

# Analysis of Querétaro-Celaya Highway Applying Queueing Theory

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**Abstract:** Querétaro-Celaya highway is the obligatory road to the center of Mexico; the establishment of new companies in the area has generated the need to study the dynamics of the vehicular flow that transits this route. Given that the flow of traffic on a highway is a stochastic phenomenon, it is necessary to apply tools that take into account the randomness of the system to measure performance. Queueing theory models capture the random nature of the phenomenon and provide direct information about the relationship between the variables of the system. In this work, the flow of vehicles on the Querétaro-Celaya highway is analyzed with a macroscopic approach and using analytical models of queueing theory; it is concluded that currently the flow on this road is non-congested. The method is an alternative for institutions that do not have specialized packages to carry out studies of this nature.

**Key words:** Traffic flow, fundamental diagram, queueing theory, cycle time.

## 1. Introduction

The flow of traffic is a phenomenon that is found in various areas: factories, information systems and transport. In the case of transportation and cities; vehicular and pedestrian traffic is of great interest in the design of facilities such as corridors, streets and roads. Traffic flow on a highway is a stochastic phenomenon: the number of vehicles entering or leaving a road is a parameter that changes every day; this makes it necessary to use tools that take into account the randomness of the long-term operation to measure the performance of a road. There are several simulation packages; however, these have the drawback of requiring a high computational effort, which often translates into the time needed to perform a single simulation [1]. Queueing theory models also capture the random nature of the vehicular flow and directly provide information about the relationship between the variables of the system [2, 3]. This paper

shows the procedure to carry out an analysis of the vehicular traffic flow of Querétaro-Celaya highway; with a macroscopic approach and using M/M/1/K expressions from queueing theory, where the first M means that the distribution of probability of the arrivals are Poisson or Exponential, the second M means that the time of the service is Poisson or Exponential. The number 1 means that there is only one server and the K means the limit of the length of the queue.

No evidence of this approach was found for the road network in Mexico. The contributions of the present work are:

(1) Given that there is no reported evidence of this method in the Mexican Republic; it helps to show an alternative tool for the analysis of the road system in Mexico, and it is aimed at those institutions and entities interested in carrying out a similar study and that do not have access to specialized packages.

(2) Continue to show the queueing theory application to analyze vehicle flow in road networks in this case systems with finite capacity.

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(3) The calculations to obtain the properties of the network are made using an algorithm different from the one used regularly in the literature.

## 2. Previous Work

Queueing theory models have been applied for several years in Traffic Engineering for the analysis of intersections and coordination of traffic lights [2]; as examples are the seminal works of May and Keller [4] and Newell [5]; The most recent implementations are those by CastroGarcía [6], Viti and van Zuylen [7], Comert and Cetiin [8] and Sumalee, Zhong, Pan, and Sze-to [9]. Exist several models of waiting lines, better known as Possion models, among which we can cite models with a single server with infinite capacity (M/M/1):(DG/inf/inf), model with a single server with Limited capacity (M/M/1):(DG/N/inf), where N is the limit of the system, model with several servers with infinite system capacity (M/M/c):(DG/inf/inf), where c means number of the server, model with several servers with limited capacity of the system (M/M/c):(DG/N/inf), among others [10].

In the case of vehicular flow; in the series of papers by Heide-mann [11] the queueing theory models are applied for the calculation of parameters such as traffic density; Jain and MacGregor Smith [2] apply M/G/c/c models to traffic flow networks. Vandaele, van Woensel, and Verbruggen [12] propose a set of expressions for vehicular flow using the equations of the M/M/1, M/G/1 and G/G/1 systems; with these expressions they obtain flow-density, speed-flow and speed-density graphs, they assumed that a freeway has enough capacity for any number of cars. Ortiz-Triviño and Serrano-Rivera [13] develop a simulation model based on open net-works to analyze the transportation system in Bogotá.

Van Woensel and Vandaele [14] apply M/G/1/K model; arguing that roads have capacity for a finite number of cars; they also extend the work to road networks analyzing split, merge and tandem configurations; some examples are shown analyzing

the effect of the service and demand variability to approximate the speed. Osorio and Berliaire [15] propose a model with finite capacity in the queue for road networks with high congestion and traffic lights. Baykal-Gursoy, Xiao and Osbay [16] consider the effect of traffic interruptions in the flow; although the analysis is performed for a single segment and it is assumed that there is no restriction in the queue, so they use the M/M/1 model; Chen and Osorio [17] use network models with M/M/1/K stations to estimate the travel time in a road network.

In Wang, Gong and Wu [18] the vehicular flow of a university in China is analyzed; in Roy, Gupta and De Koster [19] they use the expressions of queueing theory to study the vehicular flow in a maritime terminal. Guerouahane, Aissani, Farhi and Bouallouche-Medjkoune [20] apply the M/G/c/c models for bidirectional flow.

### 2.1 Traffic Flow Modeling

The theory of traffic flow studies the dynamic properties of traffic on roads. There are three approaches: microscopic, macroscopic and mesoscopic. In the macroscopic approach, the vehicular flow is explained through three parameters: flow ( $q$ , veh/hr), density ( $k$ , veh/km, veh/mi) and speed ( $v$ , km/hr, m/seg). The traffic flow is calculated with the following expression:

$$q = kv \quad (2.1.1)$$

It is necessary to determine two parameters to obtain the third. Figs. 1-3 show the way in which each of the parameters of the Eq. (2.1.1) interact through the, speed-density, speed-flow and flow-density graphs.

If a linear behavior is assumed; then the relationship between density and speed is as in Fig. 1. For a density value less than  $k_{cap}$  the speed corresponds to a free flow; when this density value is exceeded, it is considered that the flow is congested.

The relationships between speed and flow along the section are showed in Fig. 2. The segment of the graph

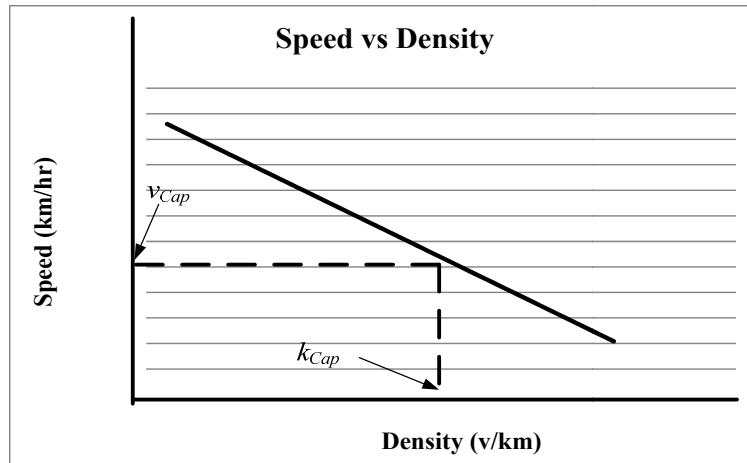


Fig. 1 Speed vs. density.

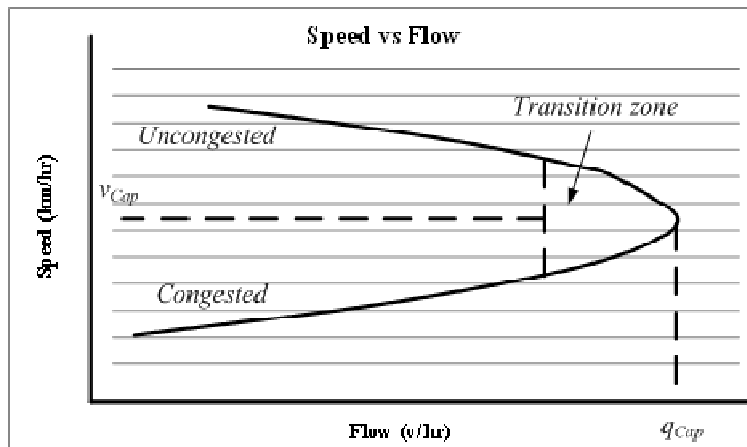


Fig. 2 Speed vs. flow.

above the line corresponds to a non-saturated vehicular flow and where the conditions allow circulation without set-backs; the speed in this segment is close to the maximum speed.

The lower part of the curve represents a situation where the flow is obstructed generating a reduction in the speed of the vehicles; this section corresponds to conditions of saturated flow on the road; finally, there is a transition zone in which; once the point with the problem is left behind; the flow passes to the zone of non-saturated vehicular flow [1, 21].

The Fig. 3 shows the relationship between flow and density;  $q_{cap}$  represents the highest rate of traffic flow that the highway is capable of handling, this is referred as capacity of the roadway; the traffic density that corresponds to this capacity flow is  $k_{cap}$  [21]. At

low density the flow is in the zone of free flow, as the flow increases, the density increases its value; there is a critical value of density ( $k_{cap}$ ) that corresponds to the transition from a zone of free flow to one of congested flow.

### 2.2 Traffic Flow as a Queueing System

A road can be divided into sections or segments of equal length ( $L$ ); in said section the vehicles move using a section of length  $l$ . The maximum density ( $k_{cap}$ ) is the critical value of density: the number of vehicles that can be accommodated in a segment of length  $l$  (then  $k_{cap} = l^{-1}$ ) [12, 14]. Each section has enough space for a certain number of vehicles; as the number of cars circulating increases then the density (number of vehicles per kilometer) also increases; at

the same time, the speed will decrease. The cycle time in the system ( $CTS$ ) is the time a car takes to travel through a segment; and is the sum of the wait time ( $CTq$ ) and the service time, the expression is:

$$CT_s = CT_q + t_s \tag{1}$$

With the information of the demand, the maximum density and speed, then it is feasible to estimate the cycle time within a road segment as well as the relative speed.

### 2.3 Queueing Systems

A queueing system can be described as a process where customers arrive at the facilities to request a service; when the server is empty they are served immediately; otherwise they form in a queue and wait to be served; when the attention process ends, the customer leaves the system. The terms “customer” and

“clients” are used in a general way and do not necessarily imply that they are people (Fig. 4).

In addition to the customers, a queueing system consists of the following elements [22]:

- Servers: Those responsible for serving customers; it can be a single server or several servers placed in parallel.
- The queue: Customers waiting to be served.
- The attention policy: Refers to the order in which clients are selected for the service when a queue has been formed; the most common is First In—First Out.
- Number of Spaces in the Queue: in some processes there is a physical limitation of the allowed number of customers in the row; when all available spaces are occupied; the entrance to more clients is not allowed until a place is available for at least one of them.

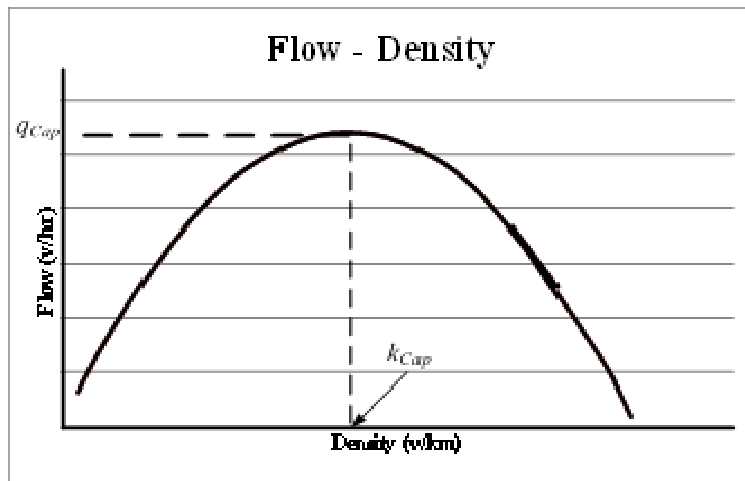


Fig. 3 Flow vs. density.

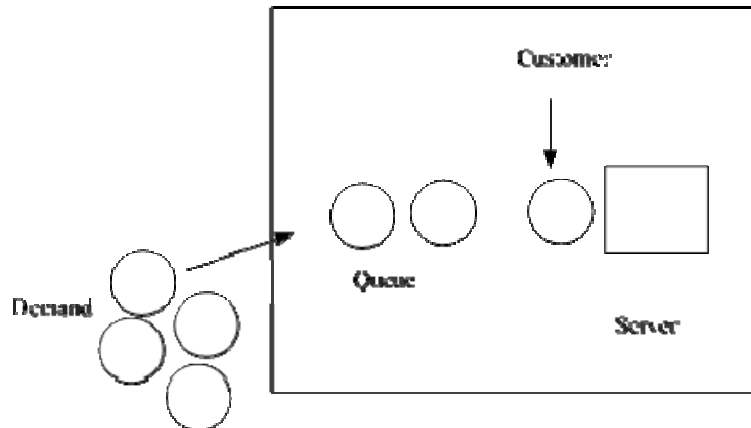


Fig. 4 Queueing system.

2.4 Finite Capacity Single Server Queuing Systems

Suppose that the customers arrive at a system with an average of  $\lambda$  clients per unit of time; the time between arrivals is a random variable that follows an exponential probability distribution. On the other hand, the station has a single server; the service time is a random variable that is distributed exponentially with mean  $t_s$ . Finally; the capacity of the system is finite with  $K$  spaces available for customers. According to the Kendall notation; this class of systems is of the M/M/1/K type; in the Table 1 the expressions to calculate the performance measures are shown [23].

2.5 Takahashi, Miyahara and Hasegawa (1980) Approximation [24]

The approximation method described in the Fig. 5, was proposed to estimate the cycle time and work in process of M/M/1/K stations with network arrangement. In this figure there are 1, 2, ..., n stations in the line; station  $i$  has enough space for the pieces waiting to be processed; there is a buffer in front of the station  $i + 1$  with  $b_j$  available spaces where the pieces wait to be processed; the capacity of each station is  $K_i = b_j + 1$ . Each station processes one piece at a time.

Let the average residence time ( $t_{r,i}$ ) be the average length of time a customer stays at a station. The

residence time is the sum of the average process time required by the customer ( $t_{s,i}$ ) plus the average time in which the next station has all its places occupied. ( $t_{K,i+1}$ ) [25]:

$$t_{r,i} = t_{s,i} + t_{K,i+1} \tag{2}$$

Note in Eq. (2) that if at station  $i + 1$  there is at least one place; then the piece will remain in station  $i$  only the period corresponding to the service; that is  $t_{r,i} = t_{s,i}$ .

From the point of view of station  $i$ , the blocking periods occur randomly. The average time that a station remains blocked is:

$$t_{K,i} = p_{K,i} t_{r,i} \tag{3}$$

In Eq. (3),  $p_{K,i}$  is the probability that all places are occupied [24]. Substituting in Eq. (4):

$$t_{r,i} = t_{s,i} + p_{K,i+1} t_{r,i+1} \tag{4}$$

The average time that station  $i + 1$  has occupied all spaces is  $p_{K,i+1} t_{r,i+1}$ ; if it is a system with M/M/1/K stations; then  $p_{K,i}$  is calculated with the following Eq. (5):

$$p_{K,i} = \frac{\rho_i^{K_i}(1-\rho_i)}{(1-\rho_i^{K_i+1})} \tag{5}$$

Substituting in Eq. (6):

$$t_{r,i} = t_{s,i} + \frac{\rho_{i+1}^{K_{i+1}}(1-\rho_{i+1})}{(1-\rho_{i+1}^{K_{i+1}})} t_{r,i+1} \tag{6}$$

where,  $\rho_{i+1} = \lambda_{e,i} t_{r,i+1}$ . The output flow in M/M/1/K stations is denoted as  $\lambda_{e,i}$ ; in a series system, the output flow of station  $i$  becomes the input stream of station  $i + 1$  [25].

Table 1 Performance measures of M/M/1/K queuing systems.

| Measure                                     | Formula   |
|---|---|
| Utilization/congestion ( $\rho$ )           | $\rho = \lambda t_s$ (2.4.1)  |
| Work in process (WIP <sub>s</sub> )         | $WIP_s = \frac{\rho}{1-\rho} - \left( \frac{\rho^{(K+1)}(K+1)}{1-\rho^{(K+1)}} \right)$ (2.4.2) |
| Blocking probability ( $p_K$ )              | $P_K = \left( \frac{\rho^K(1-\rho)}{1-\rho^{(K+1)}} \right)$ (2.4.3)                            |
| Throughput ( $\lambda_e$ )                  | $\lambda_e = \lambda(1 - p_K)$ (2.4.4)  |
| Cycle time in the system (CT <sub>s</sub> ) | $CT_s = \frac{WIP_s}{\lambda_e}$ (2.4.5)  |

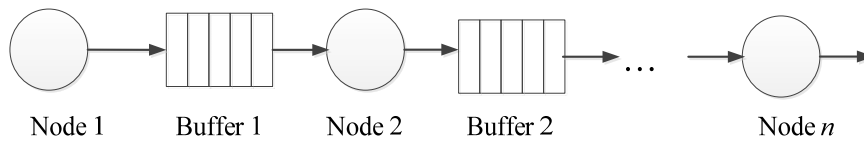


Fig. 5 Queuing series with buffers.

To obtain  $p_{K,i}$ , it is necessary to determine  $\rho_i$  and in turn we must determine  $t_{r,i}$ . In the first iteration it is assumed that  $\lambda_l = \lambda_{e,i}$  for all stations; then  $p_{k,i}$  is calculated; subsequently  $\lambda_{e,i}$  is updated, starting at station  $n$  and proceeding back to station 1; the update process is repeated until the difference in the value of  $t_{r,i}$  in two consecutive iterations is less than a certain threshold. When the update process ends; the cycle time and the work in process are calculated.

### 3. Material and Methods

In this research, the characterization of the Celaya-Querétaro road section was made with two approaches: the first by queuing theory and the second, by simulation.

The following steps were followed:

- (1) Data collection: Lengths of road segments, through vehicles and average speeds.
- (2) With the information collected, the system was characterized by waiting lines using the M/M/1/k model.
- (3) The system was characterized by a simulation model built in the Promodel package.
- (4) Cycle time was calculated using the analytical and simulation model.
- (5) The analytical result was validated by simulation comparing the outputs of the simulation model and the analog model, performing a hypothesis test.
- (6) Important parameters were calculated to measure the level of service of the road section as a child: Speed-Density curve, Speed-Flow curve, Flow density curve, Number of vehicles vs. Speed.

### 4. Results

Investments in the central zone of Mexico are driving the generation of new commercial and distribution routes, which in turn triggers investments in highway infrastructure. In the central zone of Mexico, there is the highway that connects the cities of Querétaro, Celaya, Salamanca, Irapuato and León. It is expected that the arrival of new companies of the automotive industry to the region, will bring with it an increase in the number of vehicles that circulate in the area.

In this work, the analysis is carried out in the Querétaro-Celaya section; since it is the route of entrance towards the center of México. The information was collected through the portal of the Ministry of Communications and Transportation (SCT), to know the information on the characteristics of the demand and distribution of the vehicle fleet that transits through the area; as well as its average speed. This information is the one corresponding to the year 2016 and it is registered by permanent vehicle counting stations throughout the country (Table 2).

The selected stations correspond to the highway 045 from Villa Pueblito (Querétaro) to Celaya. It is a segment of approximately 39.5 kilometers. The information of the SCT provides the "Total Daily Average" (TDA); therefore the information was converted on an hourly basis. The database provides the estimated average speed for each segment of the Queretaro-Celaya highway. From the information of SCT [26], it is obtained that the average inflow is 696 vehicles/hour registered at the entrance of the first

**Table 2 Station, length and average speed [26].**

| Segment  | Length (km) | Average speed (km/hr, from SCT) |
|--|-------------|---------------------------------|
| Exit to Libramiento Celaya East—Villa Apaseo el Grande | 5.2         | 79                              |
| Villa Apaseo el Grande—Villa Apaseo el Alto            | 10.7        | 96                              |
| Villa Apaseo el Alto—Los Ángeles                       | 17.7        | 89                              |
| Los Ángeles—Villa Pueblito                             | 5.9         | 92                              |
| Total length   | 39.5        |                                 |

counting station; this flow will be the average demand for service. It is assumed that the maximum density ( $k_{max}$ ) is 120 vehicles in a segment of 3 km [14]. The highway is 39.5 km long, dividing by the length of each segment  $39.5/3 \text{ km} = 13.16 \approx 13$  M/M/1/K nodes connected in series; it is assumed that the capacity of each node is  $K = 120$  vehicles. The maximum density is  $k_{max} = 120$  vehicles/segment. The maximum allowed speed is 90 km/hour. It is also assumed that:

- (1) The traffic flow has no interruptions.
- (2) There is no difference between lanes.
- (3) Cars are only considered in the flow.
- (4) The number of vehicles entering through node 1 follows a Poisson distribution; on the other hand, the service time in each of the 13 nodes is a random variable that follows an exponential probability distribution.

(5) It is assumed that the flow that enters through node 1 is the same one that leaves the road through node 13.

Then the service time is  $t_s = 1/(120)(90)$ . Takahashi, Miyahara & Hasegawa [24] approximation method was coded in Scilab Language to evaluate the cycle time. It should be noted that a simulation model was built in Promodel to validate the analytical results. Table 3 shows the results obtained by the mathematical model. According to the analytical results the trip lasts 2.12 minutes  $\times$  13 segments = 27.56 minutes.

To validate; the comparison of the analytical results with the exit statistics of the simulation model was made; 13 simulation runs were made; which ensure a

confidence level of 95% and a maximum error of 0.1 units of the travel time of the section analyzed. The average time obtained by the model is 27.53 minutes. Subsequently, these output data were entered into the Minitab Software, to perform the hypothesis test and calculate a confidence inter-val. For this purpose; the following hypothesis was established:

H0:  $\mu = 27.56$  minutes

H1:  $\mu \neq 27.56$  minutes

According to the results; the value of 27.56 minutes obtained analytically is included in the confidence interval, which indicates that the average of the results of the simulation model is the same as that observed in the analytical model (Table 4).

Finally the  $p$  value is 0.493. Once the analytical result was validated; we proceeded to obtain the characteristics of the flow in this road section.

On average there will be around 100.36 vehicles circulating between Querétaro and Celaya. The average speed on the road is calculated:  $39.5 \text{ km}/(27.66 \text{ min}/60 \text{ min}) = 85.68 \text{ km/hr}$ ; applying the Eq. (1), the average density of 8.11 vehicles/km is obtained. The data from the SCT [26] estimate an average speed of 89 km/hr, the difference is 2%. According to the results; it is considered that the level of service of the segment studied is of type B [1]. Based on the speed obtained with the queuing model, the speed-density, flow-density and flow-speed curves are constructed.

#### 4.1 Speed-Density Curve

The cycle time for system input values in the range

**Table 3 Analytical results.**

| Segment | Average cycle time in the system (minutes) | Average#vehicles/segment |
|---------|--|--------------------------|
| 1-13    | 2.12                                       | 7.72                     |

**Table 4 Test of mean.**

T of the one sample: cycle time  
 Test of  $\mu = 27.56$  vs.  $\neq 27.56$

| Variable   | N  | Mean    | Est. Desv. | Standar error of the mean | IC de 95%          | T     | P     |
|------------|----|---------|------------|---------------------------|--------------------|-------|-------|
| Cycle time | 16 | 27.5362 | 0.1350     | 0.0338                    | (27.4643, 27.6082) | -0.70 | 0.493 |

500-2,700 vehicles was obtained; for each scenario, the total distance was divided by the cycle time to obtain the average speed ( $v$ ). The average density for each case was obtained with the following expression [21, 27]:

$$k = k_{max} \left( 1 - \frac{v_N}{v} \right) \quad (7)$$

In this case  $k_{max} = 120$  vehicles; the nominal speed ( $v_N$ ) is that indicated by the traffic regulations: 90 km/hr; in the current conditions; the average density is 8 veh/km, this average density value is found in the area of non-congested vehicular flow. In Fig. 6 obtained with the Eq. (7), it is observed that the critical density is  $kc_{ap} \approx 60$  veh/km.

#### 4.2 Speed-Flow Curve

With the average speed; the flow of vehicles with the following expression was calculated [21, 27]:

$$q = k \left( v - \frac{v^2}{v_N} \right) \quad (8)$$

where  $q$  is the flow,  $k$  is density,  $v$  is speed and  $v_N$  is the nominal speed (90 km/hr). As can be seen in Fig. 7, as the flow approaches 2,700 veh/hr, the average speed begins to decrease; in this part of the graph it is considered that the vehicular flow is non-congested. The transition zone to congested traffic starts when flow is approximately to 2,250 veh/hr; in this phase the speed is around 45 km/hr. In the current conditions

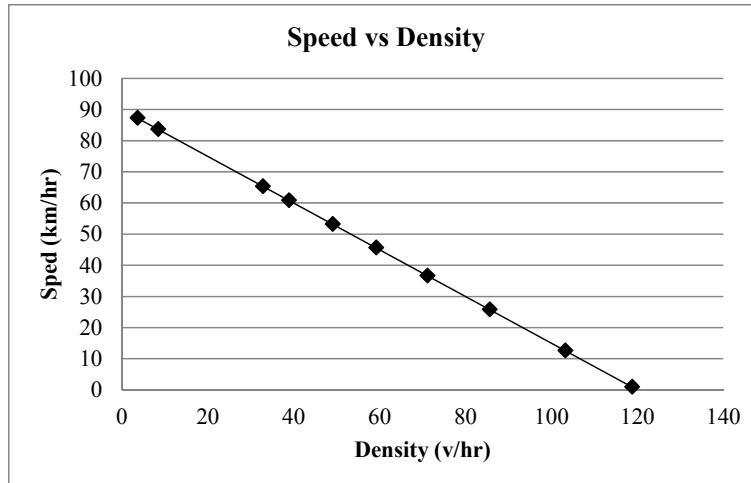


Fig. 6 Speed vs. density.

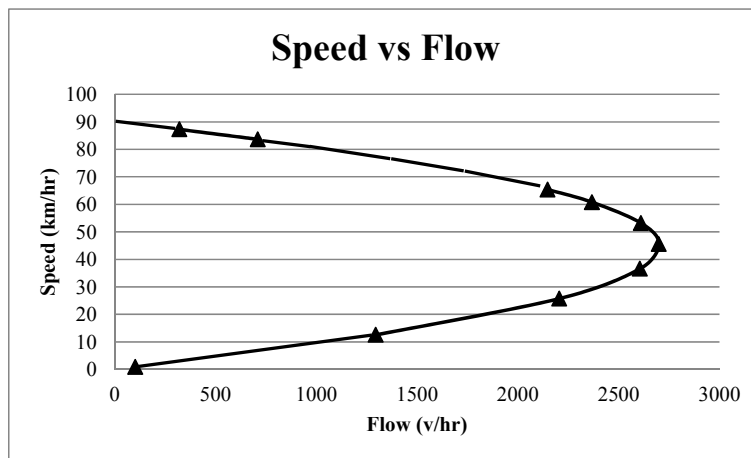


Fig. 7 Speed vs. flow.



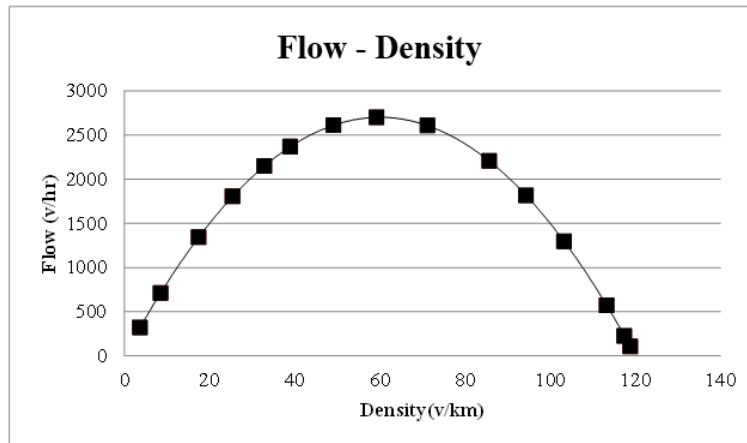


Fig. 8 Flow vs. density.

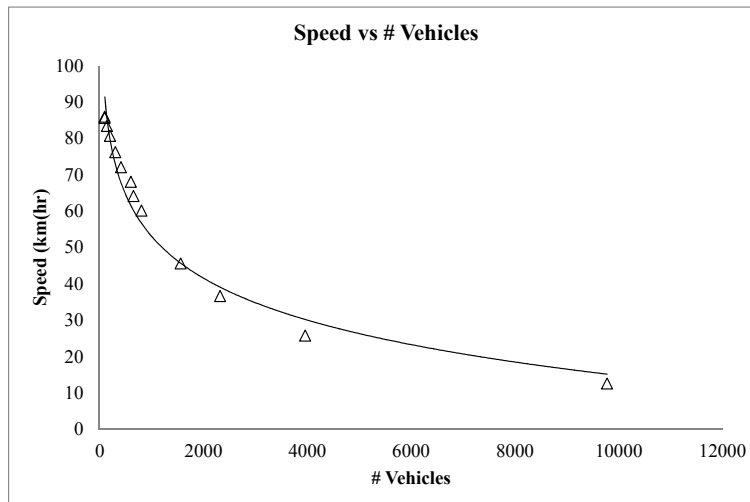


Fig. 9 Speed vs. number of vehicles.

of traffic flow in the area; it is considered that this section on average presents a vehicular flow without disturbances that generate congested flow (it is assumed that only vehicles circulate).

4.3 Flow-Density Curve

Fig. 8 shows that the critical density  $k_{cap} \approx 60$  veh/km corresponds to a flow of approximately to 2,700 veh/hr.

4.4 Number of Vehicles vs. Speed

The behavior of the average number of vehicles on the road with respect to the average speed is showed in the Fig. 9. The average number of vehicles is obtained by multiplying the WIPS by density.

For the case of the base demand; the average

number of vehicles circulating is 100.36 for the Querétaro-Celaya highway. The way the speed degrades as more cars on the road is appreciated. In this case, the transition to congested flow starts when the number of cars is greater than 1,560 vehicles, where the critical speed is approximately 45.5 km/hr.

5. Discussion

With this work, we demonstrate that using an analytical approach like queueing theory or through a simulation approach; it is possible to obtain similar results to carry out studies related to the flow of vehicles.

It is important to carry out this type of analysis to understand the current situation, improve the situation and foresee the future behavior in the face of a growth

in the vehicle fleet that makes use of the roads and in this way maintain the road networks providing an efficient service for the benefit of the users and in the productivity of the region and the country.

## 6. Conclusions

The vehicular flow is a stochastic process which is viable to model applying queueing theory expressions; since the roads have a finite capacity for the number of cars; the M/M/1/K models are very convenient for this system. The Queretaro-Celaya highway is an obligatory route for the flow of goods that are directed towards the center of the country; hence the convenience of obtaining a set of measures that quantify the current conditions of vehicle flow and the capacity to accommodate an increasing demand for vehicles.

A model representing this highway was developed assuming that it behaves as a queueing system with serial arrangement and finite capacity. In what corresponds to the average speed; there is a difference with respect to the SCT database of 2%. From the data of the SCT it was obtained that currently the flow conditions are favorable for this highway. This model assumes that the time of service and the time between arrivals are random events and are distributed exponentially; in other words; in terms of variability, it is assumed that it is moderate.

An improvement for the model is to consider that the service time has an average  $t_s$  and standard deviation; and quantify in this way the effect of variability on the results obtained.

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