

# **Research on Prediction of Ship Manoeuvrability**

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**Abstract:** Based on the manoeuvring MMG (Mathematical Modeling Group) and the Runge-Kutta Method, a mathematical model for simulation of ship manoeuvrability is established. On this basis, a prediction program is compiled on platform Visual Basic 6.0. The turning motion, Zig-zag and crash stopping ability of a container ship are simulated by this program. Compared with the model test results, the error is less than 10%. In view of the admissible accuracy, the program is capable of predicting ship manoeuvrability.

Key words: MMG, maneuverability, prediction, program.

# 1. Introduction

In the ship designing field, ship manoeuvrability was not paid too much attention to because rapidity is the primary consideration. While this neglected performance is considered more and more recently because researchers notice that it is also one of the most important ship hydrodynamic performances, it is very useful to evaluate the navigational safety.

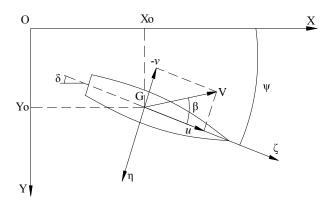
In 2002, IMO Resolution MSC. 137 (76) "Standards for Ship Manoeuvrability" was released [1]. The criteria of the turning ability, initial turning ability, yaw-checking and course-keeping abilities and stopping ability are defined in the standards. All the design, construction, repair and operation of ships are responsible to apply to the standards since 2004.

In order to examine the ship manoeuvrability efficiently and rapidly, free running model test technique is the mainstream method. And so many reputational manoeuvring tanks in the world have implemented countless related tests with the help of the theoretical basis and mature facilities. It is a waste of time, human and financial resources if some ship's manoeuvrability cannot satisfy the standards because there must be excess modification and at least another time test. So it is crucial to predict the ship manoeuvrability before the model test.

Based on the manoeuvring MMG (Mathematical Modeling Group), the Runge-Kutta Method and some other research, a calculator program is proposed in this paper to predict the ship manoeuvrability. In the last, a calculation example of a container ship is used to test the feasibility of the program.

#### 2. Ship Manoeuvring Mathematical Model

#### 2.1 Coordinate System



The coordinate system consists of the space fixed coordinate O-XY and motion coordinate G- $\zeta\eta$ , point G is the intersection of longitudinal section, midship section and the height of centre of gravity.

#### 2.2 Mathematical Model

Based on the MMG and the interaction of ship, propeller and rudder, equations of ship manoeuvring motion are established.

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$$\begin{cases} \left(m + \lambda_{11}\right) \dot{u} - \left(m + \lambda_{22}\right) vr = X_H + X_P + X_R \\ \left(m + \lambda_{22}\right) \dot{v} + \left(m + \lambda_{11}\right) ur = Y_H + Y_P + Y_R \\ \left(I_Z + \lambda_{66}\right) \dot{r} = N_H + N_P + N_R \\ 2\pi \left(I_P + J_P\right) \dot{n}_P = Q_P + Q_E + Q_f \\ r = \dot{\psi} \\ \dot{X}_G = u \cos(\psi) - v \sin(\psi) \\ \dot{Y}_G = u \sin(\psi) + v \cos(\psi) \end{cases}$$

$$(1)$$

where, *m* is ship's mass, *u* is longitudinal speed, *v* is transverse speed, *r* is angular velocity,  $n_P$  is main engine revolution,  $\psi$  is heading angle, *X*, *Y*, *N* are longitudinal force, transverse force and turning moment respectively, *H*, *P* and *R* represent hull, propeller and rudder respectively,  $\lambda_{11}$ ,  $\lambda_{22}$  and  $\lambda_{66}$  are added mass and added inertia,  $I_Z$  is rotary inertia of ship,  $I_P$  and  $J_P$  are rotary inertia and added inertia of propeller and shafting respectively,  $Q_p$  is main engine

Then:

torque, 
$$Q_E$$
 is torque consumed by propeller, and  $Q_f$  is torque consumed by shaft friction.

#### 2.3 Solution of Mathematical Model and Calculation

The equations above can be solved by the Runge-Kutta Method as follows:

$$\begin{cases} \dot{u} = \left[ \left( m + \lambda_{22} \right) vr + X_H + X_P + X_R \right] / \left( m + \lambda_{11} \right) \\ \dot{v} = \left[ - \left( m + \lambda_{11} \right) ur + Y_H + Y_P + Y_R \right] / \left( m + \lambda_{22} \right) \\ \dot{r} = \left( N_H + N_P + N_R \right) / \left( I_Z + \lambda_{66} \right) \\ \dot{n}_P = \left( Q_P + Q_E + Q_f \right) / \left( 2\pi \left( I_P + J_P \right) \right) \end{cases}$$
(2)

The  $\dot{u}$ ,  $\dot{v}$ ,  $\dot{r}$ ,  $\dot{n}_P$  are given by:

$$\begin{cases} \dot{u} = f_{1}(t, u, v, r) \\ \dot{v} = f_{2}(t, u, v, r) \\ \dot{r} = f_{3}(t, u, v, r) \\ \dot{n}_{P} = f_{4}(t, u, v, r) \end{cases}$$
(3)

$$\begin{cases} u_{i+1} = u_i + \frac{\Delta t}{6} \left( K_{11} + 2K_{12} + 2K_{13} + K_{14} \right) \\ v_{i+1} = v_i + \frac{\Delta t}{6} \left( K_{21} + 2K_{22} + 2K_{23} + K_{24} \right) \\ r_{i+1} = r_i + \frac{\Delta t}{6} \left( K_{31} + 2K_{32} + 2K_{33} + K_{34} \right) \\ n_{p_{i+1}} = n_{p_i} + \frac{\Delta t}{6} \left( K_{41} + 2K_{42} + 2K_{43} + K_{44} \right) \end{cases}$$

$$(4)$$

There into:

$$\begin{cases}
K_{j1} = f_{j} \left( u_{i}, v_{i}, r_{i} \right) \\
K_{j2} = f_{j} \left( u_{i} + \frac{\Delta t}{2} K_{j1}, v_{i} + \frac{\Delta t}{2} K_{j1}, r_{i} + \frac{\Delta t}{2} K_{j1} \right) \\
K_{j3} = f_{j} \left( u_{i} + \frac{\Delta t}{2} K_{j2}, v_{i} + \frac{\Delta t}{2} K_{j2}, r_{i} + \frac{\Delta t}{2} K_{j2} \right) \\
K_{j4} = f_{j} \left( u_{i} + \frac{\Delta t}{2} K_{j3}, v_{i} + \frac{\Delta t}{2} K_{j3}, r_{i} + \frac{\Delta t}{2} K_{j3} \right) \\
j = 1, 2, 3, 4
\end{cases}$$
(5)

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The initial condition is that:  $u(0) = V_0$ , v(0) = 0, r(0) = 0,  $n_p(0) = n_{p0}$ , where  $V_0$  is the initial ship speed,  $n_{p0}$  is the main engine revolution. The manoeuvrability of turning motion, Zig-zag and crash stopping can be calculated by iterative solution.

# 3. Hydrodynamic Calculation

#### 3.1 Added Mass and Added Inertia

ZHOU [2] obtained the equations below by regressing the results of the model tests.

$$\begin{vmatrix} \lambda_{11} = m \frac{1}{100} \bigg[ 0.398 + 11.988C_b \bigg( 1 + 3.73 \frac{d}{B} \bigg) - 2.89C_b \frac{L}{B} \bigg( 1 + 1.13 \frac{d}{B} \bigg) + 0.175C_b \bigg( \frac{L}{B} \bigg)^2 \bigg( 1 + 0.541 \frac{d}{B} \bigg) - 1.107 \frac{L}{B} \frac{d}{B} \bigg] \\ \lambda_{22} = m \bigg[ 0.882 - 0.54C_b \bigg( 1 - 1.6 \frac{d}{B} \bigg) - 0.156 \big( 1 - 0.673C_b \big) \frac{L}{B} + 0.826 \frac{d}{B} \frac{L}{B} \bigg( 1 - 0.678 \frac{d}{B} \bigg) - 0.638 \frac{d}{B} \frac{L}{B} \bigg( 1 - 0.669 \frac{d}{B} \bigg) \bigg] \\ \lambda_{66} = m \frac{L^2}{10000} \bigg[ 33 - 76.85C_b \big( 1 - 0.784C_b \big) + 3.43 \frac{L}{B} \big( 1 - 0.63C_b \big) \bigg]^2 \\ I_z = \frac{1}{16} m L^2 \\ I_p = \frac{W}{g} \bigg( \frac{kD_p}{2} \bigg)^2 \\ J_p = (0.2 \sim 0.3) I_p \end{aligned}$$

$$(6)$$

where, *L* is ship length, *B* is ship breadth, *d* is draught,  $C_b$  is block coefficient, *W* is propeller weight,  $D_P$  is propeller diameter, *k* is 0.42 when propeller type is integral, 0.39 when propeller type is combined.

#### 3.2 Resistance

Ship resistance can be provided by the model test, the formula could be written as:

$$X_{H} = a_{0} + a_{1}Fr + a_{2}Fr^{2} + a_{3}Fr^{3}$$
<sup>(7)</sup>

where, Fr is Froude number;  $a_0$ - $a_3$  are coefficients.

## 3.3 Longitudinal Force and Moment

$$\begin{cases} Y_{H} = \frac{1}{2} \rho L dV^{2} \left( Y_{v}' v + Y_{r}' r + Y_{|v|v}' |v| v + Y_{|v|r}' |v| r + Y_{|r|r}' |r| r \right) \\ N_{H} = \frac{1}{2} \rho L^{2} dV^{2} \left( N_{v}' v + N_{r}' r + N_{|r|r}' |r| r + N_{vvr}' v^{2} r + N_{rrv}' r^{2} v \right) \\ v = \frac{v(i)}{V} \\ r = \frac{r(i)L}{V} \end{cases}$$
(8)

where, Y' and N' are hydrodynamic derivatives of ship manoeuvring motion [2].

#### 3.4 Propeller Thrust

The transverse force  $Y_P$  generated by propeller and moment  $N_P$  are often negligible as they are usually small amount relative to longitudinal force  $X_P$ .

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$$\begin{cases} Y_{p} = 0, N_{p} = 0 \\ X_{p} = (1 - t_{p}) \rho n_{p}^{2} D_{p}^{4} K_{T} (J_{p}) \\ Q_{p} = -\rho n_{p} (i)^{2} D_{p}^{5} K_{Q} (J_{p}) \\ K_{T} (J_{p}) = b_{0} + b_{1} J_{p} + b_{2} J_{p} + b_{3} J_{p} \\ K_{Q} (J_{p}) = c_{0} + c_{1} J_{p} + c_{2} J_{p} + c_{3} J_{p} \end{cases}$$

$$(9)$$

where  $K_{\rm T}$  is propeller thrust coefficient,  $K_{\rm Q}$  is torque coefficient,  $b_0$ - $b_3$  and  $c_0$ - $c_3$  are coefficients.

# 3.5 Rudder Force and Moment

$$\begin{cases} F_N = \frac{1}{2} \rho C_N A_R V_R^2 \\ X_R = -(1 - t_R) F_N \sin(\delta) \\ Y_R = -(1 + a_H) F_N \cos(\delta) \\ N_R = -(1 + a_H x_R) F_N \cos(\delta) \end{cases}$$
(10)

where,  $C_N$  is rudder normal force coefficient,  $A_R$  is rudder area,  $t_R$  is reduction factor of rudder force,  $a_H$  is correction factor of the transverse force acting on ship generated by steering,  $x_R$  is the distance between the centre of the transverse force and centre of ship gravity,  $\delta$  is rudder angle.

# 4. Calculation of Crash Stopping

Crash stopping ability is one of the criteria in IMO standards. The ship speed slows down as the propeller rotating in opposite direction. The flow in wake field is complicated and the rudder has little influence in the process [3]. The mathematical model in crash stopping motion can be written as:

$$\begin{cases} \left(m + \lambda_{11}\right) \dot{u} - \left(m + \lambda_{22}\right) vr = (1 - t_{P})T + X_{H} \\ \left(m + \lambda_{22}\right) \dot{v} + \left(m + \lambda_{11}\right) ur = Y_{H} + Y_{P} \\ \left(I_{Z} + \lambda_{66}\right) \dot{r} = N_{H} + N_{P} \\ 2\pi \left(I_{P} + J_{P}\right) \dot{n}_{P} = Q_{P} + Q_{E} + Q_{f} \\ Y_{P} = Y_{P}' \left\{ \frac{1}{2} \rho \frac{1}{4} \pi D_{P} \left[ \left(1 - w_{P}\right)^{2} u^{2} + \left(0.7 \pi n D_{P}\right)^{2} \right] \right\} \end{cases}$$

$$(11)$$

where,  $\omega_p$  is the wake fraction,  $Y_P$ ,  $N_P$  are transverse force and moment as the propeller rotating oppositely respectively. The dimensionless quantities in the formula are given by the expression:

$$\begin{cases} Y'_{P} = k_{0} + k_{1}J_{P} + k_{2}J_{P}^{2} + k_{3}\beta \\ N'_{P} = m_{0} + m_{1}J_{P} + m_{2}J_{P}^{2} + m_{3}\beta \end{cases}$$
(12)

where, the coefficients  $k_0$ - $k_3$ ,  $m_0$ - $m_3$  and  $\beta$  are mainly related to ship parameters [3].

# 5. Manoeuvrability Prediction of a Container Ship

The manoeuvring performance of a container ship in scantling and ballast draft is calculated by the program compiled on platform Visual Basic 6.0 by the formulas above. Ship dimensions and coefficients are listed in Table 1. The calculation value is contrasted by the model test results.

# 5.1 Turning Test

The comparison result of calculation and test of

#### Table 1 Ship dimensions and coefficients.

turning ability is shown in Table 2. It is seen that the error is within 10%. Besides, both the prediction or the test value meets the IMO requirements.

### 5.2 Zig-zag Test

The computing and the model test data of Zig-zag performance are well satisfied with the IMO standards as listed in Table 3. The results indicate that the difference between the two methods is less than 10%.

#### 5.3 Crash Stopping Test

The error of crash stopping ability between the program

Particular		Symbol	Conditions		TT '4
			Scantling	Ballast	Unit
Length between perpendiculars		$L_{PP}$	286.0		m
Breadth		В	48.2		m
Draught moulded on FP		Tf	14.8	4.6	m
Draught moulded on AP		Та	14.8	10.3	m
Block coefficient		Cb	0.703	0.615	-
Displacement		$\bigtriangledown$	143,390	63,172	m <sup>3</sup>
Rudder area		A <sub>R</sub>	67.1		$m^2$
Aspect ratio		λ	1.41		-
Propeller diameter		$D_P$	9.5		m
Pitch ratio		Р	9.9047		m
Number of propeller		-	1		-
Number of blades		-	6		-
Speed		$\mathbf{V}_0$	22.2	23.9	kn
Table 2 Turni	ing tests.				
Conditions	Item	Experiment	Calculation	Error/%	IMO
Scantling	D/L	2.245	2.153	-4.1	$\frac{D_T/L \le 5.0}{A_d/L \le 4.5}$
	$D_T/L$	2.477	2.357	-4.9	
	$A_d/L$	2.967	2.994	0.9	
Ballast	D/L	4.008	3.958	-1.2	
	$D_T/L$	4.303	4.035	-6.2	
	$A_d/L$	3.617	3.334	-7.8	
Table 3 Zig-za	ag tests				
Conditions	Item	Experiment	Calculation	Error/%	IMO
Scantling	Az'	1.415	1.382	-2.3	$\begin{array}{l} Az^{\prime} \leq 2.5 \\ \theta_{ov10-1} \leq 17.5 \\ \theta_{ov10-2} \leq 36.3 \\ \theta_{ov20-1} \leq 25.0 \end{array}$
	$\theta_{ov10-1}$	16.6	16.3	-1.8	
	$\theta_{ov10-2}$	36.1	35.3	-2.2	
	θ <sub>ov20-1</sub>	20.2	22.0	8.9	
Ballast	Az'	1.546	1.521	-1.6	Az' ≤ 2.5
	$\theta_{ov10-1}$	3.5	3.7	5.7	$\begin{array}{l} \theta_{\rm ov10-1} \leq 2.5 \\ \theta_{\rm ov10-1} \leq 16.6 \\ \theta_{\rm ov10-2} \leq 34.9 \\ \theta_{\rm ov20-1} \leq 25.0 \end{array}$
	θ <sub>ov10-2</sub>	4.1	4.4	7.3	
	$\theta_{ov20-1}$	8.7	9.0	3.4	

Conditions	Item	Experiment	Calculation	Error/%	IMO
Scantling	S/L	5.776	5.420	-6.2	$S/L \leq 15$
Ballast		6.028	6.500	7.8	

Table 4Crash stopping tests.

prediction and the model test results is less than 10% which can been seen in Table 4.

# 6. Conclusions

A program used to predict the ship manoeuvrability is compiled on the foundation of the MMG and the Runge-Kutta Method. The calculation data of the turning motion, Zig-zag and crash stopping ability of a container ship are contrasted with model test value and the result demonstrates that the error is less than 10%. This manoeuvring performance calculation program can be very good at project applications.

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