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E. Cartan's and A. Connnes' Supersymmetry and Differences of Understanding Physical Phenomena

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Abstract: Einstein claimed that one cannot define global time, and proposed defining local time additionally. Such approach was adopted also by E. Cartan, in which fermions are described by spinors with 16 bases and interact with vectors with 8 bases, that consists of a couple of 4 dimensional vectors x_i (i = 1, ···, 4) and x_i (i = 1, ···, 4). In Cartan's theory, spinors and vectors transform by super symmetric transformations G_{23}, G_{12}, G_{13}, G_{123} and G_{132} and bases of fermion spinors consist of ξ_0,ξ_i (i = 1, ···, 4), ξ_{1234}, ξ_{234}, ξ_{134}, ξ_{124}, ξ_{123} and $\xi_{i,j}$ ($i \neq j \in\{1,2,3,4\}$). Except G_{23}, the transformations mix spinors and vectors, and operations of G_{23} on spinors contain $G_{23}\,\xi_4 = \xi_0$ and $G_{23}\,\xi_{123} = \xi_{1234}$, and operations of G_{23} on vectors contain $G_{23}x_4 = -x_{4'}$ and $G_{23}\,x_{4'} = -x_4$. Therefore, there are 14 independent spinor bases and 7 independent vector bases, which corresponds to the number of bases of the G_2 symmetry.

From the bases of non−commutative geometry, Connes took two fibers from a point of S^3 basis, and on top of fibers allowed two times propagate following von Neumann algebra, but evolution of the system was assumed to be defined by one-parameter group of transformation.

Steenrod stated that the S^7 symmetry can be regarded as S^3 symmetry covered over S^4 symmetry, which allows decomposition of $S^7 \times R^8 \to (S^3\times R^4)\times(S^3\times R^4)$. We assume there is a space-time representation by an algebra $C(V)$ of smooth function and matrix algebra $M_n$and transformations A are expressed as $A = C(V)\otimes M_n$. In order to make total momentum space to remain 4 dimensional, the group of A becomes $SO(3 + n^2 -1,1) \sim SO(3,1)\times SO_{n^2-1}$ in Minkowski space. We choose n = 3 and construct SO_8 on $R^8\sim R^{4,4}$. We apply this model to understanding experimentally observed CP violation in $p\bar{p}\to t\bar{t}$ or $b\bar{b}$ and in $pp\to(H\to b\bar{b}) + \ell\ell + jets$ and Time Reversal Based Nonlinear Elastic Wave Spectroscopy (TR-NEWS) method.

Keywords: Cartan's supersymmetry, non-commutative geometry, Higgs mechanism, feuilletage(foliation).

1. Introduction

Hurwitz's [1] theorem states that composition algebra $x\cdot\bar{x} = Q(x)\cdot 1 = \bar{x}\cdot x$,that satisfies symmetry of quadratic forms

$$f(X,Y) = (Q(X+Y)-Q(x)-Q(Y))/2$$

and has a unit element are given by real number R, complex number $C = R+\sqrt{-1}R$, quaternionnumber $H = C + e_2C$ and octonion number or Cayley number $O = H + e_7H$.

Here, $e_0,e_1, e_2,\cdots,e_7$ are bases of the octonion.

A. Einstein's general relativity states that time cannot be defined globally. D. Hilbert derived Einstein's field equations from variational principles, but encountered a puzzle regarding energy conservation, which is related to invariance under time translation [2].

E. Noether applied transformation group theory to general relativity and created a formalism whose capability went far beyond resolving Hilbert's problem [2, 3]. Expression of propagation of time in physical processes contains subtle problems. Connes discussed interpretation of Lagrangean of the standard model in terms of non-commutative geometry [4-7]. He defined on S^3 sphere of space coordinates, two fiber points of time coordinates, and extendedQuantum Chromo Dynamics (QCD) by adding one time coordinate which does not commute with the ordinary time [4, 5].

S. Hawking's model dependent realism [8] states

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that physical properties are understood by consistent mathematical models. It is important to choose a proper framework for model building and a number system which can incorporate the complexity of Nature.

In the expression of fiber bundles (E, Π, F, G, X) [9-12], Connes defined two fibers F that are extended from a point of a base space $X = S^3$, and the group G that defines dynamics of leptons described by Dirac equation is expressed by quaternions $\in H$, and a tangential space TS^3 was identified with $S^3 \times$ R^4spacetime.

But the quaternion is a subgroup of octonion O [18, 26, 45] or Cayley number, and in asymptotic base space Y, it has S^7 symmetry. An octonion has the triality symmetry, but its structure is not well understood although there are studies in string field theory of Atiyah and Witten [13, 14] whose asymptotic form of Y is restricted to $S^3 \times S^3$, and not S^7 and as an element of TX, $S^3 \times R^4$ was studied. Cayley numbers of S^7 are defined as an ordered product of 3-sphere bundles S^3 and $S^{3\prime}$ over S^4. Therefore in TX there are two fibers $S^3 \times R^4$ and $S^{3\prime} \times R^4$.Cartan[15] derived triality symmetry of spinors on S^3 and $S^{3\prime}$ that interact with 4 dimensional fields x and x'.

The definition of triality symmetry of Ref. [14] is different from that of Cartan, since they consider only R-symmetry and K-Symmetry among fermions, while Cartan defined fermions by spinors ξ and vectors by$x$ and x' and there are 5 categories of supersymmetric transformations: G_{23}, G_{12}, G_{13}, G_{123} and G_{132}, and except G_{23} there are transformations of $\xi$to x or x' and vice versa. The fermion spinorsξ has 16 bases ξ_0,ξ_i (i =1,⋯,4), ξ_{1234}, ξ_{234}, ξ_{134}, ξ_{124}, ξ_{123} and $\xi_{ij}(i<j) \in \{1, 2, 3, 4\}$. Since $G_{23}\xi_4 = \xi_0$ and $G_{23}\xi_{123} = \xi_{1234}$, there are 14 independent spinor bases. Vector x and x'have respectively 4 bases (x_i (i = 1, ⋯, 4), $x_{i'}$ (i' = 1, ⋯, 4)). Since $G_{23}x_{4'} = -x_4$ and $G_{23}x_4 = -x_{4'}$ there are 7 independent bases.

Following the suggestion of Bleuler that Lorentz symmetry need not be satisfied in virtual states, we take differential of $U(1)$ defined in $S^3 \times R^4$ and $U'(1)$ defined in $S^{3\prime} \times R^4$ opposite directions. The S matrices of such systems can be derived from induced representation and adopting semidirect product of unitary transformation and non-unitary triality transformation, that exists in Cartan's supersymmetry.

We study also superpositions of sound waves which contains nonlinear $U(1)$ component and its time reversed nonlinear wave $U'(1)$. We find interesting superposition properties of memristor wave, which was studied inRefs. [37-41, 47].

In Time Reversal based Nonlinear Elastic Wave Spectroscopy (TR-NEWS) method, presence of more degrees of freedom in superposition of time reversed focused pulses can be understood, if leptons on detectors needed to follow a quaternion which was a projected subgroup of octonions.

The relation between the Quantum Electro Dynamics (QED) which was successfully described by Dirac equation and the Quantum Chromo Dynamics (QCD) which is expected to be described by Yang-Mills equation including color degrees of freedom is not so obvious. In addition to quantum mechanics, one needs to include gravitational interactions, which can be performed by changing the real degrees of freedom to the complex degrees of freedom. In quantum mechanics, two events are defined by unitary operator $U_{\alpha 1}$ and $U_{\alpha 2}$:

$$U_{\alpha 1} U_{\alpha 2} = \sigma(\alpha_1, \alpha_2)\, U_{\alpha 1 \alpha 2}$$

andnonequivalence of $U_{\alpha 1\alpha 2}$ and $U_{\alpha 2\alpha 1}$ means violation of time reversal symmetry.

Two representations

$$U = L_1 \oplus L_2 \oplus \cdots,\ U = M_1 \oplus M_2 \oplus \cdots$$

are identical if there is bounded linear operator V that satisfies $V L_x V^{-1} = M_x$,which means:

$$V L_x = M_x V$$

for $x \in H$ and x transform as $x \to hxh^{-1}$.

Induced representations[10] are useful in describing a group G which hasa closed normal subgroup N. Irreducible representations of semi-direct product G is written in the form nh.

On a space S one can define a projection-valued measure P_E,which satisfies:

$$U_x P_E U^{-1}{}_x = P_{[E]\,x}{}^{-1}$$

Where $E \subseteq S, x \in G$.

The time is a projection valued measure, and we apply the induced representation to the study of time reversal symmetry. A superposition of a non linear sound wave and time reversed non-linear sound wave can reduce improper superposition of lobes around the proper peak produced by a superposition of reflected waves[19-24, 38].

Cartan's supersymmetry [9, 10, 15] that allows asymmetry of time reversal could explain $p\bar{p}\to t\bar{t}$ forwardbackward asymmetry, and solve the problems of decay widths of Higgs boson to vector mesons. It could also explain chaotic behaviors of Memristor based ultrasonic transducer, which can be applied for realizing lossless electronic wave propagation that will be used in medical detection processes and others.

In section 2, we explain the notion of feuilletage[3] (foliation) which appears in the fiber bundle approach and derive Noether's theory.

In section 3, we show application of Steenrod's fiber bundle approach to Cartan's space time.

In section 4, we discuss on experimental results of $p\bar{p}\to t\bar{t}$ forward backward asymmetry observed at TEVATRON and anomalyobserved at CERN Large Hadron Collider (LHC)in $pp \to (H \to b\bar{b}) + \ell\ell + jets$ [32-35, 53, 54] in the new framework. Discussion on Cartan's supersymmetry, Connes' supersymmetry and conclusions are given in section 5.

2. Noether's Theory Based on the Theory of Foliation (Feuilletage)

Description of dynamics of a particle in described by space of evolution V described by the initial condition$y = (t, r, v)$ and the space of movement U described by the coordinate x and vectors dx and δx [3].

*Emmy Noether, Invariante Variationsprobleme 1918.

Let V be an n-dimensional manifold defined by x and E be am dimensional vector space spanned by tangent vectors Dx, we denote $x \to E$ as differential map from V to E. If $x \to dx$ and $x \to \delta x$ are defined for open regions $dx \in E$ and $\delta x \in E$, such that for an open region $[d, \delta]x \in E$, where

$$[d,\delta]x = d[\delta x] - \delta[dx],$$

it is also defined, then the map $x \to E$ is called foliation (feuilletage).

When $t \in R^m$ and $z \in R^{n-m}$, the mapping

$$\binom{t}{z} \to x$$

means that

$$dx \in E \Leftrightarrow dz = 0$$

when $x \to V$ is a non-null vector in R^3, one defines a "symplectic"manifold$U$ as a manifold on which a mapping of differential 2-form$\sigma : x \to \sigma$ for $x \in X$ satisfies two conditions:

(1) $\nabla\sigma$ is regular, which means that if

$\sigma(X),(Y) = \sigma(X'),(Y')$ then $(X),(Y) = (X'),(Y')$, and

(2)The exterier derivative

$$(\nabla\sigma)_{jkl} = \partial_j\sigma_{kl} + \partial_k\sigma_{lj} + \partial_l\sigma_{jk} = 0.$$

A "presymplectic" manifold V is defined as a manifold on which a differential 2-form $y \to \sigma$is defined such as

(1)$ker(\sigma)$ is a one dimensional constant > 0, and

(2)$\nabla\sigma = 0$ In this case the field$y \to ker(\sigma)$ is characteristic foliation of the form σ, and its feuilles are called feuille of V . When the field$x \to E$is a foliation of a manifold V of dimension n and dim(E) = m, we call manifold U in V is transversal to foliation, if in all points of U, its vectorial tangents are supplimentally of E, whose dimension is$n - m$. One can let a transversal manifold passes all points of V.

Foliation is called separable if on all points of V,a transversal manifold U passes which encounters with each feuille only on a point at most. The manifold U is called a transversal section.

If F and G are application from R^n to R^n and $F^{-1}\cdot G$ and its inverse $G^{-1}\cdot F$ are differential we call F and $G$are coherent.

A set A of V is called atlas when

(a) Elements of A are coherent each other;

(b) The sets of values recover V.

When one chooses an atlas A, one calls the admissible coordinate system as carte of V. Sets of all carte is atlas, which is the greatest atlas containing A.

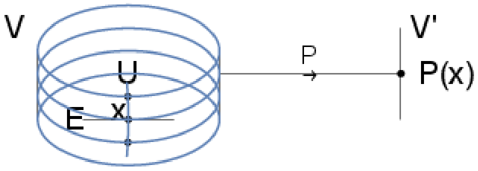

Fig. 1 Foliation (Feuilletage) mapping of $x \in V \to E$ and transverse manifold U to E.

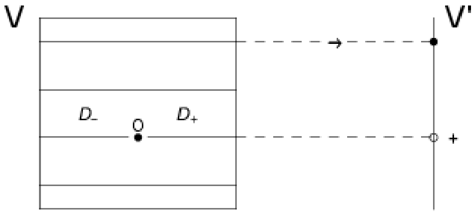

Fig. 2 Foliated separable manifolds. V is separable, but $V'$is inseparable.

A separable foliation which passes x of V as shown in Fig. 1 is written as $P(x)$ and a map of the transversal the transversal section F, P.F forms an atlas of a set V'offeuille. $V'$is called quotient manifold of V by foliation. P is called projection of V on V'.

When $P(x)$ is a feuille that passes x, the theorem of Noether indicates that the moment μ depends only on $P(x)$, and if manifold V is separable, $P(x)$ describes the symplectic manifold U, quotient of V by its foliation, and symplectomorphysm G from V to V' is still dynamical group of U. When G is a Lie group and G is its Lie algebra. The elements of vector space $G*$ dual to G is called torsion of G, andtorsion variable μ is called moment of G, which is also moment of group G that operate on U.

For an element $a \in G$, one defines the vector space $G\times V$ on V as

$$(a,x) \to aV(x),\ ZV(x) = d[aV(x)]$$

if there exists differentiable application $x \to \mu$ of Vin $G*$, such that

$$\sigma(ZV(x)) = -\nabla[\mu.Z]$$

for all Z constant in G, and

$$aV(P(x)) = P(aV(x)).$$

Even if the manifold V is separable, $V'$is not necessarily separable as shown in Fig. 2. We define R^2 space by (y, z) and define a foliation by $dz = 0$. Feuilles are given by straight lines $z = z_0/= 0$ and half straight lines

$$D^+(z = 0, y > 0),\ D^-(z = 0, y < 0).$$

Open sets that contain D^+ and that contain D^- meet together. When A is a diffeomorphism on V that respects feuilletage, there exists a permutation $\widehat{\boldsymbol{A}}$ of V' defined by $\widehat{\boldsymbol{A}}\ (P(x)) = P(A(x))$ for $x \in V$, and $\widehat{\boldsymbol{A}}$ is a diffeomorphism of V'. In Fiber Bundle approach, one defines topological space E which consists of base space X and fibers $F = \Pi^{-1}(X)$, where Π is a projection operator of an event on the base space. Relations between initial data and final data are defined by group G and a Fiber bundle is defined as a set (E, Π, F, G, X). Noether's theorem states that if V is a presymplectic manifold, μ is a moment of dynamical group of V, then μ is constant on each feuille of V. Tangent bundle TX of real linear space X is defined by the projection $\Pi TX = TX \to X$; $(x,a) \to a$ for any $a \in X$ and a sphere S^n for any non-negative integer n may be thought to be a smooth submanifold of R^{n+1} and TS^n is identified as

$$\{(x,a) \in R^{n+1} \times S^n : x\cdot a = 0\}.$$

In Cartan's theory of spacetime, there are 16 dimensional fermion spinor bases ξ_i (i = 0, 1, 2, 3, 4), $\xi_{i,j}$ ($i/= j \in \{1,2,3,4\}$) and ξ_{1234}, ξ_{234}, ξ_{134}, ξ_{124}, ξ_{123} and 8 dimensional vector bases x_i, x'_i (i = 1, 2, 3, 4). There are G_{23}, G_{12}, G_{13}, G_{123} and G_{132} transformations of fermions and vectors, and except G_{23}there are transformations of ξ to x or x' and vice versa. There istriality symmetry, and involutions are not trivial. Space-time is decomposed as $R^8 = R \oplus R^{0,7}$. There is unique direction of $e0$, but time dependent phases on feuillets defined by local dynamics are not necessary same.

3. Steenrod's Fiber Bundle Approach and Decomposition $S^7 \times R^8 \to S^3 \times R^4 + S^{3'} \times R^4$

Kaluza-Klein theory is a theory of embedding smooth functions of space time represented by algebra $C(V)$ to a smooth function A on a principal fiber bundle

$$P = V \times SU_n.$$

We assume internal structure of A is expressed by matrix algebra M_n and express $A = C(V) \otimes M_n$. In order to make total momentum space to remain 4 dimensional, the group of A reduces $SO(3+ n^2 - 1,1) \sim SO(3,1) \times SO_{n^2-1}$, in Minkowski space. By choosing $n = 3$, we can construct SO_8 algebra on $R^8 \sim R^{4,4}$. Although time propagation on two fibers $S^3 \times R^4$ and $S^{3'} \times R^4$ are local (It is misleading to specialize the time coordinate and express $R^{1,1} \times R^3$), global time is defined in the space of S^7 symmetry.

Following Cartan, we assume that the asymptotic topological space of leptons X is on S^7 bundle, which in Steenrod's description [12], given by S^3 bundles on S^4 sphere, and in TX one can define $S^3 \times R^4$ and $S^{3'} \times R^4$, in which the transformation is expressed by octonions $O \supset (H, H)$. Trautman [17] stated that for

$z_\alpha \in H$ that satisfy

$$\bar{z}_0 z_0 + \bar{z}_1 z_1 + \cdots + \bar{z}_n z_n = 1$$

defines a sequence of Hopf principal fiber bundle $S^{4n+3} \to HP_n$ with group $Sp(1) = SU(2)$ and that there exists a connection

$$Sp(2)/Sp(1) = S^7 \to S^4$$

which is obtained by adjoining points at infinity to a Quaternion.

Hopf principal fiber bundle is $\eta = (S^{2\lambda-1}, q, S^\lambda, S^{\lambda-1})$ ($\lambda = 1, 2, 4$). One may need Cartan's equation which contains octonions and have G_2 symmetry. Any octonion elements e_1, e_2 and e_3 make elements e_4, e_5, e_6 and e_7, and multiplication tables of e_i are known. We find

$$\dim G_2 = \dim S^6 + \dim S^5 + \dim S^3 = 14$$

the total dimension is the same as $SU(3)$ division of G_2.

$$8 + 3 + 3^* = 14$$

When E and F vector space of n and p dimension, respectively, and B is a bilinear form

$$B : (x, y) \in E \times F \to B(x, y) \in R \text{ or } C$$

$B(x, x)$ can be written as $Q(x)$and

$$2B(x,y) = Q(x + y) - Q(x) - Q(y).$$

The basis of Quaternion $C(Q)$ is given by 1, e_1,e_2, e_1e_2, and that of Octonions is obtained by using Cartan'striality principle and Clifford algebra [18, 26].

4. Interference of $S^3 \times R^4$ and $S^{3'} \times R^4$

Dirac spinors consist of 2 × 2 components, and each component transforms by quaternions. Cartan's spinors consist of 4 × 2 components, and they transform not by quaternions but by a kind of octonion. Since quarks have color degrees of freedom, hadronic dynamics could be described by dynamics of octonions instead of quaternions.

Foreward backward asymmetry in

$$\boldsymbol{p\bar{p}} \to \boldsymbol{t\bar{t}} + X$$

and

$$pp \to \ell\ell + (H \to \boldsymbol{q\bar{q}}(\ell\bar{\boldsymbol{\ell}}) + Y$$

The top-quark pair forward-backward asymmetry measured at Tevatron$(\boldsymbol{p\bar{p}})$ and LHC(pp) is

$$A_{FB}(\Delta y) =$$

$$(N(\Delta y > 0) - N(\Delta y < 0))/(N(\Delta y > 0) + N(\Delta y < 0))$$

where, $\Delta y = y_t - y_{\bar{t}}$. The rapidity y_t is defined as $y_t = (1/2)\log((E^t+p^t_z)/(E^t-p^t_z))$ and $y_{\bar{t}}$ is defined similarly. Experimentally $A_{FB} = 8.7 \pm 1\%$ at Tevatron was observed.

The top quark charge asymmetry at LHC for

$$pp \to \ell\ell + (H \to q\bar{q}(\ell\bar{\ell})) + Y$$

defined as

$$A_C(\Delta|y|) = (N(\Delta|y| > 0) - N(\Delta|y| < 0))/$$
$$(N(\Delta|y| > 0) + N(\Delta|y| < 0))$$

where, $\Delta|y| = |y_t| - |y_{\bar{t}}|$.

The coupling of Higgs boson to $\boldsymbol{q\bar{q}}$ and $\ell\bar{\boldsymbol{\ell}}$ is Yukawa type, and in Cartan's supersymmetry, spinors ξ_{1234} and ξ_{123} appear in propagators. Higgs bosons

$$h_0 \xi_{1234} \sum_{i=1}^{3} \xi_i$$

and

$$\boldsymbol{h_0{}'\xi_{123}\sum_{i=1}^{3}\xi_{i4}}$$

are different from those of standard model which are based on $SU(2)_L$ and $SU(2)_R$ chiral fields. Time reversal symmetry in $\boldsymbol{p\bar{p}}$ $(\boldsymbol{\ell\bar{\ell}}) \rightarrow \boldsymbol{t\bar{t}}$ can be violated, since ξ_{1234} and ξ_{123} in $\alpha^4{}_g$gluon exchange diagrams can be different. They behave like ghosts i.e. they fix the gauge of the system. The loop contribution may become important at $\boldsymbol{p\bar{p}}$ energy higher than 5 m_{Higgs} = 625GeV.

The $\boldsymbol{t\bar{t}}$ production in pp collision at 7 TeV in lepton +jets event was measured by the CERN Large Hadron Collider CMS collaboration and ATLAS collaboration. Recently results of $H \rightarrow \boldsymbol{b\bar{b}}$ in pp collision at $\sqrt{\boldsymbol{s}}$ = 13TeV was reported [53]. When all lepton channels are combined, the probability p_0 of obtaining $H \rightarrow \boldsymbol{b\bar{b}}$ decay data from background in 2-lepton data set is 0.019% as compared to expectation value of standard model 3.1%, which means that strong 2-lepton signals not from background is observed.

Since detected leptons are e and μ and τ is not included, entanglement of $S^3 \times R^4$ and $S^{3\prime} \times R^4$ of e, μ and τcould reduce background contribution in 2-lepton signals.

In CMS experiment of $\boldsymbol{t\bar{t}}$ production in pp collision at $\sqrt{\boldsymbol{s}}$ = 8 TeV, shows that in lepton + jet channels CP violation was not observed and consistent with the standard model, but CP violation in quark + jet channels is not measured. Brodsky and Wu showed that CP violation effects in simulations can be reduced by adopting the principle of maximum conformality and choosing a proper renormalization scheme [33, 54].

Since Cartan'soctonions are not Cayley numbers, but have triality symmetries, Cartan's supersymmetry was consistent with tree level Higgs boson dynamics [39-43]. A main difference of Cartan'ssupersymmery and Atiyah-Witten's supersymmetry is the presence of trialities in the octonion products. There are particle models based on non-commutative geometry which are consistent with standard model [4]. In their model, transformations by quaternion bases are adopted, but we can extend transformation by octonions and incorporate triality transformations which are $C\ell_8 \rightarrow C\ell_8$ transformation of order 2. For example, quarks $\xi_{14}\xi_{123}$ couple to x^1, and $\xi_{34}\xi_{123}$ couples to x^3 and the two coupling eventsare treated as non-commutative events, and couplings of ξ_{14} to ξ_{34} via a component of Higgs scalar $\xi_{ijk}{}^*\xi_{ijk}$ and complex vector field $x^1 + ix^3$ become possible. Quarks $\xi_1\xi_{1234}$ couple to x^1 and $\xi_2\xi_{1234}$ couple to x^2 and transition of ξ_1 to ξ_2 via Higgs scalar $\xi_i{}^*\xi_i$ and complex vector field $x^1 + ix^2$ becomes also possible.

Collider independent $\boldsymbol{t\bar{t}}$ forwardbackward asymmetries for $\boldsymbol{u\bar{u}}$, $\boldsymbol{d\bar{d}}$ $\rightarrow \boldsymbol{t\bar{t}}$ productions are studied in Refs. [50, 51]. Since b and $\boldsymbol{\bar{b}}$ are heavy and have relatively long life time, they can be detected in high energy collider experiments. Using the information of amplitudes of $t \rightarrow \boldsymbol{\bar{b}}Z$ and $\boldsymbol{\bar{t}} \rightarrow b$ $\boldsymbol{\bar{Z}}$,it is possible to study amplitudesof $b\boldsymbol{\bar{b}W\bar{W}}$ $\rightarrow \boldsymbol{t\bar{t}}$, by combining the loop of $\boldsymbol{b\bar{b}}$ Z and $\boldsymbol{\bar{b}}b\boldsymbol{\bar{Z}}$ amplitudes connected by Higgs bosons as shown in Fig.3a, as well as $\boldsymbol{\ell\bar{\ell}}$ Z $\boldsymbol{\bar{Z}} \rightarrow$ $b\boldsymbol{\bar{b}}$ and $\boldsymbol{\ell\bar{\ell}}$ Z and $\boldsymbol{\bar{\ell}}\ell\boldsymbol{\bar{Z}}$ or $q\boldsymbol{\bar{q}}Z$, amplitudes connected by the Higgs boson as shown in Fig.3b, and $\boldsymbol{\ell\bar{\ell}}Z$ or $\boldsymbol{\bar{\ell}}\ell\boldsymbol{\bar{Z}}$ amplitudes connected by the Higgs boson as shown in Fig.3c.

Experimental data of $A_{\text{FB}}(\Delta y)$ in $\boldsymbol{p\bar{p}} \rightarrow \boldsymbol{t\bar{t}}$ +*jets*, indicate that in the region of $\Delta y = y_t - y_{\bar{t}} < 0$, the production is suppressed and in the region of $\Delta y = y_t - y_{\bar{t}} > 0$, the production is enhanced.It means that $\boldsymbol{\bar{t}}$ production through ξ_{1234} propagation is suppressed, and t production through ξ_{123} propagation is enhanced. Recent CMS experimental result of $pp \rightarrow \ell\ell + \boldsymbol{t\bar{t}}$ + *jets* at $\sqrt{\boldsymbol{s}}$ = 8 TeV [35], shows that in *lepton* + *jet* channels, CP violation was not observed and consistent with the standard model, but CP violation in *quark* + *jet* channels are not measured.

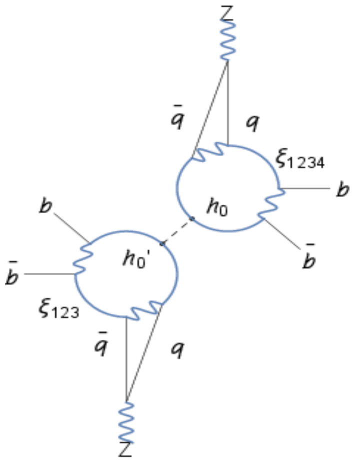

Fig. 3a $b\bar{b} \rightarrow Z(q,\bar{q})\bar{Z}(q,\bar{q}) \rightarrow b\bar{b}$.

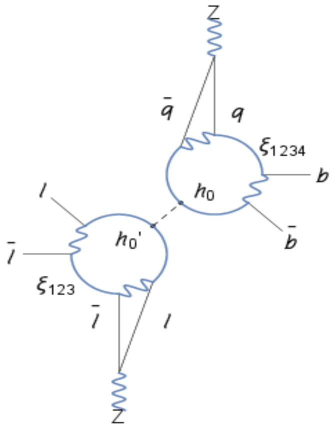

Fig. 3b $b\bar{b} \rightarrow \bar{Z}(q,\bar{q})\, Z(\ell,\bar{\ell}) \rightarrow \ell\bar{\ell}$.

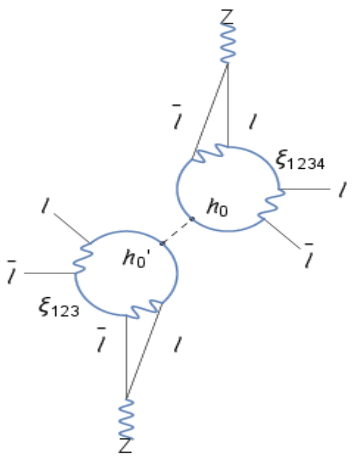

Fig. 3c $\ell\bar{\ell} \rightarrow Z(\ell,\bar{\ell})\bar{Z}(\bar{\ell},\ \ell) \rightarrow \ell\bar{\ell}$.

It means that $\bar{t}$ production through ξ_{1234} propagation is suppressed, and t production through ξ_{123} propagation is enhanced. Recent CMS experimental result of

$$pp \rightarrow \ell\ell + t\bar{t} + jets$$

at $\sqrt{s}$ = 8 TeV [35], shows that in *lepton* + *jet* channels, CP violation was not observed and consistent with the standard model, but CP violation in *quark* + *jet* channels are not measured.

5. Discussion and Conclusions

We showed that $S^3 \times R^4$ and $S^{3\prime} \times R^4$ that appear in the treatment of fermion in T X represented by octonions but not quaternions and adopting non-commutative geometry which allows creation of a phase θ by exchange of generators U and V of a ring A_θ allows a uniform understanding of physics related to time reversal symmetry. Coupling to vector field X is described by $\varphi^T XC\psi$.

In particle physics, Cartan's supersymmetry contains triality symmetry and more general than Atiyah-Witten's supersymmetry [14]. Scalar component of Cartan contains $\xi_0\xi_{1234} \in \Phi$ and $\xi_4\xi_{123} \in \Psi$, and couplings to vector fields X and X' are expressed by $\varphi^T CX\psi + \varphi^T CX'\psi$, which contains $x^1(-\xi_{14}\xi_{123} + \xi_{1234}\xi_1)$ as an example [15].

Memristors presented by Chua [44] create and maintain a safe flow of electrical current across a device, but unlike a resister, it would "remember" charges even when it lost power. Its current creates chaotic behavior [45] and time derivative of charge and magnetic flux are crucial [46]. We found that interference of solutions on $S^3 \times R^4$ feuillet and $S^{3\prime} \times R^4$ feuillet, and modification of Maxwell equation by introducing a magnetic monopole using octonion approach [47] is not necessary. Time Reversal (TR) based Nonlinear Elastic Wave Spectroscopy (NEWS) methods developed by Dos Santos and his group, allows suppressions of noise and enhancement of signals and can be applied to measure local complex damaged systems [25].

Differences of propagation of a spinor ξ_{123} and ξ_{1234} are expected to occur also in chaotic systems, and in high energy collider system in Higgs boson decay into $q\bar{q}$.

To conclude, in order to evade the no go theorem [48], importance of supersymmetry was realized [49]. But it is not evident that super symmetry of Connes based on non-commutative geometry allows to ignore superposition of solutions on different manifolds, and the same problem remains in the Tomita-Takesaki-Connes' theorem [52]. By using the induced representation of Mackey [27], the triality symmetry of Cartan's supersymmetry allows superposition of amplitude with additional phase to the original amplitude. Cartan's theory is based on Steenrod algebra, while that of Connes and collaborators are based on C^* algebra.

After Noether, people speak of "two theorems" depending on whether one accepts distinction of global time and local time or not [2]. We pointed out that algebra depends on the number system on which our theory is constructed, and physical phenomena are understood in different ways. We think the theory of Cartan's supersymmetry which contains octonions of two time components of Minkowskispacetime is a promissing base for constructing a renormalizable field theory including gravitation like Kaluza-Klein theory.

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