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The Effect of Constructivist Instructional-Based Mathematics Course on the Attitude Toward Geometry of Pre-Service Elementary School Teachers

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The purpose of this study is to investigate the attitude toward geometry of pre-service elementary school teachers and to explore the effects of the constructivist instructional-based mathematics course on the attitude. The math course used in this research is taught in a constructivist manner. A constructivist instructional-based mathematics course was designed to utilize hands-on activities, manipulatives, problem-solving approaches, and cooperative learning environment. In the beginning of the semester, an instrument, Utley Geometry Attitude Scales (UGAS), used to collect data. At the end of the semester, the UGAS was re-administered to the two classes by the researcher. The *t*-test results indicated that there was a statistically significant difference of the UGAS mean score between the pre- and post- test (p = 0.001), at the 0.01 significance level. Using α = 0.01 as the pre-study determined level of testing, there was sufficient evidence to reject the null hypothesis, regarding differences in the measure of pre-service elementary school teachers' attitudes toward geometry collected at the beginning and end of the constructivist instructional methods based on mathematics course. Furthermore, the *t*-test results indicated that there was a statistically significant difference of the UGAS sub-scale mean score between the pre- and post- test (p = 0.001), at the 0.01 significance level. Using α = 0.01 as the pre-study determined level of testing, students demonstrated the significant change in UGAS sub-scales. The results clearly show an improved attitude toward geometry. This study demonstrated that constructivist instructional-based mathematics course had improved the attitude toward geometry. The National Council of Teachers of Mathematics (NCTM) has established goals involving students' dispositions toward mathematics that include value, self-confidence, and interest. By studying pre-service elementary teachers' attitudes toward geometry and the experiences that have played a crucial role in the development of these attitudes, teacher educators can use this information to develop training programs aimed at improving these attitudes.

Keywords: attitudes, geometry, pre-service teachers, constructivist, hands-on activities, manipulatives

Introduction

Geometry serves to connect the mathematics curriculum. A good understanding of geometry and its practical emphasis on spatial characteristics and relationships will help students gain increased access to abstract mathematical concepts (National Council of Teachers of Mathematics [NCTM], 2000). Mammana and Villiani (1998) discovered that students' geometry achievement was always lower than the other areas of

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mathematics. In the author's personal interactions with elementary pre-service school teachers at a mid-sized and four-year state university in a mid-sized town, Southern Minnesota, many elementary pre-service teachers held negative beliefs and attitudes about geometry learning. Many studies have considered the mathematics attitudes of teachers and some studies have indicated that negative attitudes were possessed by pre-service teachers specifically (Davies & Savell, 2000; Grootenboer, 2002; Tsao, 2014). In addition, many pre-service teachers have been found to have negative attitudes toward mathematics that had developed when they were students, thus continuing a negative cycle (Arp, 1999).

Attitudes play an important role on students' geometry achievement. It has been also observed that students shy away from the study of geometry (Betiku, 2001). This shows the negative attitude and poor performance of students in geometry. This negativity concerned the author that if these elementary pre-service school teachers continued to hold these beliefs and attitudes, they would not be well-prepared to teach geometry to their future students. However, pre-service elementary teachers demonstrate a deficiency in their understanding of geometry as well as in their skills to communicate geometry, particularly visualization (Owens & Outhred, 2006; Sundberg & Goodman, 2005). This deficiency is not age related, but related to experience and practice with geometry.

Literature Review

Presmeg's (2006) findings would imply pre-service elementary teachers' mathematics education with visualization techniques utilizing concrete manipulatives will aid the learning process. This deficiency may impact pre-service elementary teachers' attitudes toward geometry. However, the attitude has been shown to coincide with increased performance in geometry (Usiskin, 1987). Thus, improving geometry performance may also improve the attitude toward geometry. In addition, visualization improves performance and for when visualization is improved, subsequent learning is also increased (Siegler, 2003).

Pre-service elementary teachers must gain access to visualization through many and varied experiences (NCTM, 2000). It is necessary to create a geometry climate that will stimulate and motivate students to learn. Sun and Williams (2003) referred to the importance of constructivist learning in acquiring knowledge that required learner-cantered, goal-directed, and real-life problems in order to find meaningful solutions. In order to stimulate students more in math, we need to attack the classroom environment and make it more motivationally oriented. The use of manipulatives, particularly in geometry, can create a level of excitement and enjoyment for the students. Manipulatives are a form of hands-on activity. They would include using physical objects to illustrate geometrical formations and relationships.

Olatoye (2009) developed and evaluated some of such strategies directed at improving students' attitudes to geometry subjects, which include a strong relation between geometry contents and students everyday experiences (Geddes & Fortunato, 1993). Learning to integrate difference pre-existing knowledge with a variation in teaching methods can enhance cooperation and communication in the classroom and training of teachers. Examples of such physical objects would be geoboards, pattern blocks to illustrate intersecting planes and tangrams. Studies focused on the use of concrete manipulatives in a geometry setting show the value of using manipulatives to teach geometry, these studies conclude that students adapt more readily to their environment (Martin, Lukong, & Reaves, 2007), improve geometry performance (Raphael & Wahlstrom, 1989), and influence pre-service elementary teachers to use manipulatives in their future teaching (Tsao, 2004).

This study will provide an avenue for pre-service elementary teachers to experience concrete manipulatives for their future teaching. In a practical sense, it prepares teachers for their future classrooms that may only have access to concrete manipulatives. It is also hoped, as a by-product of implementing these methods, to improve the student motivation level in math and actually have them enjoy the learning process experience.

Thompson (1993) defined attitude as a learned pattern of manners that is developed through one's environment. In particular, Utley (2004) defined attitudes towards geometry as set of beliefs focusing on geometry that pre-disposes a person to respond in a certain way. The attitudes of students can be influenced by teachers' attitudes and his/her method of teaching. Primary school teachers' attitude towards geometry influences their students' attitude. Students with a positive attitude are more likely to develop their mathematical problem-solving skills. Thus, developing a positive attitude towards mathematics in case of pre-service primary school teachers is very important. Forgasiz (2005) found that students had greater confidence about their geometric skills and are aware of its usefulness. They have a more positive perception of the topic.

The math course used in this research is taught in a constructivist manner. Constructivist instructional-based mathematics course was designed to utilize manipulatives, problem-solving approaches, and cooperative learning environment. Students actively participate in problem-solving mathematical exploration. The students take part in hands-on activities utilizing manipulatives and technology, learning mathematical ideas in much the same way elementary school students learns mathematics under a standard-based curriculum. Artzt (1999) noted that with cooperative learning environment, such as that in a classroom based on problem-solving, students have the opportunity to discuss mathematical problems with group members, because their attitudes make them feel more comfortable and reassured. Furthermore, Tsao (2004) found that pre-service teachers learn mathematical content through the use of manipulatives in mathematics methods courses making significant improvement with number sense performance. This current study intends to look at the change in the attitude toward geometry of pre-service elementary school teachers during the semester they are enrolled in the second undergraduate mathematics course of the sequence.

This research would help teacher educators to explicitly address specific aspects of dispositional issues in mathematics content courses for teaching geometry, which could have the potential to support their learning of geometry. Two research questions will be answered in the study as the following:

1. What are the attitudes toward geometry of pre-service elementary school teachers entering the second mathematics methods course? In particular, how do pre-service teachers score on each of the three attitudinal components being measured: confidence of learning geometry, usefulness of studying geometry, or the enjoyment of studying geometry?

2. Does implement a constructivist instructional-based course influence pre-service elementary teachers' attitudes toward geometry?

Methodology

Research Design

The purpose of this study is to investigate the attitude toward geometry of pre-service elementary school teachers and to explore the effects the constructivist teaching based classroom on the attitude. The constructivist teaching based classroom is the required method by the institution to teaching this mathematics class. Thus, students were not asked to volunteer to be in any classroom, which does not use the approach based on constructivist teaching. The research design can be classified as a modified “one-group pre-post-test design.” The two combined classes form “one-group” with present/post-test corresponding to the pre-post-survey. This quasi-experimental design is identified as “O-X-O”. Here, the “O” signifies the collection of data through surveys and the “X” indicates the actual course instruction. While there were a continuous treatment (constructivist instructional-based classroom), in a sense, there was a point in the course where geometry was the formal topic. For the purposes of this design, the treatment (X) was considered to be the instruction and class work on informal geometry. The t-tests were used to compare the paired changes (pre-test to post-test) in Utley Geometry Attitude Scales (UGAS) across time.

Participants

The population of this study consists of pre-service elementary school teachers at a mid-sized and four-year state university in a mid-sized town, Southern Minnesota. The sample was composed of students in two sections of the second of three mathematics methods course. Fifty-six participants from these two classes completed data collection tasks during the Spring 2015 semester for the study. Among the 56 participants, eight were male and 48 were female.

Description of Course

This course is the second of three mathematics methods courses that elementary education majors must complete. The methods course utilizes constructivist instructional methods, such as the use of hands-on manipulatives, cooperative group work, problem-solving, and calculators. The constructivism perspective views learning as a product of organization. The teacher encourages this in two ways (Richardson, 1997):

1. Facilitating an environment in which students undergo a certain amount of cognitive dissonance;
2. Devising tasks that hopefully lead to a reorganization of existing cognitive maps.

This has been translated into instructional practices, such as hands-on activities (e.g., the use of manipulatives). The engagement of students in tasks that are meant to challenge their concepts and thinking processes are included. Constructivist teaching typically involves more student-centered, active learning experiences, more student-student and student-teacher interaction, and more work utilizing concrete materials and solving realistic problems (Shuell, 1996).

The course syllabus states the following purpose for the course: The purpose of this course is to provide opportunities for pre-service teachers to examine their understanding of various mathematics topics and to construct a vision of mathematics that considers the goals and assumptions of the current reform movement in mathematics education. Content, methods, and materials for teaching elementary school mathematics will be examined with a focus on problem-solving, number theory, informal geometry, measurement, and statistics. In this course, pre-service teachers were engaged in a variety of teaching/learning activities. These included lectures, discussions, the use of hands-on, manipulatives, cooperative learning activities, question and answer sessions, student demonstrations/explanations, and problem-solving. Pre-service teachers were expected to present results and problem solutions to their peers. The methods course used the text *Mathematics for Elementary Teachers With Activities* by Sybilla Beckmann (2013). The course covered eight chapters of the textbook, which include one chapter of number theory, one chapter of statistics chapter, and six chapters of geometry. The following part shows the sample of hands-on activity with cooperative group work.

Making and Analyzing Prisms and Pyramid

The first step of the activity is that students will need toothpicks and miniature marshmallows for this activity, and make the shapes listed below. In each case, students will visualize the shape first and predict how many toothpicks and marshmallows you will need to make it. Figure 1 shows examples of pentagonal prism and hexagonal prism.

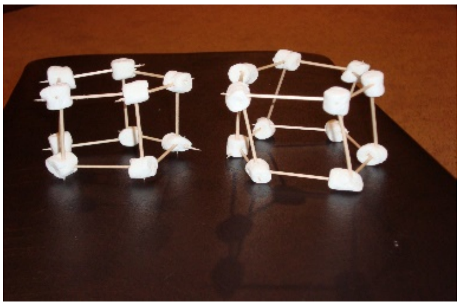

Figure 1. Using toothpicks and miniature marshmallows to contract a pentagonal prism and a hexagonal prism.

1. A triangular prism;
2. A pentagonal prism;
3. A pyramid with a triangle base;
4. A pyramid with a square base;
5. A pyramid with a hexagonal base.

The second step of activity is that student will answer the questions below without using a model.

1. How many faces does a pyramid with a hexagon base have? What shapes are the faces? Explain briefly;
2. How many edges does a pyramid with a hexagon base have? Explain;
3. How many corners (vertices) does a pyramid with a hexagon base have? Explain;
4. How many faces does a pentagonal prism have? What shapes are the faces? Explain briefly;
5. How many edges does a pentagonal prism have? Explain;
6. How many corners (vertices) does a pentagonal prism have? Explain.

Instrument and Procedure

The UGAS was developed after extensive review of a variety of existing instruments used to measure attitudes to mathematics (Utley, 2004) and was designed to measure the attitudes of undergraduate college students toward geometry.

Table 1

Sample of UGAS

Sub-scales	Statements
Usefulness of study of geometry	I believe that I will need geometry for my future. Geometry has no relevance in my life.*
Confidence to learn geometry	I am sure that I can learn geometry concepts. I lack confidence in my ability to solve geometry problems.*
Enjoyment of learning geometry	When I leave class with a geometry question unanswered, I continue to think about it. Geometry problems are boring.*

Note. * Item negatively worded.

The UGAS consist of 32 items self-report scale, including 17 positively and 15 negatively worded items, which was developed by Juliana Utley and was used with his permission in this study. The UGAS uses a Likert type scale where in the subjects respond, on a scale of 1 to 5 (1 = "Strongly disagree," 2 = "Disagree," 3 = "Neutral," 4 = "Agree," and 5 = "Strongly agree."), to their degree of agreement with a statement. Table 1 shows a sample of UGAS items. A high score represents more indicative of an overall positive attitude toward geometry.

The UGAS included three sub-scales as confidence of learning geometry, usefulness of studying geometry, and the enjoyment of studying geometry. Table 2 shows three sub-scales with items of UGAS. Possible student scores on UGAS range from 32 to 160. Higher scores on UGAS indicate more favorable attitudes towards geometry. A measure of internal consistency was calculated for the UGAS and each of its sub-scales. The Cronbach's alpha reliability coefficient for the UGAS is 0.96. The Cronbach's coefficient alpha was 0.95 for the confidence sub-scale, 0.93 for the usefulness sub-scale, and 0.92 for the enjoyment sub-scale. The last question is concerned with gender.

Table 2

Sub-Scales of UGAS

Sub-scales	Contained items
Confidence to learn geometry	2, 10, 12, 17, 19, 20, 24, 26, 28, and 31
Usefulness of studying geometry	1, 5, 8, 9, 11, 13, 18, 21, 22, 27, 29, and 32
Enjoyment of learning geometry	3, 4, 6, 7, 14, 15, 16, 23, 25, and 30

In the beginning of the semester, instruments used to collect data—UGAS. At the end of the semester (14 weeks later), the UGAS was re-administered to the two classes by the researcher. Participants were informed of the purpose of the study and solicited to participate. All participants were informed that there would be no penalty for refusal to participate in the study and those who agree to participate were ensured the confidentiality of their responses. Participants were finished completing the UGAS within 15 minutes. The *t*-tests were used to compare the paired changes (pre-test to post-test) in UGAS.

Results and Discussion

Table 3 displays pre-test/post-test means on the UGAS by sub-scales for the 56 participants. The UGAS sub-scale data on the pre-test mean scores of the confidence of learning geometry scale, usefulness of studying geometry scale, and the enjoyment of studying geometry scale were 3.29, 3.65, and 2.83, respectively. The UGAS sub-scale data on the post-test mean scores of the confidence of learning geometry scale, usefulness of studying geometry scale, and the enjoyment of studying geometry scale were 3.71, 3.91, and 3.05, respectively. Results showed that approach based on constructivist teaching had a positive impact on attitudes toward geometry with the three sub-cales.

Table 3

Sub-Scale of Pre-Test/Post-Test Means of UGAS (N = 56)

Sub-scales	Quantity	Pre-test mean	Post-test mean
Confidence to learn geometry	12	3.29	3.71
Usefulness of geometry	10	3.65	3.91
Enjoyment of learning geometry	10	2.83	3.05

Note. Negatively items have been scored reversely.

Table 4 displays pre-test/post-test mean scores of UGAS sub-scales item by item. Results showed that 30 items mean score of UGAS average increased.

Table 4

Pre-Test/Post-Test Means of UGAS Responses (N = 56)

Sub-scales	Item No.	Pre-test mean	Post-test mean
Confidence of learning geometry	1. I am sure that I can learn geometry concepts.	4.40	4.55
	5. I often have trouble solving geometry problems.	2.82	3.39
	8. I am confident I can get good grades in geometry.	4.05	4.08
	9. When I cannot figure out a geometry problem, I feel as though I am lost and cannot find my way out.	2.92	3.55
	11. I lack confidence in my ability to solve geometry problems.	3.23	3.73
	13. I feel sure of myself when doing geometry problems.	2.91	3.61
	18. For some reason even though I study, geometry seems unusually hard for me.	2.83	3.28
	21. Geometry problems often scare me.	3.28	3.84
	22. I am confident that if I work long enough on a geometry problem, I will be able to solve it.	4.03	4.04
	27. Geometry tests usually seem difficult.	2.66	2.98
	29. I can usually make sense of geometry concepts.	3.35	4.00
	32. I have a lot of confidence when it comes to studying geometry.	2.92	3.38
Usefulness of studying geometry	2. I believe that I will need geometry for my future.	3.80	4.11
	10. Geometry has no relevance in my life.	3.66	3.91
	12. Geometry is not a practical subject to study.	3.78	4.04
	17. I can see ways of using geometry concepts to solve everyday problems.	3.26	3.82
	19. Geometry is not worthwhile to study.	3.64	4.00
	20. I often see geometry in everyday things.	3.25	3.42
	24. I will need a firm understanding of geometry in my future work.	3.71	3.96
	26. I do not expect to use geometry when I get out of school.	3.82	3.92
	28. I will not need geometry for my future.	3.98	4.00
	31. Geometry is a practical subject to study.	3.64	3.95
Enjoyment of learning geometry	3. Geometry problems are boring.	2.89	3.13
	4. When I leave class with a geometry question unanswered, I continue to think about it.	2.98	2.75
	6. When I start solving a geometry problem, I find it hard to stop working on it.	3.03	2.81
	7. Time drags during geometry class.	2.83	3.07
	14. Geometry is fun.	2.71	3.02
	15. I just try to get my homework done for geometry class in order to get a grade.	2.66	2.92
	16. Geometry is an interesting subject to study.	2.85	3.23
	23. Solving geometry problems is enjoyable.	2.69	2.95
	25. Working out geometry problems does not appeal to me.	2.78	3.18
	30. Geometry has many interesting topics to study.	2.94	3.45

Note. Negatively items have been scored reversely.

Results showed that approach based on constructivist teaching had a positive impact on attitudes toward geometry. There is an unexpected result and it needed future investigations to find out the reasons. That shows that Items 4 and 6 mean score of UGAS average decreased with 0.23 and 0.22, respectively. The UGAS sub-scale data on the pre-test mean scores of the confidence of learning geometry scale were 3.29 and six of the items had mean scores below the neutral position of 3.0, and six items had mean scores above 3.0. The UGAS sub-scale data on the post-test mean scores of the confidence of learning geometry scale were 3.71 and only one item had mean scores below the neutral position of 3.0, and 11 items had mean scores above 3.0. Forgasiz (2005) found that students had greater confidence about their geometric skills and are aware of its usefulness, and they have a more positive perception of the topic. The pre- and post- test mean score of the usefulness of geometry learning sub-scale were 3.65 and 3.91. This represents attitudes that are somewhat positive, indicating that they had moderately positive attitudes scores about usefulness of geometry currently and in relationship to their future education, vocation, or other activities.

The UGAS sub-scale data on the pre-test mean scores of the confidence of enjoyment geometry scale were 2.85 and nine of the 10 enjoyment items had mean scores less than the neutral position of 3.0, reflecting negative attitudes concerning liking and enjoying geometry. This indicates a negative attitude and these future elementary school teachers did not especially enjoy geometry. The UGAS sub-scale data on the post-test mean scores of the enjoyment of learning geometry scale were 3.05 and four items had mean scores below the neutral position of 3.0, and six items had mean scores above 3.0. This result implies that pre-service teachers did not like solving geometrical problems.

Table 5 summarizes the *t*-test results between the mean scores on UGAS and sub-scales of UGAS in the pre- and post- test. The *t*-test results indicated that there was a statistically significant difference between the UGAS mean score of the pre- and post- test ($p = 0.001$), at the 0.01 significance level. Using $\alpha = 0.01$ as the pre-study determined level of testing, there was sufficient evidence to reject the null hypothesis regarding differences in the measure of pre-service elementary school teachers' attitudes toward geometry collected at the beginning and end of the constructivist instructional methods based on mathematics course.

Table 5

The T-Test Results on the UGAS and Sub-Scales of UGAS

	Mean	*Std.* error	*t*-value
UGAS	9.66	1.65	5.82^*
Confidence to learn geometry	4.91	0.85	5.77^*
Usefulness of geometry	2.62	0.62	4.20^*
Enjoyment of learning geometry	2.12	0.57	3.69^*

Note. * $p < 0.001$.

Furthermore, the *t*-test results indicated that there was a statistically significant difference between the UGAS sub-scale mean score of the pre- and post- test ($p = 0.001$), at the 0.01 significance level. Using $\alpha = 0.01$ as the pre-study determined level of testing, students demonstrated significant change in UGAS sub-scales. The results showed a significant difference within groups from pre-test to post-test before and after the intervention in pre-service elementary teachers' attitude toward geometry. The above survey was given in order to document the changes students' attitude with regard to their feelings about geometry. The results clearly show an improved attitude toward geometry. This study demonstrated that use of hands-on, manipulatives, and cooperative learning activities had improved the attitude toward geometry. This highlights the effectiveness of concrete manipulatives to help improve pre-service elementary teachers' attitude toward mathematics (Putney & Cass, 1998), adding to Sowell's (1989) findings that knowledgeable teachers who incorporate concrete teaching aids facilitate improved attitudes toward mathematics.

Conclusions

As mathematics teacher educators seek to design programs of study that will prepare mathematics teachers for their future classrooms, the issues of content and pedagogy always emerge. Improving pre-service teachers' attitudes toward geometry is an important concern for university education courses in order to facilitate positive mathematics attitudes in future elementary students. Improving the attitudes of pre-service elementary school teachers is a crucial step in breaking the cycle of teachers with negative attitudes fostering negative attitudes in their own students (Philippou & Christou, 1998). As reported in result section, statistically significant changes from pre-test to post-test were found in UGAS, and change was significant at the $\alpha = 0.001$ level. Students definitely improved in students' attitudes toward learning geometry.

This result indicates the focus of teaching is student thinking and mathematical activity, taking part in hands-on activities, utilizing manipulatives, and learning mathematical ideas, so that pre-service elementary teachers may improve their attitudes toward learning geometry. For example, manipulatives may prove to produce an effect not only for two-dimensional (2D) visualization, but also for the attitude toward geometry. Concrete manipulatives can then become a tool to aid instruction of pre-service elementary teachers' future students in geometry. The NCTM (2000) and innovative educators (Tsao, 2004; Estes, 2004) had advocated a shift from traditional teacher-centered instruction to a student-centered format.

The NCTM has established goals involving students' dispositions toward mathematics that include value, self-confidence, and interest. By studying pre-service elementary teachers' attitudes toward geometry and the experiences that have played a crucial role in the development of these attitudes, teacher educators can use this information to develop training programs aimed at improving these attitudes. As mathematics teacher educators, we are conscious that we have a long way to go, develop, and evaluate strategies directed at improving students' attitudes to geometry subjects, which include a strong relation between geometry contents and students everyday experiences. Learning to integrate difference pre-existing knowledge with a variation in teaching methods can enhance cooperation and communication in the classroom and training of teachers, but the goal is to make them become more knowledgeable an d confident in geometry, and developing positive the attitudes.

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