A Selection Process as a Game

Manuel Alberto M. Ferreira

Instituto Universitário de Lisboa (ISCTE-IUL), Lisbon, Portugal

Maria Cristina Peixoto de Matos

Instituto Politécnico de Viseu (IPV)-Escola Superior deTecnologia e Gestão de Viseu (ESTGV), Viseu, Portugal

The aim of this paper is to exemplify how to solve the problem of selecting a candidate up to his/her acceptance trough game theory. The originality this paper proposes is how this problem will be approached: It will be treated as a single game which is made up of two parts, going as far as to state that the payoffs in the first part of the game will be the mediators of the second part of the game.

Keywords: selection process, game theory, two-part game, code form

Introduction

A presidential decree reduced the number of candidates to the vice-presidency to three people. Each of the three candidates are ranked on a scale from 1 (lowest) to 10 (highest). The presidential board attributed 10 points, 8 points, and 5 points to the candidate classified in 1st place, 2nd place, and 3rd place, respectively. The probabilities candidate i (i = 1, 2, 3) accepting the j-th offer to run for the vice-presidency have been defined, considering that the first j-1 offers to the others have been declined, and are denoted by p_{ij} and the respective values are in Table 1.

Table 1

Acceptance Probabilities

Player 1	$p_{11} = 0.5$	$p_{12} = 0.2$	$p_{13} = 0.0$
Player 2	$p_{21} = 0.9$	$p_{22} = 0.5$	$p_{23} = 0.2$
Player 3	$p_{31} = 1.0$	$p_{32} = 0.8$	$p_{33} = 0.4$

The question is: What is the order in which the three potential candidates can be offered the vice-presidential nomination if the presidential decree imposes the expected number of points maximization, supposing that no candidate is requested more than once and, each time a candidate rejects, another one is requested, until at least one has accepted or all have rejected?

Game Details

This game which is made up of two parts—a selection process and an acceptance process—attests that the payoffs in the first part of the game (potential candidates) will be the intermediaries of the second (decision elements). Thus:

Players

Candidate classified in 1st place—1;
Candidate classified in 2nd place—2;
Candidate classified in 3rd place—3;
Presidential Board—4.

Strategies[1]

Potential candidates. The strategy of each potential candidate is to accept the offer A or to reject the offer R.

Presidential board. The presidential board wants to establish the order in which the potential candidates will be invited to maximize the expected number of points. In this way, the strategy for the presidential board will be the order in which the three potential candidates can be offered the vice-presidential nomination until at least one has accepted or all have rejected the offer P.

Payoffs

Presidential board. The presidential board payoff is the expected number of points of each possibility in the order of the proposal presented to the potential candidates.

Potential candidates. For these players it is possible to define: If the player accepts the offer, he/she gets the "total prize", that is, he/she gets payoff 1. On the other hand, if he/she rejects the proposal, he/she does not get anything so his/her payoff will be 0.

This game is an example of game theory application to human resources management (on this subject see also Andrade, Ferreira, & Filipe, 2012; Ferreira et al., 2008; 2012; 2014; 2016; Ferreira, 1991; 2014; Filipe, Ferreira, & Andrade, 2012; Matos & Ferreira, 2005; Matos, Ferreira, & Filipe, 2018).

Code Form Representation for the Game

To represent the game, code form game representation (see Ferreira & Matos, 2018; Matos & Ferreira, 2002a; 2002b; 2003; 2005) will be used. Code form is a table where the strategies that are available to any player, are codified. The table is built containing the whole game information. Table 2 is the code form representation for the game now considered. Reading from left to the right, the first column indicates the period number and the second column indicates the move[2] number. The following columns mention who moves when and in what circumstances and what action is played when somebody is called upon to move. Last column indicates the payoffs vector in accordance with the strategies chosen by the players. It is easy to check that the order in which the three potential candidates can be offered the vice-presidential nomination must be (follow symbols in bold):

To invite in the first place the candidate classified in 2nd place, 2; if he/she rejects the proposal, the candidate classified in third place, 3, should be invited and if he/she does not accept, the candidate classified in first place, 1 should be invited. The expected number of points is $7.6 = 7.2 + 0.4 + 0 + 0$.

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[1] See Bicchieri et al., 1999.
[2] See Benoit and Krishna, 1985 and Eberwein, 2000.

Table 2

Code Form Game

1	1	(4, P)			
1	2		(1, A, 0.5)		(5, 1, 0, 0)
			(1, R, 0.5)		
	3		(2, A, 0.5)		(2, 0, 1, 0)
			(2, R, 0.5)		
	4			(3, A, 0.4)	(0.5, 0, 0, 1)
				(3, R, 0.6)	(0, 0, 0, 0)
	3		(3, A, 0.8)		(2, 0, 0, 1)
			(3, R, 0.2)		
	4			(2, A, 0.2)	(0.16, 0, 1, 0)
				(2, R, 0.8)	(0, 0, 0, 0)
	2		**(2, A, 0.9)**		(**7.2**, 0, 1, 0)
			(2, R, 0.1)		
	3		(1, A, 0.2)		(0.2, 1, 0, 0)
			(1, R, 0.8)		
	4			(3, A, 0.4)	(0.16, 0, 0, 1)
				(3, R, 0.6)	(0, 0, 0, 0)
	3		**(3, A, 0.8)**		(**0.4**, 0, 0, 1)
			(3, R, 0.2)		
	4			**(1, A, 0)**	(**0**, 1, 0, 0)
				(1, R, 1)	(**0**, 0, 0, 0)
	2		(3, A, 1)		(5, 0, 0, 1)
			(3, R, 0)		
	3		(1, A, 0.2)		(0, 1, 0, 0)
			(1, R, 0.8)		
	4			(2, A, 0.2)	(0, 0, 1, 0)
				(2, R, 0.8)	(0, 0, 0, 0)
	3		(2, A, 0.5)		(0, 0, 1, 0)
			(2, R, 0.5)		
	4			(1, A, 0)	(0, 1, 0, 0)
				(1, R, 1)	(0, 0, 0, 0)

Note that the winning strategy (from the presidential board point of view) is weak in guarantying the existence of a vice-president in the end of the selection process. Indeed, if the first two candidates refuse the vice-presidency, it will be vacant since the best-classified candidate will certainly refuse when asked in third place.

This is an example of how, when it comes to modelling human behaviour, all criteria are debatable and have strengths and weaknesses. You have to be very careful in your choice, and the quantitative aspects are not necessarily the most important.

Conclusions

The analyzed game illustrates that the player who has decision power is not always the one who decides the game. In other words, the presidential board is the player who dominates the situation; therefore, they

decide who the best candidate is. But, indeed, the ones who actually decide the game are the potential candidates when either they accept or they refuse the proposal. Really, the potential candidates are the ones who determine the game's outcome; starting from a situation of weakness they gain the control of the game. They are the result of the first part of the game and become the deciding elements of the second part of the game.

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