and designation of the metric result of the solution of the problem, then the seventh stage of the developed algorithm is carried out.

Otherwise, the value of the counter j of the newly entered plane of projections $\Delta^j, j = 1, M$ increases by unit ($j = j + 1$).

The complex drawing of a new plane of projections $\Delta^j, j = 1, M$ is constructed and new points $K^{ij}, i = 1, N, j = 1, M$ for the newly obtained plane of projections are defined for a new geometrical image, that is, the sixth stage of the given algorithm is repeated.

D6. Steps D6.1, D6.2, D6.3, D6.4, D6.5 of the

developed algorithm are executed as steps D1, D2, D3, D4, D5 for the next part of the curvilinear surface— $\Delta^j, j = 1, M$.

To the counter j of this next part $\Delta^j, j = 1, M$ of the curvilinear surface the first unit value $j = 1$ is assigned.

D6.1. The next part of the curvilinear surface is allocated— $\Delta^j, j = 1, M$. This allocated part directly borders with the previously allocated part of the curvilinear surface.

D6.2. The curvilinear surface located between the allocated generating lines is replaced with a flat figure— $a^j, j = 1, M$.

D6.3. True size of the obtained flat figure is determined— $b^j, j = 1, M$.

D6.4. The flat figure of the true size is constructed on a plane (drawing)— $K^{ij}, i = 1, N, j = 1, M$.

D6.5. Verification of the termination of the construction of the development of the surface is carried out— $K^{ij}, i = 1, N, j = 1, M$.

If all identical flat figures of the true size corresponding to the allocated part $\Delta^j, j = 1, M$ of the curvilinear developed surface coincide with a plane, and other parts ($\Delta^j, j = 1, M$) are not going to be allocated, then the seventh stage is carried out.

If all identical flat figures of the true size corresponding to the allocated part $\Delta^j, j = 1, M$ of the curvilinear developed surface coincide with a plane, and other parts ($\Delta^j, j = 1, M$) are going to be allocated, then the sixth stage of the developed algorithm is carried out.

The value of the counter j of the allocated parts $\Delta^j, j = 1, M$ is increased by the unit value ($j = j + 1$). The complex drawing of an approximating flat figure $a^j, j = 1, M$ is constructed. The true size of the obtained flat figure is determined— $b^j, j = 1, M$. The flat figure of the true size is constructed on a plane (drawing)— $K^{ij}, i = 1, N, j = 1, M$. That is, the sixth stage of the developed algorithm is repeated.

A6. Additional points $K^{ij}, i = 1, L, j = 1, M$ for unequivocal allocation and designation of the end result of the solution of the axonometric problem are

defined. Number L belongs to a set of natural numbers.

To the counter j , a second unit value is assigned for the next orthogonal projection Δ^j , $j = 1, M$ of the product— $j = 1$.

A6.1. An orthogonal projection is allocated in the complex projective drawing (or prototype photos). This orthogonal projection Δ^j , $j = 1, M$ (the front view, the top view, the left-side view, etc.) contains the maximum information on the design features of the product (an aperture, a lug, planes, facets, flutes, etc.) in comparison with remained orthogonal projections Δ^j , $j = 2, M$.

For the newly allocated orthogonal projection Δ^j , $j = 1, M$, steps A6.2, A6.3, A6.4 the developed algorithm are carried out in full analogy with steps A2, A3, A4.

A6.2. A geometrical analysis of a new orthogonal projection Δ^j , $j = 1, M$ is carried out for the definition of characteristic geometrical images (circles, arches, pieces of straight lines, points).

For the construction of axonometry of the characteristic geometrical images, their characteristic points are allocated— K^{ij} ($K^{ij}_1, K^{ij}_2, K^{ij}_3$), $i = 1, L; j = 1, M$.

The number L of new characteristic points K^{ij} ($K^{ij}_1, K^{ij}_2, K^{ij}_3$), $i = 1, L; j = 1, M$ does not include characteristic points K^i (K_1^i, K_2^i, K_3^i), $i = 1, N$ for which axonometric projections $(K(x_K, y_K, z_K))^i$, $i = 1, N$ have been already constructed.

The original coordinates x^{ij}, y^{ij}, z^{ij} , $i = 1, L; j = 1, M$ are defined for each characteristic point K^{ij} (x^{ij}, y^{ij}, z^{ij}), $i = 1, L; j = 1, M$ of the analyzed orthogonal projection Δ .

Thus, on the considered iteration of the solution of a typical geometrical problem for an auxiliary image Δ^j , $j = 1, M$, the first group of auxiliary actions a^j , $j = 1, M$ is carried out. As a result of these actions a^j , $j = 1, M$, the original coordinates x^{ij}, y^{ij}, z^{ij} , $i = 1, L; j = 1, M$ of the additional characteristic points K^{ij} ($K^{ij}_1, K^{ij}_2, K^{ij}_3$), $i = 1, L; j = 1, M$ of the product are defined.

A6.3. At this stage of construction of axonometry of the product by means of natural coefficients of distortion k_x, k_y, k_z for coordinate axes Ox, Oy, Oz, the secondary axonometric coordinates $(x_K, y_K, z_K)^{ij}$, $i = 1, L; j = 1, M$ of the axonometric projection $(K)^{ij}$, $i = 1, L; j = 1, M$ into primary orthogonal coordinates $(x_K, y_K, z_K)^{ij}$, $i = 1, L; j = 1, M$ points $K^{ij}(x^{ij}, y^{ij}, z^{ij})$, $i = 1, L; j = 1, M$ are defined as:

$$x_K = k_x \cdot x^{ij}, \quad y_K = k_y \cdot y^{ij}, \quad z_K = k_z \cdot z^{ij}.$$

The results b^j , $j = 1, M$ of this stage for the problem of construction of axonometry are the secondary axonometric coordinates $(x_K, y_K, z_K)^{ij}$, $i = 1, L; j = 1, M$ for each characteristic point $K^{ij}(x^{ij}, y^{ij}, z^{ij})$, $i = 1, L; j = 1, M$ — $(K(x_K, y_K, z_K))^{ij}$, $i = 1, L; j = 1, M$.

Thus, at this stage of the solution of a typical geometrical problem for the auxiliary image Δ^j , $j = 1, M$, the second group of auxiliary actions b^j , $j = 1, M$ is carried out. The result of these actions b^j , $j = 1, M$ is the determination of the secondary axonometric coordinates $(x_K, y_K, z_K)^{ij}$, $i = 1, L; j = 1, M$ of characteristic points $K^{ij}(x^{ij}, y^{ij}, z^{ij})$, $i = 1, L; j = 1, M$ of the product.

A6.4. Axonometric projections of the characteristic points are constructed— $K^{ij}(x^{ij}, y^{ij}, z^{ij})$, $i = 1, L; j = 1, M$.

Connecting the axonometric projections $(K(x_K, y_K, z_K))^{ij}$, $i = 1, L; j = 1, M$ of the characteristic points $K^{ij}(x^{ij}, y^{ij}, z^{ij})$, $i = 1, L; j = 1, M$ by a thin line, one can allocate the constructed projections of the geometrical images belonging to the orthogonal projection Δ^j , $j = 1, M$ of the product.

A result of the fourth stage includes the axonometric projections $(K(x_K, y_K, z_K))^{ij}$, $i = 1, L; j = 1, M$ of the additional characteristic points $K^{ij}(x^{ij}, y^{ij}, z^{ij})$, $i = 1, L; j = 1, M$ and allocated by a thin line, the additional axonometric projections of the geometrical images belonging to an orthogonal projection Δ^j , $j = 1, M$ of the product.

Thus, the fourth stage of the solution of a typical geometrical problem for the auxiliary image Δ^j , $j = 1, M$ includes the construction of the additional

necessary projections of geometrical images.

A6.5. A decision on the unambiguity of the construction of the end result for the problem is made.

If quantities of points K^{ij} , $i = 1, N + L, j = 1, M$, belonging to geometrical images of orthogonal projections Δ^j , $j = 0, M$ are enough for unequivocal allocation and a designation of result of the solution of an axonometric problem, the seventh stage of the given algorithm is carried out.

Otherwise, the value of the counter j of the additional orthogonal projections Δ^j , $j = 2, M$ is increased by the value unit ($j = j + 1$). The new orthogonal projection Δ^j , $j = 2, M$ is allocated. The auxiliary procedures $a^j, b^j, j = 1, M$ are carried out. After this, a decision on the unambiguity of construction of the end result is made. That is, the sixth stage of the developed algorithm is repeated.

G6. Thus, at the sixth stage of solving a typical geometrical problem additional points K^{ij} , $i = 1, N, j = 1, M$ of the new constructed geometrical image for unequivocal allocation and a designation of the end result for solving the problem are defined.

P7. The result of the solution of the positional problem is allocated and designated by the connection of the obtained points K^{ij} , $i = 1, N, j = 1, M$ of threefold incidence by a smooth line taking into account their visibility on the complex drawing.

M7. The result of the solution of the problem, i.e. a geometrical image with the demanded metric characteristic, is allocated and designated. The size of the required metric characteristic is determined.

D7. All the constructed that far adjacent flat figures are allocated with smooth curves and designated— K^i , $i = 1, N; K^{ij}$, $i = 1, N, j = 1, M$. The constructed set of adjacent flat figures is an approximate development of the considered curvilinear surface.

A7. The result of the solution of the axonometric problem is allocated and designated by connection of obtained points K^{ij} , $i = 1, N + L + P, j = 1, M$ by a smooth line taking into account their visibility on the projective.

The connecting and overall dimensions are added to the constructed axonometry of the product.

G7. Thus, at the seventh stage of the solution of a typical geometrical problem the end result is allocated and designated.

P. The developed flow-chart of the algorithm of solution of positional problems on mutual intersecting of geometrical images reflects repetition of the first five stages at the sixth stage for new intermediaries Δ^j , $j = 1, M$ [2].

M. The developed flow-chart of the algorithm of solution of metric problems reflects repetition of first five stages at the sixth stage for new planes of projections Δ^j , $j = 1, M$ [4].

D. The developed flow-chart of the algorithm of the construction of a development of a surface reflects repetition of first five stages at the sixth stage for the next fragment of a curvilinear surface— Δ^j , $j = 1, M$ [3].

A. Thus, the developed flow-chart of the algorithm of the construction of axonometry of the product reflects repetition of first five stages at the sixth stage for new orthogonal projections Δ^j , $j = 1, M$ (Fig. 1).

G. Thus, the developed flow-chart of the general algorithm of the solution of typical geometrical problems reflects repetition of first five stages at the sixth stage for new images Δ^j , $j = 1, M$ (Fig. 1).

3. Results and Discussion

3.1 Results

The flow-chart of the general algorithm of the solution of typical geometrical problems (Fig. 1) corresponds to the flow-chart of the algorithm of the solution of positional problems on mutual intersecting of geometrical images [2], the flow-chart of algorithm of the solution of metric problems on definition of the demanded metric characteristic [4], the flow-chart of algorithm of construction of a development of a surface [3] and the flow-chart of the algorithm of the construction of an axonometry of a product.

The symbolical designations used in blocks are kept

identical to facilitate the complete comparison of the flow-chart of the general algorithm of the solution of typical geometrical problems (Fig. 1), the flow-chart of the algorithm of the solution of positional problems on mutual intersecting of geometrical images [2], the flow-chart of the algorithm of the solution of metric problems [4], the flow-chart of the algorithm of construction of development of a surface [3] and the flow-chart of the algorithm of construction of an axonometry of the product.

The flow-chart of the developed algorithm contains only standard logic blocks of operators for programming using a language of high level.

3.2 Algorithm Efficiency

Efficiency of the general algorithm is verified by many students and engineers dealing with the solution of practical problems.

For example, using the algorithm of the proposed structure, students solve the positional problem of the construction of the intersecting line of two surfaces of rotation (Fig. 2).

Results of the solution of the problem are shown in

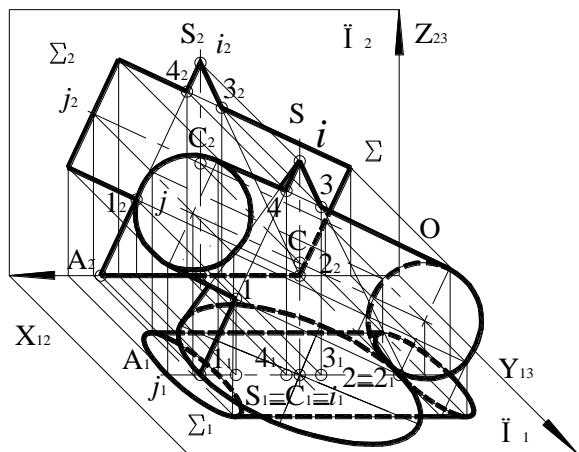


Fig. 2 Three- and two-dimensional models of initial geometrical images of a cone and a cylinder.

Fig. 3.

In tutorial study and/or in an exam, students with the help of the developed algorithm of structure can solve in a timely manner a metric problem of determining the true size of segment ΔABC (Fig. 4) of a plane.

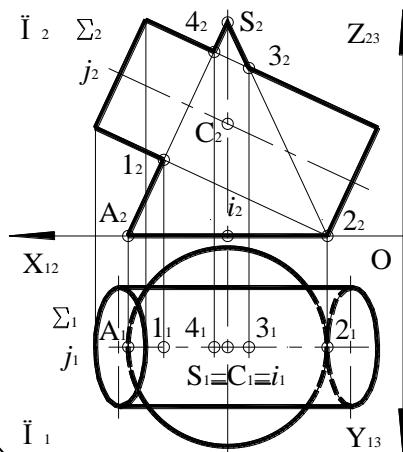
Results of solution of a problem are shown in Fig. 5 for three- and two-dimensional geometrical models.

For example, using the algorithm of the developed structure in practical training of students, the problem of construction of the development of a curvilinear surface of inclined circular cone S (Fig. 6) can be solved. Students receive result for time, which you had.

Results of solving a problem are shown together with approximation of geometrical model of inclined circular cone S by tetrahedral model of a pyramid (Fig. 7).

There is a problem of constructing axonometric views of a product (Fig. 8).

Using the algorithm of the developed structure, engineers solve the problem of constructing axonometric views of a product (Fig. 9).



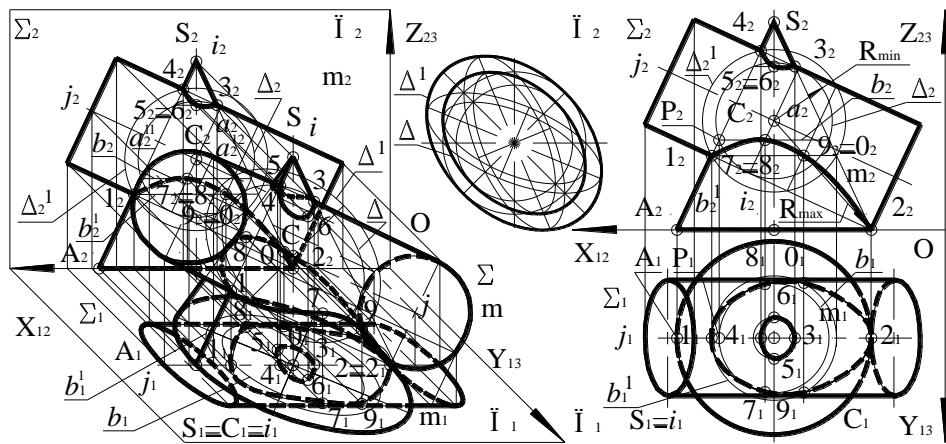


Fig. 3 Geometrical models of the construction of the line of intersection of two surfaces of rotation.

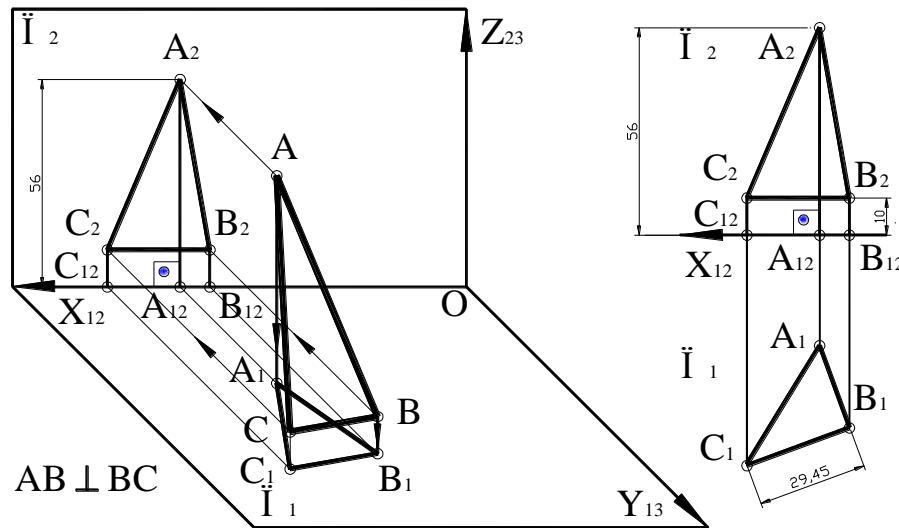


Fig. 4 Geometrical models of segment ΔABC of a plane.

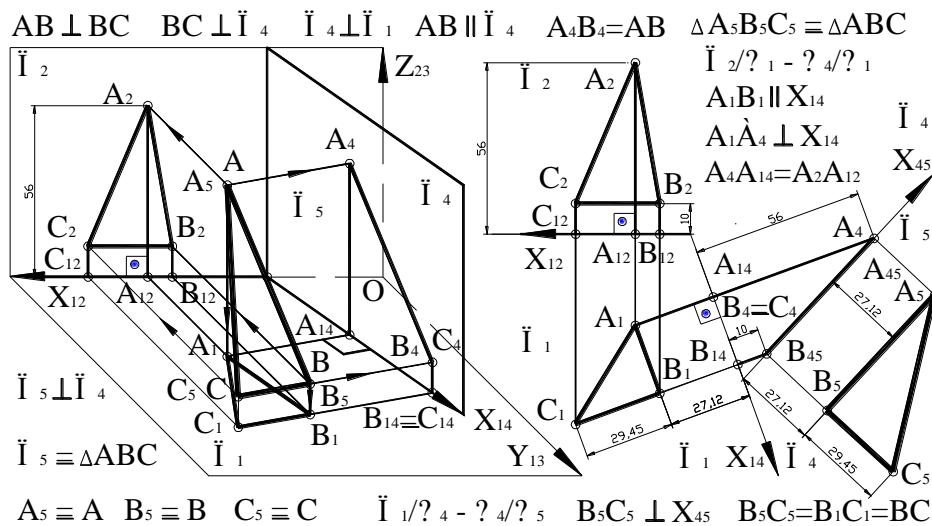


Fig. 5 The complex drawings of segment ΔABC of a plane in the created system of planes of projections Π₄/Π₁ and Π₄/Π₅.

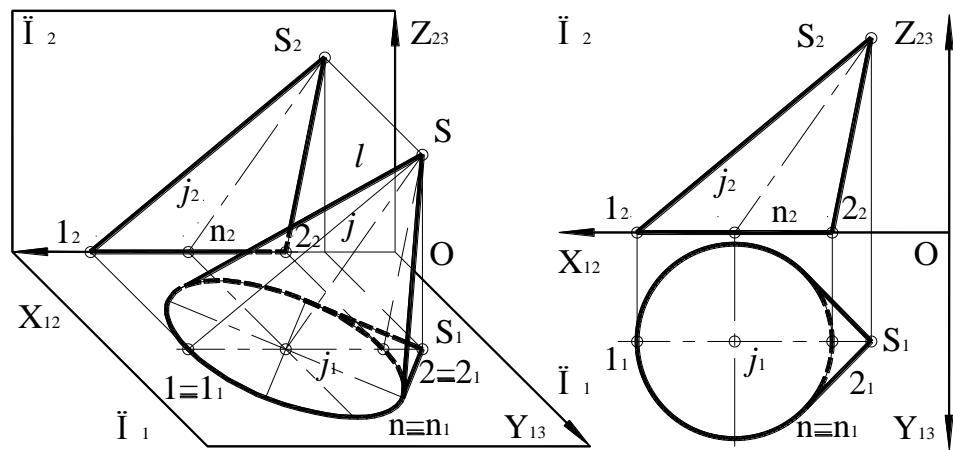


Fig. 6 Geometrical models of inclined cone S.

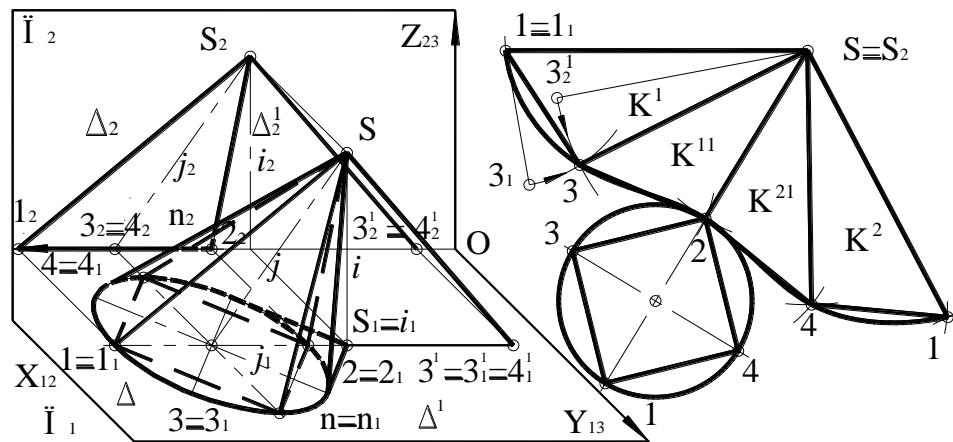


Fig. 7 The development of a curvilinear surface of an inclined circular cone.



Fig. 8 The problems of constructing axonometric views of a product.

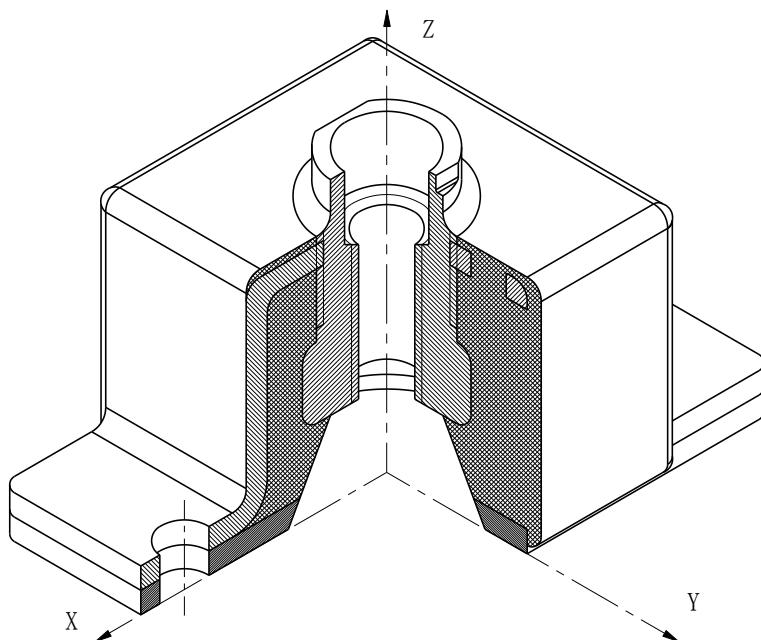


Fig. 9 Axonometric view of a product.

4. Conclusions

(1) The developed algorithm generalizes known approaches of solving typical geometrical problems [5, 21, 28].

(2) The flow-chart of general algorithm for solving typical geometrical problems (Fig. 1) corresponds to a flow-chart of algorithm for solving positional problems on mutual intersecting of geometrical images [2], a flow-chart of algorithm for solving metric problems by defining the necessary metric characteristic [4], a flow-chart of algorithm for constructing development of a surface [3], and a flow-chart of algorithm for constructing the axonometric views of a product [20].

(3) Using proposed general algorithm in the form of standard logic blocks facilitates students' and specialists' understanding of a geometrical essence of the studied phenomenon and an essence of the method for solving a problem [6-28].

(4) The developed general approach to the solution of typical geometrical problems considers the iterative nature of used methods [21].

(5) The introduced formal general algorithm allows students to master their skills of independent work.

(6) Practical engineers can use the developed general approach for solving new difficult real-world problems [28].

(7) Synthesized general algorithm, as the main contribution to geometry, allows, on deductive foundations—"from the general to quotient", to teach and study engineering geometry.

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