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Application of the Phase Difference of the Variable Components of the Stokes Vector for Measuring Linear Birefringence

Igor Linchevskyi and Ira Mizyunska

Faculty of Physics and Mathematics, Institute of Igor Sikorsky Kyiv, Polytechnic National Technical University of Ukraine, Kiev 03056, Ukraine

Abstract: To investigate the magneto-optical properties of new materials, a method for measuring linear birefringence based on a change in the initial phase of the variable component of the signal at the output of the photopolarimeter was developed. For this purpose, there was used a magneto-optical modulator, at the output of which the variable components of the Stokes vector have different initial phases. The advantage of the method is the absence of rotating optical elements, the independence of the measurement results with a change in the intensity in the optical circuit, which simplifies the measurement process. As an example, the results of measurements of magnetic linear birefringence in a film of yttrium garnet ferrite are given. The minimal increment of the phase shift between optical waves polarized along orthogonal linear polarizations (the Cotton-Mouton effect), which was measured in this experiment is 0.01 deg.

Key words: Linear birefringence, photopolarimeter, Stokes parameters, magneto-optical crystal.

1. Introduction

Control of the light polarization and measuring birefringence of materials has a significant value for fundamental and applied researches [1-3]. There are known methods for measuring of birefringence using a rotating phase plate $(\lambda/4)$ and an analyzer [4]. For such method, the accuracy of the relative wave phase shift measurements with orthogonal polarization does not exceed (1-2) deg. The lower the accuracy of measurements is provided by the method of the linear birefringence measurement [5] which is based on the measurement of the Stokes vector amplitude differences after crossing the medium with the known Mueller matrix.

Methods of the linear birefringence measurement which use a dual-beam interferometer [6] there are more precise. Here measurement error wave phase shift with orthogonal polarization reaches 10^{-5}. However, this sensitivity is hardly reached, especially with multimode laser since the Mach-Zehnder interferometer is sensitive to the influence of the environment (temperature changes, vibrations).

In order to increase the accuracy of the polarization measurements the optical radiation is preliminary modulated. There exists modulators based on the photoelastic effect (MPE) [7, 8], which are used in including spectropolarimetry [9] and for detection of weak signals from circular polarization [10]. The usage of the MPE allows measuring the linear birefringence by passing the rotating optical elements. The MPE method uses two quarter-wave plates, but one knows that these components are very rarely calibrated. Obviously, this method is not achromatic.

Birefringence measurements often use the spectral analysis of the electrical signal at the output of the photodetector [11]. The disadvantage of this method is the dependence of the results on the modulation amplitude of the wave phase difference with

Corresponding author: Igor Linchevskyi, doctor, professor, research fields: magneto-optical modulators and magnetomechanical resonance.

orthogonal polarizations at the outlet [12].

Attention is drawn to the method using a polarimetric device with Faraday modulation of the light beam [13]. It allows one to determine with high precision (to within some 0.05 deg) of the sample the birefringence. This method can easily be automated. The disadvantage is the use of an achromatic quarter of the wave plate with high accuracy (phase difference with accuracy 0.01 deg).

One of the common factors limiting the measurement accuracy of linear birefringence in the methods and devices discussed above is the conversion of the measured parameter to the corresponding intensity changes of the signal. This is especially true for small absolute values of relative wave phase shift, for example, in magnetic films.

Thus, it is necessary to create a method for measuring linear birefringence, which has a high sensitivity and does not require the use of achromatic and rotating optical elements [13].

To solve this problem we present a new method of birefringence measurement using a polarimetric device with MOM (magneto-optical modulator). New in the method is the determination of the birefringence of the sample under study through the phase difference of the electrical signals at the output of the photodetector and the reference oscillator.

2. Experimental Setup

In accordance with the principle of superposition when two harmonic oscillations with the same frequency are added, the initial phase ϑ of the resulting oscillation is determined by the amplitudes u_1, u_2 and phases δ_1, δ_2 of the original oscillations

$$\vartheta = \arctan\left(u_1 \sin\delta_1 + u_2 \sin\delta_2\right) / \left(u_1 \cos\delta_1 + u_2 \cos\delta_2\right) \quad (1)$$

The purpose of the MOM application is to obtain the radiation the Stokes vector components of which are amplitude modulated and have different phases at the frequency 2ω.

The modulation of the angle of the polarization plane rotation (due to the Faraday effect) is realized with the help of an alternating magnetic field component (with a circular frequency 2ω) directed parallel to the axis of light propagation. Simultaneously, by means of the second component of the magnetic field (with a frequency ω) directed perpendicularly to the direction of the light propagation, the modulation of the light ellipticity is performed (due to the Cotton Mouton effect).

Between the variable fields, an initial phase shift is pre-created. The harmonic component of the photodetector current with the frequency 2ω is characterized by a phase shift towards the control magnetic field. This phase shift depends on the parameters of the linear birefringence of the test sample.

Let us consider the operation of the MOM, in which the MOC (magneto-optical crystal) is placed simultaneously in an alternating magnetic field the strength vector component of which are represented in the following form: $\mathbf{h}_z = \mathbf{h}_{0z}\cos\left(2\omega t + \alpha\right)$, where α is the initial phase and $\mathbf{h}_x = \mathbf{h}_{0x}\cos\left(\omega t - \pi/4\right)$. The direction of $\mathbf{h}_z$ vector coincides with the direction of light in MOC and $\mathbf{h}_x$—vector is perpendicular to it (Fig. 1).

Radiation at the output of the Stokes vector can be presented as $(V_2) = [M](V_1)$, where (V_1) is the optical Stokes vector at the input of the MOM, $[M] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_1 & -s_1c_2 & s_1s_2 \\ 0 & s_1 & c_1c_2 & -c_1s_2 \\ 0 & 0 & s_2 & c_1 \end{bmatrix}$ is the MOC Mueller matrix, $c_1 = \cos\left(2a_1H_{0z}\left(\cos\left(2\omega t + \alpha\right)\right)\Delta z_1\right)$, $c_2 = \cos\left(b_1\left(H_{0y}\cos\left(\omega t - \pi/4\right)\right)^2\Delta z_1\right)$, $s_1 = \sin\left(2a_1H_{0z}\left(\cos\left(2\omega t + \alpha\right)\right)\Delta z_1\right)$, $s_2 = \sin\left(b_1\left(H_{0y}\cos\left(\omega t - \pi/4\right)\right)^2\Delta z_1\right)$, Δz_1 —thickness, a_1 and b_1—coefficients characterizing the Faraday effect and the linear magnetic birefringence (the Cotton-Mouton effect) of the MOC.

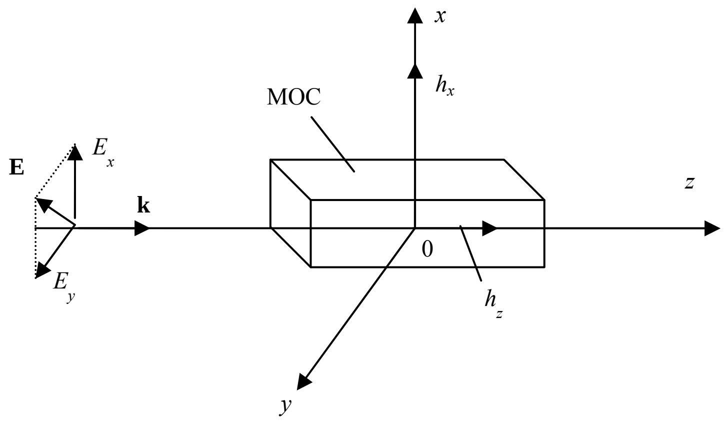

Fig. 1 The MOM scheme. 1—MOC, E, E_x, E_y—vector of electric field and its projections, k—an optical wave vector, $\mathbf{h}_x$, $\mathbf{h}_z$—alternating magnetic field tension vector.

For definiteness, we assume that the optical radiation at the input of the MOM is linearly polarized, the polarization plane azimuth of which has a 45^0 angle and the Stokes vector of which has the following form $(V_1)=(1\quad 0\quad 1\quad 0)$. Then

$$(V_2)=\begin{pmatrix}1\\-c_2s_1\\c_1c_2\\s_2\end{pmatrix}.$$

Note that all subsequent measurements are carried out at the frequency 2ω. To explain the appearance of the variable component at the frequency 2ω in the components of the vector (V_2), we use the expansion in terms of Bessel functions:

$$\begin{cases}c_1=2J_1\left(2a_1H_{0z}\Delta z_1\right)\cos\left(2\omega t+\alpha\right),\\ c_2=\cos\left(b_1H_{0y}^2\Delta z_1/2\right)J_0\left(b_1H_{0y}^2\Delta z_1/2\right)+\\ +2\sin(b_1H_{0y}^2\Delta z_1/2)J_1\left(b_1H_{0y}^2\Delta z_1/2\right)\sin\left(2\omega t\right),\\ s_1=2J_1\left(2a_1H_{0z}\Delta z_1\right)\cos\left(2\omega t+\alpha\right),\\ s_2=2\cos\left(b_1H_{0y}^2\Delta z_1/2\right)J_1\left(b_1H_{0y}^2\Delta z_1/2\right)\sin\left(2\omega t\right)\end{cases}\quad(2)$$

The vector (V_2) can be represented as a sum of two vectors

$$(V_2)=\begin{pmatrix}1\\0\\0\\0\end{pmatrix}+\left(\tilde{V}_2\right)\quad(3)$$

where,

$$\left(\tilde{V}_2\right)=2\left(\cos\left(b_1H_{0y}^2\Delta z_1/2\right)\right)\times$$
$$\times\begin{pmatrix}0\\-J_0\left(b_1H_{0y}^2\Delta z_1/2\right)J_1\left(2a_1H_{0z}\Delta z_1\right)\cos\left(2\omega t+\alpha\right)\\ J_0\left(b_1H_{0y}^2\Delta z_1/2\right)J_1\left(2a_1H_{0z}\Delta z_1\right)\cos\left(2\omega t+\alpha\right)\\ J_1\left(b_1H_{0y}^2\Delta z_1/2\right)\sin\left(2\omega t\right)\end{pmatrix}$$

is a variable component of the Stokes vector at the frequency 2ω.

Turning to the complex form of writing, we obtain the Eq. (4)

$$\left(\tilde{V}_2\right)=2\cos\left(b_1H_{0y}^2\Delta z/2\right)\begin{pmatrix}0\\-me^{i\alpha}\\me^{i\alpha}\\is\end{pmatrix}e^{i2\omega t}\quad(4)$$

where, $m=J_0\left(b_1H_{0y}^2\Delta z/2\right)J_1\left(2a_1H_{0z}\Delta z\right)$, $s=J_1\left(b_1H_{0y}^2\Delta z/2\right)$.

Vector components $\left(\tilde{V}_2\right)$ characterize the variable components at the frequency 2ω with the corresponding amplitudes and phase delays towards the control magnetic field.

The radiation after the MOM is used to measure the linear birefringence of the test sample. In our experimental setup, the measured parameter (phase shift introduced between optical waves polarized along orthogonal linear polarizations (the Cotton-Mouton effect)) is determined based on the results of measurements of the phase delay of the photodetector current at the frequency 2ω.

The measurement scheme is presented in Fig. 2. The laser radiation passes through the MOM, which contains the MOC placed inside the solenoid. A homogeneous magnetic field $\mathbf{h}_z$ is created inside the solenoid. A magnetic field $\mathbf{h}_x$ perpendicular to the direction of light propagation is created by the Helmholtz coils. Alternating current with a circular frequency 2ω from the generator is fed to the solenoid through a resistor R_1. The resistor R_1 together with the solenoid self-inductance L_1 forms a phase-shifting circuit. The current experiences a phase shift α through the solenoid winding in voltage at the output of the generator to the angle $\alpha = -\arctan(2\omega L_1/R_1)$. As a result magnetic field $\mathbf{h}_z$ is changed by the law $\mathbf{h}_{0z}\cos(2\omega t + \alpha)$.

From the output of the generator a sinusoidal signal with a circular frequency 2ω is fed to the frequency divider. From its output the signal with a circular frequency ω goes to the coils through the resistor R_2. With the help of coils the magnetic field $\mathbf{h}_x$ is formed. The resistor R_2 together with the coils self-inductance L_2 is form a phase-shifting circuit. As a result, the current through the coils undergoes a phase shift for the voltage at the output of the frequency divider by the angle $\psi = -\arctan(\omega L_2/R_2)$. By appropriate selection of the circuit parameters, it is not difficult to provide the condition $\psi = -\pi/4$. This ensures the required value of the magnetic field strength function $\mathbf{h}_x = \mathbf{h}_{0x}\cos(\omega t - \pi/4)$.

Optical radiation from the output of the MOM passes through the test sample and enters the polarization analyzer. The linear birefringence in the test sample is produced by a transverse constant magnetic field (the Cotton Mouton effect). The phase (quarter-wave) plate is used as an additional element in measurements to refine the Stokes vector parameters at the output of the MOM. Such measurements are made once before the measurements of the test medium parameters.

After the analyzer the optical radiation, which contains a variable component of the intensity with the frequency 2ω, is fed to the input of the photodetector, where it is converted into a corresponding alternating electrical signal.

The photodetector signal processing circuit comprises a selective photocurrent amplifier with signal amplitude limiting function, a synchronous detector and a registering device. The amplitude of the alternating signal at the output of the amplifier-limiter is unchanged and has the form of a meander with the frequency 2ω. The variable signal at the output of the photodetector has amplitude close to its maximum. The use of the signal limiting mode helps to suppress

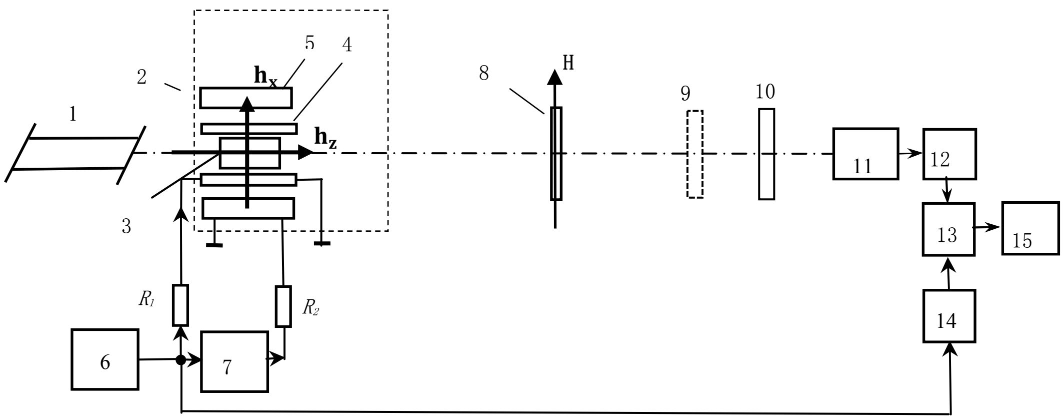

Fig. 2 Experimental setup. 1—laser, 2—MOM, 3—MOC, 4—solenoid, 5—coils, 6—generator, 7—frequency divider 1:2, 8—test sample, 9—phase plate ($\lambda/4$), 10—polarization analyzer, 11—photodetector, 12—selective amplifier (with amplitude limiting function), 13—synchronous detector, 14—phase-shifting device ($\pi/2$), 15—registering device, $\mathbf{H}$—vector of a constant magnetic field intensity.

noise and thereby increase the signal/noise ratio. Simultaneously, a reference voltage from the generator is applied to the second input of the synchronous detector. The initial phase of the reference signal is primary changed to the angle $\pi/2$ using a phase-shifting device. Thus, it is achieved that the voltage at the output of the synchronous detector is proportional $\sin\varphi$, where φ —the phase shift between the photodetector current and the generator voltage. This makes it possible to increase the sensitivity of the setup for small values of the angle φ .

Suppose that the test sample, in general, has linear and quadratic magneto-optical effects. After passing through test sample, with, the Stokes vector at the output of the analyzer with azimuth β:

$$(V_3) = [p_a][\mathrm{M}_1](V_2)\,, \qquad (5)$$

where,

$$[p_a] = \frac{1}{2}\begin{bmatrix} 1 & \cos 2\beta & \sin 2\beta & 0 \\ \cos 2\beta & \cos^2 2\beta & \cos 2\beta \sin 2\beta & 0 \\ \sin 2\beta & \cos 2\beta \sin 2\beta & \sin^2 2\beta & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$[M_1] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C_1 & -S_1C_2 & S_1S_2 \\ 0 & S_1 & C_1C_2 & -C_1S_2 \\ 0 & 0 & S_2 & C_1 \end{bmatrix}$$ —the Mueller matrix of the analyzer and test sample respectfully; $C_1 = \cos(2a_F), C_2 = \cos\theta_{CM}, S_1 = \sin(2a_F), S_2 = \sin\theta_{CM}$, $\theta_{CM} = b_{CM}H^2\Delta z/H_s^2$ —phase shift between polarized orthogonal optical waves (the Cotton-Mouton effect) in the test sample, b_{CM} —coefficient characterizing the Cotton-Mouton effect, H_s —saturation field, Δz —thickness of the test sample, a_F —angle of rotation of the plane of polarization due to the Faraday effect (in the presence).

If we assume that the analyzer's azimuth is $\beta = \pi/4$, the variable component of the light intensity at the input of the photodetector

$$\tilde{I} = \left((C_1C_2 - S_1)me^{-i\alpha} + iC_1S_2s\right)e^{i2\omega t} \qquad (6)$$

After the corresponding conversions the Eq. (6) is described with a formula

$$\tilde{I} = \sqrt{\begin{matrix}\left[(C_1C_2 - S_1)m\cos\alpha\right]^2 + \\ +\left[C_1S_2s - (C_1C_2 - S_1)m\sin\alpha\right]^2\end{matrix}}\; e^{i(2\omega t + \varphi)},$$

where,

$$\varphi = \arctan\left(\frac{C_1S_2s - (C_1C_2 - S_1)m\sin\alpha}{(C_1C_2 - S_1)m\cos\alpha}\right) \qquad (7)$$

If the test samplehas the Faraday effect, then to determine the phase shift for the Cotton-Mouton effect, we should use the relation:

$$\theta_{CM} = \arctan\frac{m\left(1 - \mathrm{tg}(2\alpha_F)\right)(\mathrm{tg}\alpha + \mathrm{tg}\varphi)\cos\alpha}{s} \qquad (8)$$

where the expression $\mathrm{tg}(2\alpha_F) = S_1/C_1$ determines ξ —a polarization plane rotation angle due to the Faraday effect. In the absence of the Faraday effector the Faraday effect is small ($\mathrm{tg}(2\alpha_F) \ll 1$), the Eq. (8) takes the form of:

$$\theta_{CM} = \arctan\frac{m(\mathrm{tg}\alpha + \mathrm{tg}\varphi)\cos\alpha}{s} \qquad (9)$$

Before carrying out the measurements with the test sample, it is necessary to perform calibration measurements to refine the parameters of the Stokes vector (V_2) at the output of the MOM. Of course, we can restrict ourselves to the corresponding calculation in accordance with the Eq. (4). It is necessary to measure the amplitudes of the vector variables (V_2) in the absence of the medium under study. When measuring the amplitudes m and s of the vector variables (V_2), it is necessary to take into account the amplitudes of the signals at the output of the selective amplifier. In these measurements, the amplitude-limiting function in the selective amplifier must be switched off. If the azimuth of the analyzer $\beta = \pi/4$, the variable component of the intensity at the input of the photodetector will be

$$I_1 = \frac{m}{2}\cos(2\omega t + \alpha) \qquad (10)$$

When installing a quarter-wave plate with a zero azimuth in front of the analyzer, for which $\beta = \pi/4$, the variable intensity

$$I_2 = s\sin(2\omega t) \qquad (11)$$

In accordance with Eqs. (8) and (9), it is sufficient to determine the ratio m/s in terms of the amplitude ratio I_1/I_2.

In the MOM a crystal made of yttrium garnet ferrite with dimensions 4 × 4 × 8 mm was used. The linear modulation frequency $f = \omega/2\pi$ was 10 kHz.

The source of radiation was a helium neon laser ($\lambda = 1.15\mu m$, power 5 mW). As the test sample, an yttrium garnet ferrite film with a thickness $\Delta z = 9\mu m$ on the gadolinium gallium garnet substrate (thickness 450 μm) was used. To create a linear magnetic birefringence, the sample was placed perpendicular to the direction of light propagationin a constant magnetic field with the intensity H that varied in the range 0-H_s, H_s is the saturation field strength along the easy magnetization axis. In the experiment the vector **H** was parallel to the plane (111) and perpendicular to the axis of easy magnetization. The magnetic saturation mode in the plane (111) can be obtained for the field values H>H_s [13]. For the test sample, the maximum value of the angle of rotation of the plane of polarization (Faraday effect) does not exceed 0.15 deg. Therefore, in accordance with Eq. (8) we neglected the influence of the Faraday effect on the results of measuring the linear magnetic birefringence.

3. Experimental Results

In the experiment, taking into account the refined parameters of the normalized variables of the Stokes vector at the output of the MOM, a ratio $m/s = 1.8$ was obtained. The delay angle is $\alpha = -10^0$.

Fig. 3 shows the experimental (curve a) and the calculated (curve b) dependence of the phase shift for the Cotton-Mouton effect value of the testsample on the square of the magnetic field strength. To obtain the theoretical dependence of the phase shift for the Cotton-Mouton effect, it was assumed that $\theta_{CM} = b_{0CM}(H/H_s)^2 \Delta z$, where b_{0CM} is the specific phase shift for the Cotton Mouton effect in the yttrium garnet ferrite ($b_{0CM} = 160$ deg/cm, $H_s = 10^3$ A/m) [15].

The graph of the experimental dependence of thetest sample linear birefringence actually reflects the change in the magnetization in the film plane [14]. The difference (within 8%) of the experimental dependence on the theoretical is due to the influence of the substrate on which the layer of the magneto-optical crystal is grown. The effect of the substrate influence is manifested better at low values of the magnetic energy [15].

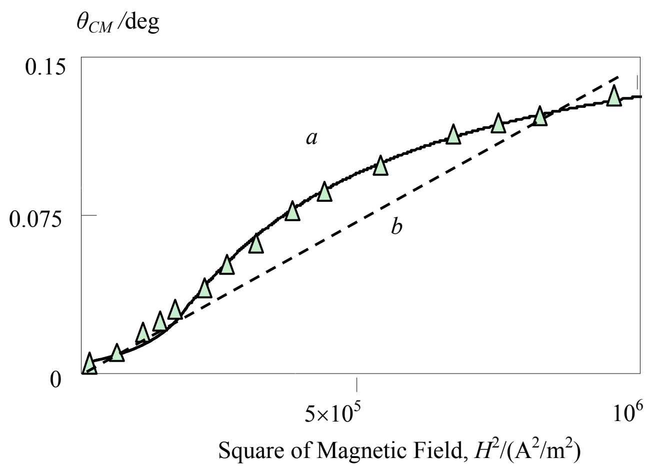

Fig. 3 The phase shift between optical waves polarized along orthogonal linear polarizations (the Cotton-Mouton effect) effect dependence for the yttrium garnet ferrite film from the square of the magnetic field strength H: (a)—experimental, (b)—theoretical.

The minimal increment of the phase shift waves polarized along orthogonal linear polarizations (the Cotton-Mouton effect), which was measured in this experiment (sensitivity of the method) is 0.01 deg.

4. Conclusions

A technique is presented and an experimental setup for measurement of linear birefringence is developed using the MOM at the output of which the variable components of the Stokes vector have different initial phases at the modulation frequency. The advantage of the considered method of measuring the linear birefringence is the absence of the need for additional adjustments, which takes place, when compensators and rotating optical elementsare used. This made it possible simultaneously to reduce the measurement errors associated with changes in the intensity in the optical scheme.

The proposed method for measuring linear birefringence through changes in the initial phase shift of the electrical signal at the output of the photodetector made it possible to increase the signal/noise ratio. The results of the magnetic linear birefringence measurements (the Cotton Mouton effect) in a film of yttrium garnet ferrite are presented.

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