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Evaluation of Constants in Fracture Mechanics by Means Non-destructive Testing of Al+3.5%Mg Alloys

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Abstract: The constants in fracture mechanics are fracture toughness (for plane strain), critical value of *J*—integral , critical value of crack tip opening displacement. Mechanical testing under certain conditions to determine them is used. They are complicated, long and expensive procedures. In practice, it is interesting to determine the constant in fracture mechanics for elements of constructions by means of non-destructive testing (NDT) (non-destructive evaluations, NDE) from Al+3.5%Mg alloys (according EN 1706).

Key words: Constant in fracture mechanics, Al+3.5%Mg alloys, ultrasonic, hardness.

1. Introduction

In fracture mechanics [1-5] the following constants: fracture toughness (crack resistance)—K_{Ic}, critical value of *J*—integral—J_{Ic}, critical value of crack tip opening displacement—δ_{Ic} are used. The values of ($K_{Ic}, J_{Ic}, \delta_{Ic}$) by means of mechanical testing are obtained. For aluminum alloys (Al+3.5%Mg alloys EN 1706, Europe Union Norm) ASTM B 645-10:2015 (USA) is used. For mechanical testing the requirement is placed

$$B \geq 5.0 * \left(\frac{K_{Ic}}{\sigma_{YS}}\right)^2 \quad (1)$$

where *B* is thickness of test specimen, σ_{YS} is yield stress for material under test. Often, it is not possible to prepare a test sample from the subject which is tested.

The mechanical testing is difficult to realize, long and expensive. Often, it is not possible to prepare a test sample from the subject which is tested. This leads to necessity to develop a method of NDE for $(K_{Ic}; J_{Ic}; \delta_{Ic})$. The relation type is $(K_{Ic}; J_{Ic}; \delta_{Ic}) \Leftrightarrow (\overline{D}; E; \nu; \psi; \sigma_{YS}; \sigma_{UTS})$, where $\overline{D}$ —average grain size, $(E; \nu)$—elasticity modulus, ψ—relative contraction, σ_{YS}—yield stress, σ_{UTS}—ultimate tensile stress. The relations types $(E; \nu; \psi; \sigma_{YS}; \sigma_{UTS}) \Leftrightarrow (V_L; V_T; \alpha_L; HB)$, are known [2, 6], where $(V_L; V_T; \alpha_L; HB)$ are velocity, attenuation of longitudinal, transverse ultrasonic waves and hardness.

Therefore $(V_L; V_T; \alpha_L; HB) \Rightarrow (K_{Ic}; J_{Ic}; \delta_{Ic})$ is to be obtained.

2. NDE of Constants in Fracture Mechanics

2.1 NDE of the Constant—K_{Ic}

In crack mechanics there is Andreykiv's dependency [7]

$$\left(\frac{K_{Ic}^{A}}{\sigma_{YS}}\right)^2 \approx \frac{\sqrt{3}}{3} . \xi(\overline{D}) \frac{E}{1-\nu^2} . \psi \quad (2)$$

The NDE of the constant K_{Ic} by means is considered. In Eq. (2) $(\overline{D}; E; \nu; \psi)$ by means NDE are obtain, $\xi(\overline{D})$ is structural function, $\xi(\overline{D}) = \overline{D}.\left[\sigma_0 + K_y (\overline{D})^{-1/2}\right]^{-1}$. For Al+3.5% Mg alloys

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the reference values are $K_y \sim 8{,}5 MPa.mm^{1/2}$; $\sigma_0 \sim 50{,}0 MPa$ [8]. The yield stress σ_{YS} by Hall-Peth's relationship [7] is calculated by:

$$\sigma_{YS}, MPa = \sigma_0 + K_y\left(\overline{D}, mm\right)^{-1/2} \quad (3)$$

Elasticity characteristics E; ν respectively (Young's module, Poison's coefficient), according to ASTM E 494:2015, are calculated by:

$$\frac{E}{\rho.V_T^2} = \frac{3-4(V_T/V_L)^2}{1-(V_T/V_L)^2}; \quad \nu = \frac{0.5-(V_T/V_L)^2}{1-(V_T/V_L)^2} \quad (4)$$

To obtain the equation for $\overline{D}, mm$ Hall–Petch's relationship Eq. (3) and Bussinesq's dependency [6] are considered:

$$\sigma_{YS}, MPa = k_{Buss}.\varphi(\nu).\left(HB, kgf/mm^2\right) \quad (5)$$

where, according to EN 1706, $k_{Buus} \in (2.12-3.53)$, $med.k_{Buus} \sim 2.8$, $\varphi(\nu) = \left\{\frac{1}{2}(1-2.\nu) + \frac{2}{9}(1+\nu)[2.(1+\nu)]^{1/2}\right\}$ [6].

From the system Eq. (3), Eq. (5) is obtained as:

$$K_y.\left(\overline{D}\right)^{-1/2} + \left[\sigma_0 - k_{Buss}.\varphi(\nu).HB\right] = 0 \quad (6)$$

Eq. (6) is non-linear regarding $\overline{D}, mm$. It is solved by bisection method, ZEROIN algorithm [9] and on-line compiler for C++. The number of iteration is $N \sim \log_2^2\left[(X_R - X_L)/TOL\right]$, where $(X_R : X_L)$ is searching interval, the accuracy (machine zero) is TOL = 10^{-6}. In this case the searching interval (0, 1]. the number of iterations is ~170 (the time of work of ZEROIN is ~3-4 s),

$$(\psi, \%/100) = m.\left(V_L, mm/\mu s\right) + b \quad (7)$$

where ψ —relative contraction and *HB*, kgf/mm^2—Brinel's hardness, R^2~99.9%, m = 0.1016; b = 0.0038.

2.2 NDE of the constant— J_{Ic}

Fracture mechanics for J_{Ic} [1-3] are given in Table 1. They are NDE for J_{Ic} by means NDE for $\left(K_{Ic}; E; \nu\right)$.

2.3 NDE of the constant— δ_{Ic}

The results of facture mechanics for δ_{Ic} [2, 4] are given in Table 2.

where

$$L_{UTS} = \ln\left(\sec\left(\frac{\pi.\sigma_C}{2.\sigma_{UTS}}\right)\right); \quad L_{YS} = \ln\left(\sec\left(\frac{\pi.\sigma_C}{2.\sigma_{YS}}\right)\right);$$

σ_C —critical value of the stress in crack tip;

ℓ —depth of crack tip for δ_{Ic};

h_{UT} —full value of crack tip depth by ultrasonic testing is obtain $h_{UT} > \ell$.

For Al+3.5% Mg alloys there is $\sigma_C \equiv \sigma_{\max} \sim \sigma_{UTS} \approx 2.\sigma_{YS}$ (EN 1706). The relationships $\delta_{Ic} = \delta_{Ic}\left(E; \sigma_{YS}; \sigma_{UTS}; K_{Ic}\right)$, for NDE, are given in Table 3.

where

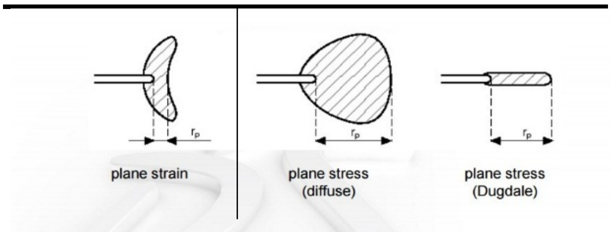
$\left(E; \sigma_{YS}; \sigma_{UTS}\right)$ and K_{Ic} are NDE;

h_{UT} —measured by means of a technique known as

Table 1 NDE of J_{Ic} [2-5].

Plastic strain	Plastic stress
$J_{Ic} = \frac{1-\nu^2}{E} K_{Ic}^2$ (8)	$J_{Ic} = \frac{1}{E} K_{Ic}^2$ (9)

Table 2 δ_{Ic}.



$\sigma_C/\sigma_{YS} \geq 1$ Full-scale plasticity	$\sigma_C/\sigma_{YS} < 1$ Small-scale plasticity
$\delta_{Ic} = \frac{K_{Ic}^2}{m.\sigma_{YS}.E}$ (10) where $m \approx 2$	$\delta_{Ic} = \frac{K_{Ic}^2}{m.\sigma_{YS}.E}$ (11) where $m = 1$
Dugdale's dependency (12) (modified) $\delta_{Ic} = \left(\frac{8.\ell}{\pi}\right)\left(\frac{\sigma_{UTS}}{E}\right)L_{UTS}$ $\frac{\ell}{h_{UT}} = \cos\left(\frac{\pi.\sigma_C}{2.\sigma_{UTS}}\right)$	Dugdale's dependency (13) $\delta_{Ic} = \left(\frac{8.\ell}{\pi}\right)\left(\frac{\sigma_{YS}}{E}\right)L_{YS}$ $\frac{\ell}{h_{UT}} = \cos\left(\frac{\pi.\sigma_C}{2.\sigma_{YS}}\right)$

Table 3 NDE of δ_{Ic}.

Plastic strain	Plastic stress
According to Wikipedia	
$\delta_{Ic} \approx \frac{K_{Ic}^2}{m.\sigma_{YS}.E}$ (14) med $m \sim 2.0$	$\delta_{Ic} = \frac{K_{Ic}^2}{\sigma_{YS}.E}$ (15)
According to Dugdale's dependencies	
$\delta_{Ic} \approx \left(\frac{\sigma_{UTS}}{E}\right)\left(\frac{h_{UT}}{250}\right)$ (16)	$\delta_{Ic} \approx \left(\frac{\sigma_{YS}}{E}\right)\left(\frac{h_{UT}}{1000}\right)$ (17)

"ultrasonic diffraction from crack tip" (TOFD method, according ASTM E 2373:2014).

About the relation $J_{Ic} = m.\sigma_{YS}.\delta_{Ic}$ in fracture mechanics [1-5]. A general theoretical relation, does not exist. It is known, that $m = 1$ for plane stress and $m > 1$ for plane strain. There are publications [2, 4, 5], where $m \in \{1.6;\ \sqrt{3};\ 3.0\}$ for plane strain. Therefore additionally theoretical research is needed.

3. Experiment

For samples from Al+3.5%Mg alloys (EN 1706) measures have been made.

Leeb hardness tester M-295, WILCON, Germany and ultrasonic flaw detector SITESCAN 150 S, SONATEST, England are used.

With this equipment realized the measurements of ultrasonic velocity, ASTM E 494:2015,

$(V_L), mm\ /\ \mu s \geq\ (6,36 \pm 0,030)$;

$(V_T), mm\ /\ \mu s \geq\ (3,15 \pm 0,015)$;

- hardness, ASTM E 10 – 2018,

$HB, Kgf\ /\ mm^2\ \geq\ (85 \div 105) \pm 10$.

4. Calculations

Reference values and results of NDE, for Al+3.5% Mg alloys, are given in Tables 4-6.

Table 4 Reference values [3, 5].

E, GPa	70
ν	0.35
σ_{YS}, MPa	5,080
ψ,%	64.%
$K_{Ic}, MPa.m^{1/2}$	27; 30; 33

Table 5 NDE of constants.

E, GPa	71.2 (64.1–78.3)*
ν	0.32 (0.29-0.35)*
σ_{YS}, MPa	70.1 (63-77)*
ψ,%	64 (58-70)*

Table 6 NDE of constants.

$K_{Ic}, MPa.m^{1/2}$ $\lvert\Delta\rvert$,%	29.0 (26.1-31.9)* ~(15; 4; 7)
$J_{Ic}, kN.m^{-1}$	12.15 (10.913.4)*
$\delta_{Ic}, \mu m$	2.73 (2.46–3.01)*

where $\lvert\Delta\rvert$, % is the error of NDE for K_{Ic}, * is the 95% confidence intervals obtained by functions AVERAGE, STDEV, TINV, CONFIDENCE from EXCEL, MS Office are calculated.

5. Classification of Field of Validity of Fracture Mechanics

It is known [1, 2], the size of the plasticity zone at the crack tip— r_P (Irwin's correction) for plain strain is

$$r_P = \frac{(1-2.\nu)^2}{\pi}\left(\frac{K_{Ic}^A}{\sigma_{YS}}\right)^2 \quad (18)$$

where for aluminum alloys [3] $(K_{Ic}/\sigma_{YS})^2 \sim (3.1 \div 3.3)mm$, NDE of Poison's ratio is $\nu = \frac{0.5-(V_T/V_L)^2}{1-(V_T/V_L)^2}$ and Eq. (18) is reduced to:

$$r_P(V_L;V_T) \approx \left[\frac{(V_T/V_L)^2}{1-(V_T/V_L)^2}\right]^2 \quad (19)$$

With Eq. (19), the classification, by means NDE, of the type of fracture [3], is presented in Table 7.

Table 7 Type of fracture.

Type of condition	Type of fracture	Fracture mechanics
$r_P(V_L;V_T) \to 0$	Crispy	Linear
$r_P(V_L;V_T) \leq (h_{UT};L)$	Quasi-crispy	Linear
$r_P(V_L;V_T) \sim (h_{UT};L)$	Quasi-viscous	Non-linear
$r_P(V_L;V_T) >> (h_{UT};L)$	Viscous	Non-linear

where $(V_L;V_T)$, according to ASTM E 494:2015 are measured, $(L;h_{UT})$ are respectively sizes of the depth of crack (measured according to ASTM E 2373:2014) and of the size of body for testing (measure by means micrometer).

For element of construction from Al+3.5%Mg alloys $r_P(6.36;3.15)=0.106$ is obtained. If $(h_{UT}/L)\leq 1/10$ is accepted, then the type of fracture, in the test material, is quasi-crispy. Therefore the condition for linear fracture mechanics (Table 7) is fulfilled.

6. Conclusion

In this article the possibilities of NDE of constants $(K_{Ic};J_{Ic};\delta_{Ic})$, for constructive elements from Al+3.5% (EN 1706), are considered. The algorithm is:

(1) Measurements of $(V_L;V_T;\alpha_L;HB)$.

(2) NDE of the mechanical properties $(E;\nu;\psi;\sigma_{YS};\sigma_{UTS})$ and the structural characteristic $\overline{D}$

$$(V_L;V_T;\alpha_L;HB)\Rightarrow(E;\nu;\psi;\sigma_{YS};\sigma_{UTS};\overline{D}).$$

(3) NDE of constants $(K_{Ic};J_{Ic};\delta_{Ic})$

$$(V_L;V_T;\alpha_L;HB)\Rightarrow(\overline{D};E;\nu;\psi)\Rightarrow K_{Ic};$$

$$(V_L;V_T;\alpha_L;HB)\Rightarrow(E;\nu;K_{Ic})\Rightarrow J_{Ic};$$

$$(V_L;V_T;\alpha_L;HB)\Rightarrow(E;\sigma_{YS};K_{Ic};m)\Rightarrow\delta_{Ic}.$$

The following norms are used:

EN 1706:2017, Aluminium and aluminium alloys-Castings-Chemical composition and mechanical properties.

ASTM B 645:2010, Standard Practice for Linear-Elastic Plane-Strain Fracture Toughness Testing of Aluminum Alloys.

ASTM E 2373:2014, Standard Practice for Use of the Ultrasonic Time of Flight Diffraction (TOFD) Technique

ASTM E 494:2015, Standard Practice for Measuring Ultrasonic Velocity in Materials.

ASTM E 10:2018, Standard Test Method for Brinell Hardness of Metallic Materials.

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