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Non-regular Employment of Women, Fertility Rate, and Economic Growth

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This study analyzes the interaction between non-regular employment of women and economic growth patterns by an overlapping-generations model. Declining Birthrate White Paper-Cabinet Office (2013) shows the ideal number of children per household is 2.42 compared to the current number of 2.07, which is the lowest so far in Japan. The main reason households do not have the ideal number of children is "the costs burden of childcare and education" and the ratio amounts to 60.4%. In recent years in Japan, households in which both the husband and the wife work are increasing, whereas those in which only the husband works are decreasing. Additionally, although women have same educational background and abilities as men, most women become non-regular employees after marriage and childbirth, which reduces household income. In such a situation, raising the rate of pension insurance will be a big burden for the household and the declining birthrate may be caused by high levels of educational expenditure and pension insurance. The Japanese government has discussed raising the wages of non-regular employees. This paper finds that a rise in the wage rate of non-regular employment is needed under the public pension policy that raises the rate of pension insurance, and it must be at an adequate level. That is, there is a high risk that this policy will have a negative effect on Japan's economic growth if an adequate level is not achieved.

Keywords: overlapping-generations, employment of women, fertility rate, public pension policy, human capital, economic growth

Introduction

This study is motivated by the declining birthrate, aging population, and rising rate of pension insurance in Japan. Declining Birthrate White Paper-Cabinet Office (2013) shows the ideal number of children per household is 2.42 compared to the current number of 2.07, which is the lowest so far. The main reason why households do not have the ideal number of children is "the costs burden of childcare and education" and the ratio amounts to 60.4%.

In recent years in Japan, households in which both the husband and the wife work are increasing, whereas those in which only the husband works are decreasing.

Additionally, although women have same educational background and abilities as men, most women become non-regular employees after marriage and childbirth, which reduces household income. In such a situation, raising the rate of pension insurance will be a big burden for the household and the declining birthrate

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may be caused by high levels of educational expenditure and pension insurance. Therefore, the Japanese government has discussed raising the wages of non-regular employees.

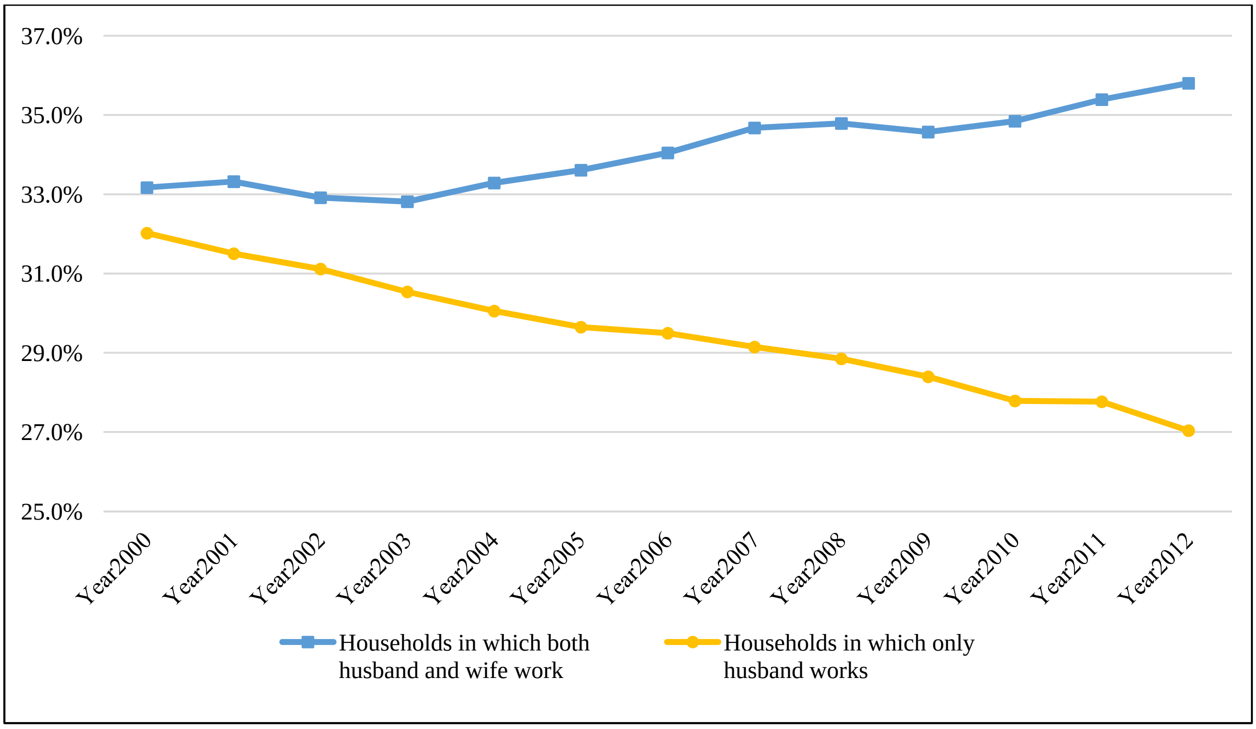

Figure 1. Households in which both husband and wife work and those in which only the husband works. Source: Statistic Bureau, Ministry of Internal Affairs and Communications in Japanese.

This paper analyzes the interaction between the non-regular employment of women and economic growth patterns and finds that the rise of the wage rate of non-regular employees has a positive effect on both the fertility rate and nationwide human capital accumulation under the public pension policy, if the level is adequate.

Literature Review

This study analyzes the interaction between the non-regular employment of women and economic growth patterns using the overlapping-generations model, mainly based on Galor and Weil (1996), Van Groezen, Leers, and Mejidam (2003), and Cardak (2004). Galor and Weil (1996) consider two types of individuals, men and women, and the influence of the labor gender gap on fertility and economic growth patterns. Van Groezen, Leers, and Mejidam (2003) analyze the effect of endogenous fertility on pension finance[1]. They consider a small open economy where the individual's labor income level is equal to the wage rate. Moreover, they assume that the individual's utility level is determined only by his/her own consumption levels and the number of his/her children. However, this paper considers human capital accumulation and assumes that an individual's income level is determined not only by wage rate but also by human capital level, based on Galor and Tsiddon (1996; 1997). Galor and Tsiddon (1996; 1997) also consider a small open economy, as do Van Groezen, Leers, and Mejidam (2003), and analyze the interaction between human capital accumulation and economic growth

[1] Groezen, Leers, and Mejidam (2003) analyze the effect of endogenous fertility on pension finance under a pay-as-you-go pension system, which is the same as Nishimura and Zhang (1992), Peters (1995), and Kato (1999).

patterns. They assume that the human capital level is determined by parental human capital levels and educational expenditure, but their assumption that the educational expenditure is financed by unrestricted international lending is not realistic. Moreover, in Galor and Tsiddon (1996; 1997) and Van Groezen, Leers, and Mejidam (2003), they assume that there is only one type of individual and do not consider the influence of the labor gender gap.

This paper considers two types of individuals, men and women, based on Galor and Weil (1996), and assumes that household utility level is influenced not only by their own consumption levels and number of children but also by educational expenditure on children and leisure, based on Cardak (2004). The household assigns its leisure to childcare. An individual's human capital level is determined by parental human capital levels, educational expenditure by his/her parents, and parental childcare time. This paper also assumes that men and women are in regular and non-regular employment, respectively, and although a man can not determine the childcare time by himself, a woman can determine it by herself. Moreover, this paper considers the effect of a rise in the wage rate of non-regular employees under a public pension policy that raises the pension insurance rate on fertility rate, human capital accumulation, and economic growth patterns. This paper assumes that although men are imposed an annuity, women are not.

The second section surveys the basic model and derives the optimal number of children and the human capital production function. The third section analyzes the effects of a rise of the wage rate of non-regular employees and the public pension policy to raise the pension insurance rate on fertility rate, human capital accumulation, and economic growth patterns.

The Basic Model

Consider a small open overlapping-generations economy that operates in a perfectly competitive world in which economic activity extends over an infinite discrete time. Individuals of generation t live three periods: t, $t+1$, and $t+2$. This paper assumes that all men and women marry and their population sizes are the same to simplify the analysis. Moreover, the children of individuals in one generation are born in the second period.

The Goods Market

Production occurs within a period according to a constant-returns-to-scale. The output produced at time t, Y_t is determined by equation (1):

$$Y_t = F\left(K_t, \{(1-l)+(1-l_t)\}\frac{H_t}{2}\right) = F\left(K_t \frac{2-l-l_t}{2} H_t\right) \tag{1}$$

where K_t and H_t are the quantities of capital and efficient labor employed in the production at time t, respectively, l is the leisure that the government determines in a period, and l_t is the leisure that individuals of generation $t-1$ determine at time t.

The output per producer at time t, $f(k_t)$ is determined by equation (2):

$$f(k_t) \equiv \frac{Y_t}{H_t};\ k_t \equiv \frac{K_t}{H_t} \tag{2}$$

where $f(k_t)$ is strictly monotonic increasing and strictly concave, satisfying the neoclassical boundary conditions. Producers operate in a perfectly competitive environment. Given, the wage rate and the rate of return to capital at time t, r_t, w_t, respectively.

Producer's profit at time t, π_t is determined by equation (3):

$$\pi_t = H_t f(k_t) - \frac{(1-l)+(1-l_t)\lambda}{2} w_t H_t - r_t H_t k_t = H_t f(k_t) - \frac{1+\lambda-l-\lambda l_t}{2} w_t H_t - r_t H_t k_t \tag{3}$$

where λ is the wage rate of non-regular (women) employees for the same working time and human capital level as regular (men) employees that satisfies $0<\lambda<1$. Producers choose the level of k_t so as to maximize π_t:

$$f'(k_t) = r_t \tag{4}$$

$$\frac{1+\lambda-l-\lambda l_t}{2} w_t = f(k_t) - f'(k_t)\cdot k_t \tag{5}$$

Suppose the rate of return is stationary at a level $\bar{r}$. From equation (4), k_t is also stationary at a level $f(\bar{k}) \equiv \bar{k}$. From equation (5), the wage rate is determined by equation (6):

$$\frac{1+\lambda-l-\lambda l_t}{2} w_t = \frac{1+\lambda-l-\lambda l_t}{2} \bar{w} \tag{6}$$

Human Capital Accumulation

An individual of generation t is born to parents with h_t units of human capital. A household F of generation $t-1$ invests in their children e_t^F units of educational expenditure and $l+l_t$ units of their endowment toward childcare. An individual of generation t acquires h_{t+1} units of human capital:

$$h_{t+1} = \left(\frac{l+l_t}{n_t^F}\right)^{\beta} \left(\frac{e_t^F}{n_t^F}\right)^{\gamma} (h_t)^{\delta}\ ;\ \beta,\gamma,\delta \in (0,1), \beta+\gamma+\delta = 1, \beta<\delta<1-\gamma \tag{7}$$

where n_t^F is the number of children in the household F of generation $t-1$ at time t.

This paper assumes that the population sizes of men and women are the same, and the efficient labor employed in the production at time $t+1$ is determined by equation (8):

$$H_{t+1} = \frac{\prod_{j=0}^{t} n_j^F L_0 h_{t+1}}{2} = \frac{(n_0^F \times n_1^F \times n_2^F \times \cdots n_t^F) L_0 h_{t+1}}{2} \tag{8}$$

where $L_0 (>0)$ is the initial population size of the whole country.

Utility Maximization of Households

Individuals of generation t earn labor income at time $t+1$. This paper considers two groups of individuals: men and women. Their labor incomes are determined as in equation (9):

$$\begin{cases} (1-l) w_{t+1} h_{t+1} = (1-l)\bar{w} h_{t+1} & \cdots \text{regular employment}(Men) \\ (1-l_{t+1})\lambda w_{t+1} h_{t+1} = (1-l_{t+1})\lambda \bar{w} h_{t+1} & \cdots \text{non-regular employment}(Women) \end{cases} \tag{9}$$

Moreover, this paper assumes that men and women are regular and non-regular employees, respectively, and all men and women marry to simplify the analysis. The income of household F of generation t at time $t+1$, I_{t+1}^{F} is determined by equation (10):

$$I_{t+1}^{F} = \{(1-l)+(1-l_{t+1})\lambda\}\overline{w}h_{t+1} \tag{10}$$

The consumption of household F of generation t at time $t+1$, $c_{t+1}^{t,F}$ is determined by equation (11):

$$c_{t+1}^{t,F} = \{(1-\rho)(1-l)+(1-l_{t+1})\lambda\}\overline{w}h_{t+1} - e_{t+1}^{F} - s_{t+1}^{F} \equiv I_{t+1}^{D,F} - e_{t+1}^{F} - s_{t+1}^{F} \tag{11}$$

where $\rho(0<\rho<1)$ is the pension insurance rate that is determined by the government in a period, $I_{t+1}^{D,F}$, e_{t+1}^{F}, and s_{t+1}^{F} are the disposable income, educational expenditure, and savings of household F of generation t at time $t+1$. This paper assumes that individuals with regular employment are imposed pension insurance while non-regular employees are not.

The consumption of household F of generation t at time $t+2$, $c_{t+2}^{t,F}$ is determined by equation (12):

$$c_{t+2}^{t,F} = (1+\overline{r})s_{t+1}^{F} + \frac{\rho(1-l)\overline{w}\prod_{j=0}^{t+1} n_j^F L_0 h_{t+2}}{2\prod_{j=0}^{t} n_j^F L_0} = (1+\overline{r})s_{t+1}^{F} + \frac{\rho(1-l)\overline{w}n_{t+1}^{F}h_{t+2}}{2} \tag{12}$$

where n_{t+1}^{F} is the number of children in the household F of generation t at time $t+1$ and h_{t+2} is the human capital level of individuals of generation $t+1$ at time $t+2$.

Household F of generation t chooses $c_{t+1}^{t,F}$, e_{t+1}^{F}, n_{t+1}^{F}, s_{t+1}^{F}, l_{t+1}, and $c_{t+2}^{t,F}$ so as to maximize their utility in all the three periods, U^F. In Kato (1999) and Van Groezen, Leers, and Mejidam (2003), there is no intergenerational altruism. However, this paper introduces educational expenditure and childcare time and consider the case where there is intergenerational altruism.

$$\underset{\substack{c_{t+1}^{t,F}, e_{t+1}^{F}, n_{t+1}^{F}, s_{t+1}^{F},\\ l_{t+1}, c_{t+2}^{t,F}}}{Maximize}\ U^F = \alpha_1 \log\frac{l+l_{t+1}}{n_{t+1}^{F}} + \alpha_2 \log c_{t+1}^{t,F} + \alpha_3 \log n_{t+1}^{F} + \alpha \log\frac{e_{t+1}^{F}}{n_{t+1}^{F}} + \alpha_5 \log c_{t+2}^{t,F}$$

$$;\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5 \in (0,1),\ \alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5 = 1$$

$$subject\ \ to\quad c_{t+1}^{t,F} = \{(1-\rho)(1-l)+(1-l_{t+1})\lambda\}\overline{w}h_{t+1} - e_{t+1}^{F} - s_{t+1}^{F},$$

$$c_{t+2}^{t,F} = (1+\overline{r})s_{t+1}^{F} + \frac{\rho(1-l)\overline{w}n_{t+1}^{F}h_{t+2}}{2}$$

The optimal saving s_{t+1}, number of children n_{t+1}, consumption c_{t+1}^{t}, and educational expenditure e_{t+1} of household F of generation t at time $t+1$, and consumption c_{t+2}^{t} of generation t at time $t+2$ are derived as equations (13), (14), (15), (16), and (17)[2]:

$$s_{t+1} = \frac{(\alpha_3+\alpha_5-\alpha_1-\alpha_4)I_{t+1}^{D,F}}{\alpha_2+\alpha_3+\alpha_5-\alpha_1} \tag{13}$$

[2] Equations (13), (14), (15), (16), and (17) are proved in APPENDIX A.

$$n_{t+1} = \frac{2(\alpha_1 + \alpha_4 - \alpha_3)(1+\bar{r})I_{t+1}^{D,F}}{(\alpha_2 + \alpha_3 + \alpha_5 - \alpha_1)\rho(1-l)\overline{w}h_{t+2}} \tag{14}$$

$$c_{t+1}^{t} = \frac{\alpha_2 I_{t+1}^{D,F}}{\alpha_2 + \alpha_3 + \alpha_5 - \alpha_1} \tag{15}$$

$$e_{t+1} = \frac{\alpha_4 I_{t+1}^{D,F}}{\alpha_2 + \alpha_3 + \alpha_5 - \alpha_1} \tag{16}$$

$$c_{t+2}^{t} = \frac{\alpha_5 (1+\bar{r}) I_{t+1}^{D,F}}{\alpha_2 + \alpha_3 + \alpha_5 - \alpha_1} \tag{17}$$

This paper assumes $\alpha_2 + \alpha_3 + \alpha_5 > \alpha_1$ and $\alpha_3 + \alpha_5 > \alpha_1 + \alpha_4$. The optimal leisure of women of generation t at time $t+1$, l_{t+1} is derived as equation (18)[3]:

$$l_{t+1} = \frac{\alpha_1 \{(1-\rho)(1-l) + \lambda\} - (\alpha_2 + \alpha_3 + \alpha_5 - \alpha_1) l\lambda}{(\alpha_2 + \alpha_3 + \alpha_5)\lambda} \tag{18}$$

From equations (14), (16), and (18), the optimal number of children n_t and educational expenditure e_t of household F of generation $t-1$ at time t, and the optimal leisure l_t of women of generation $t-1$ at time t are derived as equations (19), (20), and (21):

$$n_t = \frac{2(\alpha_1 + \alpha_4 - \alpha_3)(1+\bar{r})I_t^{D,F}}{(\alpha_2 + \alpha_3 + \alpha_5 - \alpha_1)\rho(1-l)\overline{w}h_{t+1}} \tag{19}$$

$$e_t = \frac{\alpha_4 I_t^{D,F}}{\alpha_2 + \alpha_3 + \alpha_5 - \alpha_1} \tag{20}$$

$$l_t = \frac{\alpha_1 \{(1-\rho)(1-l) + \lambda\} - (\alpha_2 + \alpha_3 + \alpha_5 - \alpha_1) l\lambda}{(\alpha_2 + \alpha_3 + \alpha_5)\lambda} \tag{21}$$

In equations (19) and (20), $I_t^{D,F}$ is the disposable income of household F of generation $t-1$ at time t. Incorporating (19), (20), and (21) into (7), human capital production function is derived as equation (22):

$$h_{t+1} = \left\{\frac{\alpha_1 \rho (1-l)}{2(\alpha_1 + \alpha_4 - \alpha_3)(1+\bar{r})\lambda}\right\}^{\frac{\beta}{1-\beta-\gamma}} \left\{\frac{\alpha_4 \rho (1-l)\overline{w}}{2(\alpha_1 + \alpha_4 - \alpha_3)(1+\bar{r})}\right\}^{\frac{\gamma}{1-\beta-\gamma}} (h_t)^{\frac{\delta-\beta}{1-\beta-\gamma}} \tag{22}$$

From equation (22), the human capital level of steady state equilibrium h_s^* is derived as equation (23):

$$h_s^* = \left\{\frac{\alpha_1 \rho (1-l)}{2(\alpha_1 + \alpha_4 - \alpha_3)(1+\bar{r})\lambda}\right\}^{\frac{\beta}{1-\gamma-\delta}} \left\{\frac{\alpha_4 \rho (1-l)\overline{w}}{2(\alpha_1 + \alpha_4 - \alpha_3)(1+\bar{r})}\right\}^{\frac{\gamma}{1-\gamma-\delta}} \tag{23}$$

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3 Equation (18) is proved in APPENDIX B.

In equation (22), this paper assumes that $\beta < \delta < 1-\gamma$, and human capital levels of all individuals converge to h_s^*. Moreover, incorporating equation (21) and (23) into (19), the optimal number of children n_t of household F of generation $t-1$ is finally derived as equation (24):

$$n_t = \frac{2(\alpha_1 + \alpha_4 - \alpha_3)(1+\bar{r})\{(1-\rho)(1-l)+(1+l)\lambda\}}{(\alpha_2 + \alpha_3 + \alpha_5)\rho(1-l)} \tag{24}$$

Public Pension Policy, Fertility Rate, and Economic Growth

This section analyzes the effect of the public pension policy to raise the rate of pension insurance on fertility rate, human capital accumulation, and economic growth patterns. This paper assumes that the rate of pension insurance ρ rises to ρ' due to government intervention. The economy in this paper is small open and the wage rate is stationary. That is, the economic growth pattern is determined only by the efficient labor employed in production. This paper assumes that $\rho < \rho' < 1$. From equation (23), then,

$$\begin{aligned} h_s^* &= \left\{\frac{\alpha_1 \rho (1-l)}{2(\alpha_1 + \alpha_4 - \alpha_3)(1+\bar{r})\lambda}\right\}^{\frac{\beta}{1-\gamma-\delta}} \left\{\frac{\alpha_4 \rho (1-l)\overline{w}}{2(\alpha_1 + \alpha_4 - \alpha_3)(1+\bar{r})}\right\}^{\frac{\gamma}{1-\gamma-\delta}} \\ &< \left\{\frac{\alpha_1 \rho' (1-l)}{2(\alpha_1 + \alpha_4 - \alpha_3)(1+\bar{r})\lambda}\right\}^{\frac{\beta}{1-\gamma-\delta}} \left\{\frac{\alpha_4 \rho' (1-l)\overline{w}}{2(\alpha_1 + \alpha_4 - \alpha_3)(1+\bar{r})}\right\}^{\frac{\gamma}{1-\gamma-\delta}} \end{aligned} \tag{25}$$

From equation (25), this policy has positive effect on human capital accumulation, whereas, from equation (24), this policy decreases the number of children as shown below:

$$\begin{aligned} n_t &= \frac{2(\alpha_1 + \alpha_4 - \alpha_3)(1+\bar{r})\{(1-\rho)(1-l)+(1+l)\lambda\}}{(\alpha_2 + \alpha_3 + \alpha_5)\rho(1-l)} \\ &> \frac{2(\alpha_1 + \alpha_4 - \alpha_3)(1+\bar{r})\{(1-\rho')(1-l)+(1+l)\lambda\}}{(\alpha_2 + \alpha_3 + \alpha_5)\rho'(1-l)} \end{aligned} \tag{26}$$

From equations (8), (25), and (26), there is no assurance that the rise of the rate of pension insurance has a positive effect on the efficient labor employed in production.

The fall of fertility rate has a negative effect on efficient labor employed in production, whereas promoting human capital accumulation has a positive effect on efficient labor employed in the production. That is, there is no guarantee that the policy to raise the rate of pension insurance has a positive effect on economic growth of the whole country.

Moreover, based on the above analysis, this paper considers the effect of the rise of the wage rate of non-regular employees by the government for the same working time and human capital level as that of regular employees. This paper assumes that λ rises to λ'. From equation (24), the rise of λ increases the number of children; however, from equation (23), the rise of λ has a negative effect on human capital accumulation.

When λ' satisfies the following condition, the number of children increases under the public pension policy to raise the rate of pension insurance:

$$\frac{\rho'}{\rho} < \frac{(1-\rho')(1-l)+(1+l)\lambda'}{(1-\rho)(1-l)+(1+l)\lambda} \;\Rightarrow\; \lambda' > \frac{(1-l)(\rho'-\rho)+(1+l)\lambda\rho'}{\rho(1+l)} \tag{27}$$

Moreover, when λ' satisfies the following condition, the public pension policy to raise the rate of pension insurance has a positive effect on human capital accumulation:

$$\left(\frac{\rho'}{\rho}\right)^{\beta+\gamma} > \left(\frac{\lambda'}{\lambda}\right)^{\beta} \Rightarrow \lambda' < \left(\frac{\rho'}{\rho}\right)^{\frac{\beta+\gamma}{\beta}} \lambda \tag{28}$$

From equations (27) and (28), when λ' satisfies the following condition, the simultaneous enforcement of the public pension policy to raise the rate of pension insurance and the rise of the wage rate of non-regular employees increase the number of children and have a positive effect on human capital accumulation; as a result,

$$\lambda < \lambda' < \left(\frac{\rho'}{\rho}\right)^{\frac{\beta+\gamma}{\beta}} \lambda, \ \frac{(1-l)(\rho'-\rho)+(1+l)\lambda\rho'}{\rho(1+l)} < \lambda' < 1 \tag{29}$$

This paper assumes that $\{(1-l)(\rho'-\rho)+(1+l)\lambda\rho'\}/\rho(1+l) < (\rho'/\rho)^{\frac{\beta+\gamma}{\beta}} \lambda$. In this case, when λ' satisfies the following condition, the simultaneous enforcement of the public pension policy to raise the rate of pension insurance and the rise of the wage rate of non-regular employees increase the number of children and have a positive effect on human capital accumulation; as a result,

$$\begin{aligned} &\frac{(1-l)(\rho'-\rho)+(1+l)\lambda\rho'}{\rho(1+l)} < \lambda' < 1 \qquad\qquad if \ \left(\frac{\rho'}{\rho}\right)^{\frac{\beta+\gamma}{\beta}} \lambda \geq 1 \\ &\frac{(1-l)(\rho'-\rho)+(1+l)\lambda\rho'}{\rho(1+l)} < \lambda' < \left(\frac{\rho'}{\rho}\right)^{\frac{\beta+\gamma}{\beta}} \ if \ \left(\frac{\rho'}{\rho}\right)^{\frac{\beta+\gamma}{\beta}} \lambda < 1 \end{aligned} \tag{30}$$

From equations (29) and (30), the rise of the wage rate of non-regular employees for the same working time and human capital level as that of regular employees has a definite positive effect on both fertility rate and human capital accumulation under the public pension policy that raises the rate of pension insurance.

Conclusions

This study analyzes the interaction between non-regular employment of women and economic growth patterns using the overlapping-generations model, mainly based on Galor and Weil (1996), Van Groezen, Leers, and Mejidam (2003), and Cardak (2004). This paper considers two types of individuals: men and women, and the household utility level is influenced by their own consumption levels, the number of children, educational expenditure on a child, and childcare time. Moreover, this paper considers the effect of public pension policy that raises the rate of pension insurance and the rise of the wage rate of non-regular employment on fertility rate, human capital accumulation, and economic growth patterns. The main conclusions of this paper are as follow:

(a) Public pension policy that raises the rate of pension insurance surely decreases the fertility rate when the wage rate of non-regular employees for the same working time and human capital level as that of regular employees does not change.

(b) Public pension policy that raises the rate of pension insurance has a positive effect on human capital accumulation when the wage rate of non-regular employees for the same working time and human capital level as that of regular employees does not change.

(c) If the rise of the wage rate of non-regular employees for the same working time and human capital level as that of regular employees is at adequate level, it has positive effect on both fertility rate and human capital accumulation under the public pension policy that raises the rate of pension insurance.

With a seriously aging population and low birthrate, Japan needs to find ways to enhance social security. In the analysis presented in this paper, it is shown that a rise in the wage rate of non-regular employment is needed under the public pension policy that raises the rate of pension insurance, and it must be at an adequate level. That is, there is a high risk that this policy will have a negative effect on Japan's economic growth if an adequate level is not achieved.

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Appendix A

U^F is rewritten as follows:

$$U^F = \alpha_1 \log \frac{l + l_{t+1}}{n_{t+1}^F} + \alpha_2 \log c_{t+1}^{t,F} + \alpha_3 \log n_{t+1}^F + \alpha_4 \log \frac{I_{t+1}^{D,F} - c_{t+1}^F - s_{t+1}^F}{n_{t+1}^F}$$
$$+ \alpha_5 \log \left\{ (1+\bar{r}) s_{t+1}^F + \frac{\rho(1-l)\bar{w} n_{t+1}^F h_{t+2}}{2} \right\}$$

The optimal consumption at time $t+1$, c_{t+1}^t is derived as follows:

$$\frac{\partial U^F}{\partial c_{t+1}^{t,F}} = \frac{\alpha_2}{c_{t+1}^{t,F}} - \frac{\alpha_4}{I_{t+1}^{D,F} - c_{t+1}^{t,F} - s_{t+1}^F} = 0$$

$$c_{t+1}^t = \frac{\alpha_2 \left(I_{t+1}^{D,F} - s_{t+1}^F \right)}{\alpha_2 + \alpha_4} \qquad \text{(A-1)}$$

The optimal educational expenditure at time $t+1$, e_{t+1} is derived as follows:

$$e_{t+1} = I_{t+1}^{D,F} - c_{t+1}^{t,F} - s_{t+1}^{F} = \frac{\alpha_4\left(I_{t+1}^{D,F} - s_{t+1}^{F}\right)}{\alpha_2 + \alpha_4} \tag{A-2}$$

The optimal saving at time $t+1$, s_{t+1} is derived as follows:

$$\frac{\partial U^F}{\partial s_{t+1}^F} = -\frac{\alpha_2 + \alpha_4}{I_{t+1}^{D,F} - s_{t+1}^F} + \frac{\alpha_5(1+\bar{r})}{(1+\bar{r})s_{t+1}^F + \{\rho(1-l)\overline{w}n_{t+1}^F h_{t+2}\}/2} = 0$$

$$s_{t+1} = \frac{\alpha_5(1+\bar{r})I_{t+1}^{D,F} - (\alpha_2+\alpha_4)\{\rho(1-l)\overline{w}n_{t+1}^F h_{t+2}\}/2}{(\alpha_2+\alpha_4+\alpha_5)(1+\bar{r})} \tag{A-3}$$

Moreover, the optimal number of children at time $t+1$, n_{t+1} is derived as follows:

$$\frac{\partial U^F}{\partial n_{t+1}^F} = -\frac{\alpha_1}{n_{t+1}^F} + \frac{\alpha_3}{n_{t+1}^F} - \frac{\alpha_4}{n_{t+1}^F} + \frac{\alpha_5\{\rho(1-l)\overline{w}h_{t+2}\}/2}{(1+\bar{r})s_{t+1}^F + \{\rho(1-l)\overline{w}n_{t+1}^F h_{t+2}\}/2} = 0$$

$$n_{t+1} = \frac{2(\alpha_1+\alpha_4-\alpha_3)(1+\bar{r})s_{t+1}^F}{(\alpha_3+\alpha_5-\alpha_1-\alpha_4)\rho(1-l)\overline{w}h_{t+2}} \tag{A-4}$$

Incorporating (A-4) into (A-3), the optimal saving at time $t+1$, s_{t+1} is rewritten as equation (A-5):

$$s_{t+1} = \frac{(\alpha_3+\alpha_5-\alpha_1-\alpha_4)I_{t+1}^{D,F}}{\alpha_2+\alpha_3+\alpha_5-\alpha_1} \tag{A-5}$$

Incorporating (A-5) into (A-4), the optimal number of children at time $t+1$, n_{t+1} is rewritten as equation (A-6):

$$n_{t+1} = \frac{2(\alpha_1+\alpha_4-\alpha_3)(1+\bar{r})I_{t+1}^{D,F}}{(\alpha_2+\alpha_3+\alpha_5-\alpha_1)\rho(1-l)\overline{w}h_{t+2}} \tag{A-6}$$

Incorporating (A-5) into (A-1), the optimal consumption at time $t+1$, c_{t+1}^t is rewritten as equation (A-7):.

$$c_{t+1}^t = \frac{\alpha_2 I_{t+1}^{D,F}}{\alpha_2+\alpha_3+\alpha_5-\alpha_1} \tag{A-7}$$

Incorporating (A-5) into (A-2), the optimal educational expenditure at time $t+1$, e_{t+1} is rewritten as equation (A-8):

$$e_{t+1} = \frac{\alpha_4 I_{t+1}^{D,F}}{\alpha_2+\alpha_3+\alpha_5-\alpha_1} \tag{A-8}$$

From equation (A-5) and (A-6), the optimal consumption at time $t+2$, c_{t+2}^t is rewritten as equation (A-9):

$$c_{t+2}^{t} = \frac{\alpha_5 (1+\bar{r}) I_{t+1}^{D,F}}{\alpha_2 + \alpha_3 + \alpha_5 - \alpha_1} \tag{A-9}$$

Appendix B

From (A-2), (A-5), (A-6), and (A-7), U^F is rewritten as follows:

$$\begin{aligned} U^F = {} & \alpha_1 \log (l + l_{t+1}) - \alpha_1 \log \frac{2(\alpha_1 + \alpha_4 - \alpha_3)(1+\bar{r}) I_{t+1}^{D,F}}{(\alpha_2 + \alpha_3 + \alpha_5 - \alpha_1)\rho (1-l)\overline{w} h_{t+2}} + \alpha_2 \log \frac{\alpha_2 I_{t+1}^{D,F}}{\alpha_2 + \alpha_3 + \alpha_5 - \alpha_1} \\ & + \alpha_3 \log \frac{2(\alpha_1 + \alpha_4 - \alpha_3)(1+\bar{r}) I_{t+1}^{D,F}}{(\alpha_2 + \alpha_3 + \alpha_5 - \alpha_1)\rho (1-l)\overline{w} h_{t+2}} + \alpha_4 \log \frac{\alpha_4 I_{t+1}^{D,F}}{\alpha_2 + \alpha_3 + \alpha_5 - \alpha_1} \\ & - \alpha_4 \log \frac{2(\alpha_1 + \alpha_4 - \alpha_3)(1+\bar{r}) I_{t+1}^{D,F}}{(\alpha_2 + \alpha_3 + \alpha_5 - \alpha_1)\rho (1-l)\overline{w} h_{t+2}} + \alpha_5 \log \frac{\alpha_5 (1+\bar{r}) I_{t+1}^{D,F}}{\alpha_2 + \alpha_3 + \alpha_5 - \alpha_1} \end{aligned}$$

The optimal leisure at time $t+1$, l_{t+1} is derived as follows:

$$\frac{\partial U^F}{\partial l_{t+1}^i} = \frac{\alpha_1}{l + l_{t+1}} - \frac{(\alpha_2 + \alpha_3 + \alpha_5 - \alpha_1)\lambda \overline{w} h_{t+1}}{\{(1-\rho)(1-l) + (1 - l_{t+1})\lambda\}\overline{w} h_{t+1}} = 0$$

$$l_{t+1} = \frac{\alpha_1 \{(1-\rho)(1-l) + \lambda\} - (\alpha_2 + \alpha_3 + \alpha_5 - \alpha_1) l \lambda}{(\alpha_2 + \alpha_3 + \alpha_5)\lambda}$$