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A Quadrature Approach for *N*-Collinear Crack Problem in an Orthotropic Strip

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Abstract: This study presents the determination of the stress intensity factors (SIFs) at the edges of the cracks in an elastic strip weakened by *N*-collinear cracks. The problem of an orthotropic elastic strip is reduced to a system of Cauchy type singular integral equations. The system of singular integral equations is approached by a Quadrature technique. Under two different loading conditions, the results are obtained for the different cases of crack numbers. The resistance of the strip is examined by considering the orthotropic properties of the strip material. Finally, the crack interactions are clarified during the analysis.

Key words: Crack, Theory of Elasticity, System of Singular Integral Equations, Quadrature Approach

1. Introduction

In fracture mechanic, multiple crack problems are very important subjects. Because, in practice, it is uncommon to encounter a crack problem involving only a single crack. It is important to analyse the crack interactions. Many studies have been done on the interactions of multiple cracks. Some of the studies are listed below:

Rao and Reddy [1] considered a multiple crack system in a homogeneous, isotropic, and two dimensional linear-elastic body. The SIFs and energy release rates (ERRs) are approached efficiently by fractal finite element based continuum shape sensitivity analysis under mixed-mode loading condition. Hwang et. al. [2] further extended the virtual crack extension method to the general case of multiple crack systems under mixed-mode loading. They presented analytical expressions for derivatives of ERRs and SIFs and gave comparisons between present numerical solution and finite difference method. A numerical solution of a multiple crack problem in a finite plate using coupled integral equations was presented by Chen and Wang in [3]. Non-dimensional SIFs and T-stresses for different cases of the cracks (an inclined crack in a square cracked plate, two parallel cracks in a staking position, two inclined cracks in an elliptic cracked plate) were given in tabular forms. The study of Ma et.al [4] includes a multiple crack problem for an infinite plate. Two single crack problems with different tractions applied to the crack which led to the Riemann-Hilbert problem were examined. Muravin and Turkel [5] designed a multiple crack weight (MCW) method for the strongly interacting cracks. They developed an algorithm for the construction of weight functions to handle multiple interacting cracks. Variation of SIFs for double-edged collinear cracks in finite plate under normal load and Star-shaped crack in finite plate under bi-axial load was presented. Jin and Keer [6] handled a multiple crack problem on an elastic half plane. A procedure based on the distributed dislocation method was given. Three different patterns of modeling dislocation density at the crack mouth were discussed. Nondimensional SIFs corresponds to the number of the crack were determined. An elaborated analysis was done for recent developments of multiple crack problems in plane elasticity by Chen [7]. Some kinds of integral equations were suggested for the multiple crack problems in plane elasticity. The effectiveness of the integral transform method and the complex variable function method were compared for some particular cases. Different kinds of the singular integral equations for multiple crack problems and their regularization procedures were given. Zheng et.al., [8] first formulated a nonlinear complementarily problem and the boundary value problem fitted to the servo control way. They proposed an algorithm to simulate growth of multiple cracks. As examples, they examined a symmetric central straight crack, a plate with two holes and edge cracks and a plate containing 10 random cracks.

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Erbas et.al., [9] reduced a contact problem to a singular integral equation with the help of Fourier transform technique. The singular integral equation is approached by an iterative solution method and a direct asymptotic procedure for the thick and thin strip, respectively. Numerical results of pressure and moment are given for the different values of relative thickness of the strip. Erdogan et.al., discussed the numerical solution of singular integral equations [10]. Numerical examples for first and second kind singular integral equations are included. An orthotropic strip problem weakened by a crack is handled in the studies of Yusufoğlu and Turhan [11,12]. A singular integral equation is induced by the equations of elasticity theory and Fourier transform of Airy's stress function. The solution of the singular integral equation is approached by an iterative technique and Gauss Chebyshev quadrature, respectively.

In this study, an orthotropic strip weakened by N-cracks is considered. A system of singular integral equations is derived. The system of singular integral equations is approached by Gauss quadrature formulas. As examples, the cases of *N=1, N=2, N=3* are taken and the numerical results of normalized SIFs are presented.

2. Solution of mixed boundary value problem corresponding to *N*- crack

Let us consider a crack problem which presents the generalization of the *N*-collinear cracks on an elastic orthotropic strip (see Figure.1). The cracks are located along $x \in w = \bigcup_{i=1}^{N} (a_i, b_i)$ on the axis x- and the boundaries of the strip $y = \pm h$ are hinged. Uniformly distributed pressure of magnitude $q_i(x)$, $i = \overline{1, N}$ is applied to the sides of cracks.

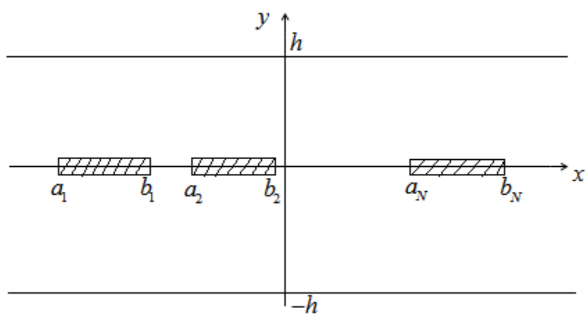


Figure 1. Geometry of *N*-collinear cracks.

Let us present a formulation of the considered problem with the help of theory of elasticity. The boundary conditions of the problem can be expressed as;

$$\upsilon(x,\pm h) = \sigma_{xy}(x,\pm h) = 0, \quad \sigma_{xy}(x,0) = 0, \quad |x| < \infty,$$
$$\upsilon(x,0) = 0, \; x \notin w; \qquad (1)$$
$$\sigma_y(x,0) = -q(x), \; x \in w,$$

in which υ is the vertical displacement; σ_{xy} and σ_y are shear and normal stress components, respectively.

According to the analogy in [11,12], considering kinematic equations, equilibrium equations, Hooke's law, Airy function and using the Fourier transform technique and Eqs. (1), the following system of singular integral equation is obtained:

$$\sum_{j=1}^{N} \int_{w} \frac{\gamma_j'(\xi)}{\xi - x} d\xi + \frac{1}{h} \sum_{j=1}^{N} \int_{w} \gamma_j'(\xi) F\left(\frac{\xi - x}{h}\right) d\xi = -\frac{\pi}{\Delta} q(x), \qquad (2)$$

where, $q(x) = q_i(x)$, $x \in (a_i, b_i)$, $i = 1,...,N$ and $\gamma_j'(x)$ are the derivatives of the function describing vertical displacements of the points on the sides of the cracks. The following additional condition arises from the continuity of the function of vertical displacements of the points on the crack sides,

$$\gamma_i(a_i) = \gamma_i(b_i) = 0. \qquad (3)$$

Hence, Eq. (3) leads to the following integral equation:

$$\int_{a_i}^{b_i} \gamma_i'(\xi) d\xi = 0, \; i = 1,...,N. \qquad (4)$$

To convert the systems of integral equations (2) and (4) into dimensionless forms, the variables $x = s_i + r_i t$, $\xi = s_j + r_j \tau$, $s_i = \dfrac{b_i + a_i}{2}$, $r_i = \dfrac{b_i - a_i}{2}$ should be used. So, the following integral equations are obtained:

$$\int_{-1}^{1} \frac{\varphi_i(\tau)}{\tau - t} d\tau + \sum_{\substack{j=1 \\ i \neq j}}^{N} \int_{-1}^{1} \frac{\varphi_j(\tau)}{\tau - \dfrac{r_i}{r_j} t + \dfrac{s_j - s_i}{r_j}} d\tau \qquad (5)$$
$$+ \sum_{j=1}^{N} \int_{-1}^{1} \varphi_j(\tau) F_{ij}^*(\tau, t) d\tau = f_i(t),$$

$$\int_{-1}^{1} \varphi_i(\tau) d\tau = 0, \qquad (6)$$

where,

$$\varphi_j(\tau) = \gamma_j'(s_j + r_j \tau),$$

$$F_{ij}^*(\tau, t) = \frac{r_j}{h} F_{ij}\left(\frac{r_j \tau - r_i t + s_j - s_i}{h}\right),$$

$$f_i(t) = -\frac{\pi q_i (s_i + r_i t)}{\Delta}.$$

The procedure given in [14] is prefered for the solution of reduced system of singular integral equations (5),(6). So, according to the index theory of Muskheleshvili [15], the solution is the form of

$$\varphi_j(t) = \frac{\phi_j(t)}{\sqrt{1-t^2}}, \quad j = 1,...,N. \tag{7}$$

Substituting Eqs. (7) into Eqs. (5) and (6), we obtain

$$\int_{-1}^{1} \frac{\phi_i(\tau)}{(\tau - t)\sqrt{1-\tau^2}} d\tau + \sum_{\substack{j=1 \\ i\neq j}}^{N} \int_{-1}^{1} \frac{\phi_j(\tau)}{\left(\tau - \frac{r_i}{r_j} t + \frac{s_j - s_i}{r_j}\right)\sqrt{1-\tau^2}} d\tau \tag{8}$$

$$+\sum_{j=1}^{N} \int_{-1}^{1} \frac{\phi_j(\tau) F_{ij}^*(\tau, t)}{\sqrt{1-\tau^2}} d\tau = f_i(t), \ i = \overline{1,N}$$

$$\int_{-1}^{1} \frac{\phi_i(\tau)}{\sqrt{1-\tau^2}} d\tau = 0, \tag{9}$$

respectively. The function $\phi_j(t)$ is defined by Lagrange interpolation polynomials

$$\phi_n^{(j)} = \sum_{m=1}^{n} \frac{\phi_j(\tau_m) T_n(t)}{(t - \tau_m) T_n'(\tau_m)},$$

where, $T_n(t)$ are Chebyshev polynomials of the first kind, τ_m, ($m = 1,2,...,n$) are the roots of the Chebyshev polynomials of the first kind and $\phi_n^{(j)}(\tau_m) = \phi_j(\tau_m)$ is valid. So, we designate the Lagrangian interpolation polynomials $\phi_n^{(j)}(t)$ as approximate solution of the system, containing from (8) and (9), namely

$$\int_{-1}^{1} \frac{\phi_n^{(i)}(\tau)}{(\tau - t)\sqrt{1-\tau^2}} d\tau + \sum_{\substack{j=1 \\ i\neq j}}^{N} \int_{-1}^{1} \frac{\phi_n^{(j)}(\tau)}{\left(\tau - \frac{r_i}{r_j} t + \frac{s_j - s_i}{r_j}\right)\sqrt{1-\tau^2}} d\tau \tag{10}$$

$$+\sum_{j=1}^{N} \int_{-1}^{1} \frac{\phi_n^{(j)}(\tau) F_{ij}^*(\tau, t)}{\sqrt{1-\tau^2}} d\tau = f_i(t),$$

$$\int_{-1}^{1} \frac{\phi_n^{(i)}(\tau)}{\sqrt{1-\tau^2}} d\tau = 0, \ i = \overline{1,N}. \tag{11}$$

By considering the known formula [16]

$$\int_{-1}^{1} \frac{T_n(\tau)}{(\tau - t)\sqrt{1-\tau^2}} d\tau = \pi U_{n-1}(t),$$

where, $U_n(t)$ is the Chebyshev polynomial of second kind, the following equality is valid

$$\int_{-1}^{1} \frac{\phi_n^{(i)}(\tau)}{(\tau - t)\sqrt{1-\tau^2}} d\tau = \pi \sum_{m=1}^{n} \frac{\phi_i(\tau_m)}{t - \tau_m} \left(\frac{U_{n-1}(t)}{T_n'(\tau_m)} - \frac{U_{n-1}(\tau_m)}{T_n'(\tau_m)} \right). \tag{12}$$

By using the following known relation for Chebyshev polynomials $\frac{U_{n-1}(\tau_m)}{T_n'(\tau_m)} = \frac{1}{n}$, Eq. (12) may be expressed as

$$\int_{-1}^{1} \frac{\phi_n^{(i)}(\tau)}{(\tau - t)\sqrt{1-\tau^2}} d\tau = \pi \sum_{m=1}^{n} \frac{\phi_i(\tau_m)}{t - \tau_m} \left(\frac{U_{n-1}(t)}{T_n'(\tau_m)} - \frac{1}{n} \right). \tag{13}$$

Taking into account (13) and using appropriate Gauss-Chebyshev integration formula for Eq. (7), next, setting $t = t_m$ (*m=1,2,...,n-1*) in equation (10), where t_m are roots of the polynomial $U_{n-1}(t)$, we obtain the following system of linear equations

$$\sum_{j=1}^{N} \sum_{m=1}^{n} \left[\frac{1}{\tau_m - (r_i / r_j) t_k + (s_j - s_i) / r_j} + F_{ij}^*(\tau_m, t_k) \right] \frac{\pi \phi_j(\tau_m)}{n} = f_i(t_k),$$

$$\sum_{m=1}^{n} \frac{\pi}{n} \phi_i(\tau_m) = 0, \quad i = 1,...,N.$$

3. Numerical results for SIFs

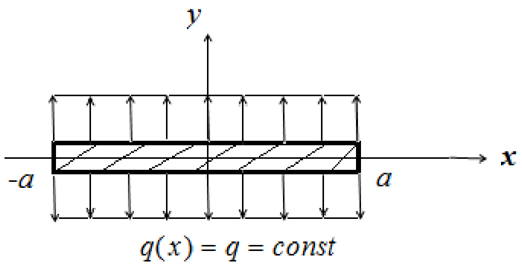
In this section, the elasticity problem is solved numerically by Gauss quadrature approach for the different cases of *N,* i.e., *N=1, N=2, N=3*. The stiffness properties of the three kinds of orthotropic materials are given in Table 1 [17].

Table 1. Stiffness properties for selected materials

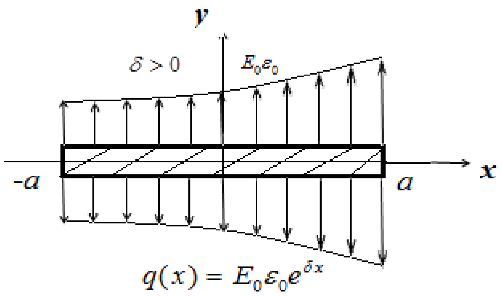
Material i	c_{11} (Pa)	c_{12} (Pa)	c_{22} (Pa)	c_{66} (Pa)
1	1.048×10^{10}	3.248×10^{9}	1.578×10^{10}	7.070×10^{9}
2	1.048×10^{10}	3.248×10^{9}	4.192×10^{10}	7.070×10^{9}
3	4.192×10^{10}	3.248×10^{9}	1.048×10^{10}	7.070×10^{9}

Material i is the name of three orthotropic materials by which the strip is made. Table 1 shows that Materials 2 and 3 are same except a 90 degree rotation of orthotropy and while Material 3 has the highest orthotropy, Material 2 has weakest orthotropy in crack line direction.

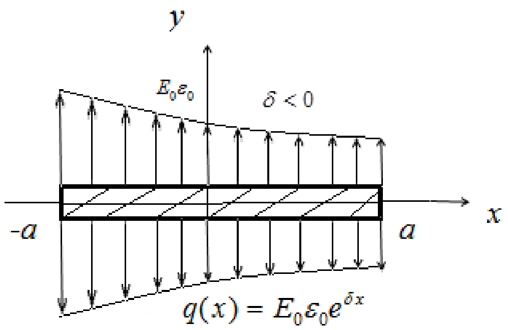
Two different loading conditions given in Figure 2 are loaded to the sides of the cracks to determine the SIFs. These conditions are uniform crack surface pressure ($q(x) = q = const$) and fixed-grip loading ($q(x)=E_0\varepsilon_0 e^{\delta x}$), where $\varepsilon_x(\infty,y)=\varepsilon_0$, $E_0=c_{11}-c_{12}^2/c_{22}$.



a) Uniform crack surface pressure



b) Fixed-grip loading.



c) Fixed-grip loading.

Figure. 2. Geometry of loading conditions.

Tables 2-3 give the normalized SIFs under uniform crack surface pressure for a strip weakened by different numbers of cracks. As is seen from Tables 2-3, cracking starts earlier when crack numbers in the strip increase. So increasing of the crack numbers leads to decreasing of strip resistance.

As is known, crack propagation occurs when $K_I \geq K_{IC}$, where K_{IC} is the fracture toughness of the strip materials. Let us show the relation between SIFs and critical load by referring the SIFs values for Material 1 in the first, second and third rows of Table 2. $K_I^{(1)}, K_I^{(2)}$ and $K_I^{(3)}$ denote the SIF values at the crack edges for strips weakened by one crack, two cracks and three cracks, respectively. From Table 2,

$K_I^{(1)}/\left(q\sqrt{\pi a}\right)=2.10824$,

$K_I^{(2)}/\left(q\sqrt{\pi a}\right)=2.26888$ and

$K_I^{(3)}/\left(q\sqrt{\pi a}\right)=2.28620$ are seen.

Accordingly,

$K_I^{(1)}/\sqrt{\pi a}=2.10824q \geq K_{IC}/\sqrt{\pi a}$ and $q \geq \dfrac{K_{IC}}{2.10824\sqrt{\pi a}}=q_C^{(1)}$. Similarly, it can be written that

$K_I^{(2)}/\sqrt{\pi a}=2.26888q \geq K_{IC}/\sqrt{\pi a}$ and $q \geq \dfrac{K_{IC}}{2.26888\sqrt{\pi a}}=q_C^{(2)}$,

$K_I^{(3)}/\sqrt{\pi a}=2.28620q \geq K_{IC}/\sqrt{\pi a}$ and $q \geq \dfrac{K_{IC}}{2.28620\sqrt{\pi a}}=q_C^{(3)}$. By comparing three relations, it has seen that critical load $q_C^{(3)}$ is minimum. It shows that, the resistance of the strip weakened by three cracks is the minimum. Similar interpretations can be done for the resistance of the strip composed of Materials 2-3 (see, Table 2), too.

Again, from Table 2, when the crack length increases, SIFs increase and critical load decreases. Additionaly, Table 2 presents the relation of normalized SIFs for all tip materials given in Table 1. Remember that from Table 1, while material 3 has the highest orthotropy, material 2 has weakest orthotropy in crack line direction. Also, it has seen from Table 2 that while the SIF values for the strip composed of Material 3 are the highest, the SIF values for the strip composed of Material 2 are the lowest. So, it can be interpreted that, when orthotropy properties of the strip increase, SIFs increase and critical load decreases. So the resistance of the strip decreases.

By comparing Table 2 and Table 3, it is concluded that, when the crack sides are loaded by uniform crack surface pressure, while the strip thickness decreases, normalized SIFs decrease, too. So critical load increases and the resistance of the strip increases.

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Table 2. The normalized SIFs under uniform crack surface pressure for a strip weakened by different numbers of cracks for $h=80a$.

N	a_1	b_1	a_2	b_2	a_3	b_3	Material i	$\frac{K_I(a_1)}{q\sqrt{\pi a}}$	$\frac{K_I(b_1)}{q\sqrt{\pi a}}$	$\frac{K_I(a_2)}{q\sqrt{\pi a}}$	$\frac{K_I(b_2)}{q\sqrt{\pi a}}$	$\frac{K_I(a_3)}{q\sqrt{\pi a}}$	$\frac{K_I(b_3)}{q\sqrt{\pi a}}$
1	-10a	-a	-	-	-	-	3	2.11244	2.11244	-	-	-	-
							1	2.10824	2.10824	-	-	-	-
							2	2.08658	2.08658	-	-	-	-
2	-10a	-a	a	10a	-	-	3	2.27884	2.61712	2.61712	2.27884	-	-
							1	2.26888	2.60551	2.60551	2.26888	-	-
							2	2.22122	2.54819	2.54819	2.22122	-	-
3	-10a	-a	a	10a	20a	30a	3	2.30064	2.65692	2.67596	2.37344	2.34032	2.28864
							1	2.28620	2.63960	2.65819	2.35751	2.32533	2.27465
							2	2.22709	2.56482	2.57957	2.28629	2.26326	2.22050
3	-10a	-5a	-3a	10a	12a	20a	3	1.92745	2.26720	2.92988	3.07422	2.74208	2.28583
							1	1.91415	2.25122	2.90883	3.05196	2.72246	2.26989
							2	1.85640	2.17887	2.80993	2.94580	2.63101	2.19935
3	a	20a	40a	60a	80a	100a	3	3.06581	3.12070	3.22432	3.22770	3.19422	3.13716
							1	3.01800	3.06586	3.15753	3.16044	3.13604	3.08632
							2	2.87238	2.89387	2.94999	2.95091	2.94895	2.92695

Table 3. The normalized SIFs under uniform crack surface pressure for a strip weakened by different numbers of cracks for $h=40a$.

N	a_1	b_1	a_2	b_2	a_3	b_3	Material i	$\frac{K_I(a_1)}{q\sqrt{\pi a}}$	$\frac{K_I(b_1)}{q\sqrt{\pi a}}$	$\frac{K_I(a_2)}{q\sqrt{\pi a}}$	$\frac{K_I(b_2)}{q\sqrt{\pi a}}$	$\frac{K_I(a_3)}{q\sqrt{\pi a}}$	$\frac{K_I(b_3)}{q\sqrt{\pi a}}$
1	-10a	-a	-	-	-	-	3	2.08658	2.08658	-	-	-	-
							1	2.07051	2.07051	-	-	-	-
							2	1.99325	1.99325	-	-	-	-
2	-10a	-a	a	10a	-	-	3	2.22122	2.54819	2.54819	2.22122	-	-
							1	2.18701	2.50677	2.50677	2.18701	-	-
							2	2.05374	2.33107	2.33107	2.05374	-	-
3	-10a	-a	a	10a	20a	30a	3	2.22709	2.56482	2.57957	2.28629	2.26326	2.22050
							1	2.18580	2.51231	2.52429	2.23628	2.22002	2.18297
							2	2.05017	2.32708	2.32961	2.06831	2.08406	2.06892
3	-10a	-5a	-3a	10a	12a	20a	3	1.85640	2.17887	2.80993	2.94580	2.63101	2.19935
							1	1.81648	2.12844	2.74047	2.87102	2.56690	2.15041
							2	1.69376	1.95556	2.47882	2.58016	2.33207	1.99270
3	a	20a	40a	60a	80a	100a	3	2.87238	2.89387	3.22432	2.95091	2.94895	2.92695
							1	2.77371	2.77920	2.81473	2.81458	2.82482	2.81959
							2	2.46166	2.45473	2.47014	2.46971	2.47531	2.48263

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Figures 3-4 show normalized SIFs correspond to material orthotrophy parameter E_1/E_2 in an ortotropic strip weakened by different cracks under uniform crack surface pressure. When material orthotrophy parameter E_1/E_2 increases SIFs increases, too. So, when the orthortopy properties of the strip material increases SIFs increases and critical load decreases. Thereby, the resistance of the strip decreases.

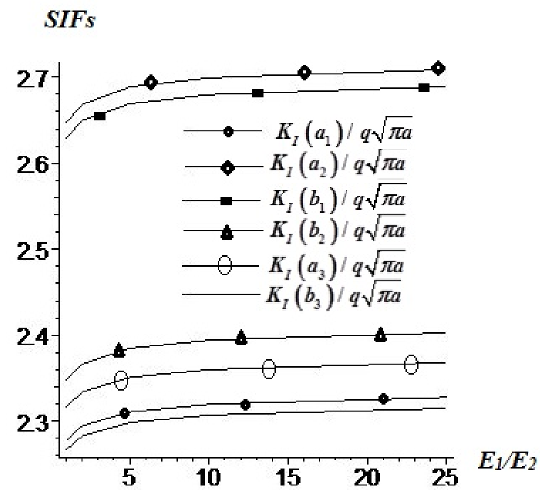

Figure 3. SIFs correspond to material orthotropic parameter E_1/E_2 in an ortotropic strip weakened by three cracks located $a_1=-10a$, $b_1=-a$, $a_2=a$, $b_2=10a$, $a_3=20a$, $b_3=30a$ under uniform crack surface pressure.

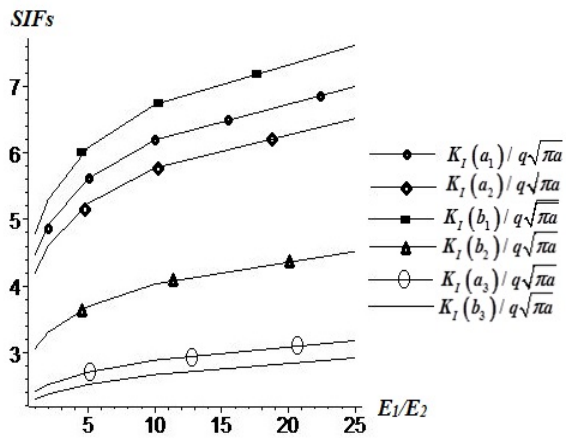

Figure 4. SIFs correspond to material orthotropic parameter E_1/E_2 in an ortotropic strip weakened by three cracks located $a_1=-120a$, $b_1=-a$, $a_2=0$, $b_2=10a$, $a_3=20a$, $b_3=30a$ under uniform crack surface pressure.

Tables 4-11 show normalized SIFs determined by an ortotropic strip weakened by different cracks under fixed-grip loading. It is assumed that same forces are applied to both two cracks. For the case of $N=1$, when $\delta h<0$, crack propagation starts at left edge (a_1), when $\delta h>0$, crack propogation stars at right edge (b_1).

In Tables 2-3, for the case of $N=1$, it is assumed that the crack is just located on $a_1=-10a$, $b_1=-a$. Because, for the same length cracks, the normalized SIF values are same under uniform crack surface pressure. But in Tables 4-9, we preffered to present two different location for the case of $N=1$. It can be concluded that, when the crack slides through $-Ox$, if $\delta h<0$, SIFs increase; on the contrary, if $\delta h>0$, SIFs decrease.

For all the cases given in Tables 4-11, independently of crack numbers, sizes and loading condition, it can be deduced that when then the material orthotropy properties of strip in the crack line increase, normalized SIFs increase, too. So, critical load decreases and the strip's resistance decreases.

Tables 4-7 shows that when $\delta h<0$, if the thickness of the strip decreases, the SIF values of the cracks located on axis $-Ox$ increase; on the contrary, the SIF values of the cracks located on axis Ox decrease. Similarly, it follows from Tables 8-11, when $\delta h>0$, if the thickness of the strip decreases, the SIF values of the cracks located on axis $-Ox$ decrease; on the contrary, the SIF values of the cracks located on axis Ox increase.

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Table 4. The normalized SIFs determined under fixed-grip loading *δh=ln0.1* for a strip weakened by different numbers of cracks for *h=80a*.

N	a_1	b_1	a_2	b_2	a_3	b_3	Material i	$\frac{K_I(a_1)}{q\sqrt{\pi a}}$	$\frac{K_I(b_1)}{q\sqrt{\pi a}}$	$\frac{K_I(a_2)}{q\sqrt{\pi a}}$	$\frac{K_I(b_2)}{q\sqrt{\pi a}}$	$\frac{K_I(a_3)}{q\sqrt{\pi a}}$	$\frac{K_I(b_3)}{q\sqrt{\pi a}}$
1	-10a	-a	-	-	-	-	3	2.64646	2.32390	-	-	-	-
							1	2.64152	2.31896	-	-	-	-
							2	2.61608	2.29355	-	-	-	-
	a	10a	-	-	-	-	3	1.92826	1.69324	-	-	-	-
							1	1.92466	1.68964	-	-	-	-
							2	1.90612	1.67112	-	-	-	-
2	-10a	-a	a	10a	-	-	3	2.79616	2.78183	2.49131	1.88048	-	-
							1	2.78602	2.77005	2.47955	1.87042	-	-
							2	2.73710	2.71172	2.42161	1.82265	-	-
3	-10a	-a	a	10a	20a	30a	3	2.80726	2.80211	2.52131	1.92882	1.27418	1.07002
							1	2.79482	2.78738	2.50634	1.91558	1.26218	1.05897
							2	2.74008	2.72012	2.43745	1.85530	1.21570	1.02021
3	a	20a	40a	60a	80a	100a	3	2.60378	1.98864	0.92460	0.67463	0.29481	0.21148
							1	2.57977	1.96318	0.90152	0.65433	0.28250	0.20194
							2	2.48162	1.85925	0.83307	0.60154	0.26073	0.18744

Table 5. The normalized SIFs determined under fixed-grip loading *δh=ln0.1* for a strip weakened by different numbers of cracks for *h=40a*.

N	a_1	b_1	a_2	b_2	a_3	b_3	Material i	$\frac{K_I(a_1)}{q\sqrt{\pi a}}$	$\frac{K_I(b_1)}{q\sqrt{\pi a}}$	$\frac{K_I(a_2)}{q\sqrt{\pi a}}$	$\frac{K_I(b_2)}{q\sqrt{\pi a}}$	$\frac{K_I(a_3)}{q\sqrt{\pi a}}$	$\frac{K_I(b_3)}{q\sqrt{\pi a}}$
1	-10a	-a	-	-	-	-	3	3.29259	2.53213	-	-	-	-
							1	3.27033	2.50990	-	-	-	-
							2	3.16308	2.40331	-	-	-	-
	a	10a	-	-	-	-	3	1.74798	1.34426	-	-	-	-
							1	1.73617	1.33247	-	-	-	-
							2	1.67923	1.27588	-	-	-	-
2	-10a	-a	a	10a	-	-	3	3.40282	2.91594	2.32950	1.51653	-	-
							1	3.36564	2.87179	2.28605	1.48136	-	-
							2	3.21251	2.68141	2.10524	1.35268	-	-
3	-10a	-a	a	10a	20a	30a	3	3.40436	2.92028	2.33766	1.53348	0.66689	0.47730
							1	3.36538	2.87326	2.29057	1.49399	0.64026	0.45626
							2	3.21166	2.68047	2.10496	1.35645	0.58450	0.41988
3	a	20a	40a	60a	80a	100a	3	2.17301	1.21992	0.24244	0.12483	0.02210	0.01084
							1	2.12330	1.17011	0.22197	0.11243	0.01980	0.00992
							2	1.93621	0.99285	0.19859	0.09705	0.01897	0.00923

Table 6. The normalized SIFs determined under fixed-grip loading *δh=ln0.2* for a strip weakened by different numbers of cracks for *h=80a*.

N	a_1	b_1	a_2	b_2	a_3	b_3	Material i	$\frac{K_I(a_1)}{q\sqrt{\pi a}}$	$\frac{K_I(b_1)}{q\sqrt{\pi a}}$	$\frac{K_I(a_2)}{q\sqrt{\pi a}}$	$\frac{K_I(b_2)}{q\sqrt{\pi a}}$	$\frac{K_I(a_3)}{q\sqrt{\pi a}}$	$\frac{K_I(b_3)}{q\sqrt{\pi a}}$
1	-10a	-a	-	-	-	-	3	2.47182	2.25708	-	-	-	-
							1	2.46711	2.25238	-	-	-	-
							2	2.44289	2.22817	-	-	-	-
	a	10a	-	-	-	-	3	1.98111	1.80900	-	-	-	-
							1	1.97734	1.80523	-	-	-	-
							2	1.95792	1.78583	-	-	-	-
2	-10a	-a	a	10a	-	-	3	2.62614	2.72796	2.52526	1.98949	-	-
							1	2.61609	2.71626	2.51358	1.97949	-	-
							2	2.56770	2.65841	2.45600	1.93190	-	-
3	-10a	-a	a	10a	20a	30a	3	2.63970	2.75272	2.56190	2.04849	1.52578	1.34135
							1	2.62685	2.73745	2.54634	2.03467	1.51313	1.32965
							2	2.57134	2.66871	2.47543	1.97221	1.46312	1.28739
3	a	20a	40a	60a	80a	100a	3	2.72729	2.26875	1.33696	1.07241	0.59932	0.47329
							1	2.69896	2.23815	1.30715	1.04525	0.58113	0.45869
							2	2.58965	2.11963	1.21563	0.96877	0.54084	0.42914

Table 7. The normalized SIFs determined under fixed-grip loading *δh=ln0.2* for a strip weakened by different numbers of cracks for *h=40a*.

N	a_1	b_1	a_2	b_2	a_3	b_3	Material i	$\frac{K_I(a_1)}{q\sqrt{\pi a}}$	$\frac{K_I(b_1)}{q\sqrt{\pi a}}$	$\frac{K_I(a_2)}{q\sqrt{\pi a}}$	$\frac{K_I(b_2)}{q\sqrt{\pi a}}$	$\frac{K_I(a_3)}{q\sqrt{\pi a}}$	$\frac{K_I(b_3)}{q\sqrt{\pi a}}$
1	-10a	-a	-	-	-	-	3	2.86555	2.38438	-	-	-	-
							1	2.84540	2.36426	-	-	-	-
							2	2.74840	2.26769	-	-	-	-
	a	10a	-	-	-	-	3	1.84073	1.53165	-	-	-	-
							1	1.82779	1.51872	-	-	-	-
							2	1.76548	1.45668	-	-	-	-
2	-10a	-a	a	10a	-	-	3	2.98195	2.78786	2.38092	1.69093	-	-
							1	2.94610	2.74504	2.33859	1.65647	-	-
							2	2.80070	2.56124	2.16138	1.52792	-	-
3	-10a	-a	a	10a	20a	30a	3	2.98424	2.79429	2.39305	1.71611	0.95441	0.75203
							1	2.94569	2.74722	2.34535	1.67539	0.92518	0.72833
							2	2.79939	2.55979	2.16092	1.53358	0.85553	0.67861
3	a	20a	40a	60a	80a	100a	3	2.35019	1.56811	0.50759	0.32121	0.09903	0.06215
							1	2.29087	1.50733	0.47657	0.29950	0.09244	0.05827
							2	2.07301	1.29635	0.42513	0.25816	0.08394	0.05111

Table 8. The normalized SIFs determined under fixed-grip loading *δh=ln5* for a strip weakened by different numbers of cracks for *h=80a*.

N	a_1	b_1	a_2	b_2	a_3	b_3	*Material i*	$\frac{K_I(a_1)}{q\sqrt{\pi a}}$	$\frac{K_I(b_1)}{q\sqrt{\pi a}}$	$\frac{K_I(a_2)}{q\sqrt{\pi a}}$	$\frac{K_I(b_2)}{q\sqrt{\pi a}}$	$\frac{K_I(a_3)}{q\sqrt{\pi a}}$	$\frac{K_I(b_3)}{q\sqrt{\pi a}}$
1	*-10a*	*-a*	-	-	-	-	*3*	1.80900	1.98111	-	-	-	-
							1	1.80523	1.97734	-	-	-	-
							2	1.78583	1.95792	-	-	-	-
	a	*10a*	-	-	-	-	*3*	2.25708	2.47182	-	-	-	-
							1	2.25238	2.46711	-	-	-	-
							2	2.22817	2.44289	-	-	-	-
2	*-10a*	*-a*	*a*	*10a*	-	-	*3*	1.98949	2.52526	2.72796	2.62614	-	-
							1	1.97949	2.51358	2.71626	2.61609	-	-
							2	1.93190	2.45600	2.65841	2.56770	-	-
3	*-10a*	*-a*	*a*	*10a*	*20a*	*30a*	*3*	2.02491	2.58986	2.82340	2.77936	3.62662	3.94279
							1	2.00763	2.56894	2.80178	2.75977	3.60764	3.92488
							2	1.94143	2.48302	2.70941	2.67342	3.52542	3.85099
3	*-100a*	*-80a*	*-60a*	*-40a*	*-20a*	*-a*	*3*	0.47329	0.59932	1.07241	1.33696	2.26875	2.72729
							1	0.45869	0.58113	1.04525	1.30715	2.23815	2.69896
							2	0.42914	0.54084	0.96877	1.21563	2.11963	2.58965

Table 9. The normalized SIFs determined under fixed-grip loading *δh=ln5* for a strip weakened by different numbers of cracks for *h=40a*.

N	a_1	b_1	a_2	b_2	a_3	b_3	*Material i*	$\frac{K_I(a_1)}{q\sqrt{\pi a}}$	$\frac{K_I(b_1)}{q\sqrt{\pi a}}$	$\frac{K_I(a_2)}{q\sqrt{\pi a}}$	$\frac{K_I(b_2)}{q\sqrt{\pi a}}$	$\frac{K_I(a_3)}{q\sqrt{\pi a}}$	$\frac{K_I(b_3)}{q\sqrt{\pi a}}$
1	*-10a*	*-a*	-	-	-	-	*3*	1.53165	1.84073	-	-	-	-
							1	1.51872	1.82779	-	-	-	-
							2	1.45668	1.76548	-	-	-	-
	a	*10a*	-	-	-	-	*3*	2.38438	2.86555	-	-	-	-
							1	2.36426	2.84540	-	-	-	-
							2	2.26769	2.74840	-	-	-	-
2	*-10a*	*-a*	*a*	*10a*	-	-	*3*	1.69093	2.38092	2.78786	2.98195	-	-
							1	1.65647	2.33859	2.74504	2.94610	-	-
							2	1.52792	2.16138	2.56124	2.80070	-	-
3	*-10a*	*-a*	*a*	*10a*	*20a*	*30a*	*3*	1.70646	2.42505	2.87118	3.15459	5.52251	6.70679
							1	1.65290	2.35290	2.79124	3.07675	5.44042	6.63108
							2	1.51816	2.15032	2.55674	2.83853	5.12749	6.34236
3	*-100a*	*-80a*	*-60a*	*-40a*	*-20a*	*-a*	*3*	0.06215	0.09903	0.32121	0.50759	1.56811	2.35019
							1	0.05827	0.09244	0.29950	0.47657	1.50733	2.29087
							2	0.05111	0.08394	0.25816	0.42513	1.29635	2.07301

Table 10. The normalized SIFs determined under fixed-grip loading $\delta h=ln10$ for a strip weakened by different numbers of cracks for $h=80a$.

N	a_1	b_1	a_2	b_2	a_3	b_3	Material i	$\frac{K_I(a_1)}{q\sqrt{\pi a}}$	$\frac{K_I(b_1)}{q\sqrt{\pi a}}$	$\frac{K_I(a_2)}{q\sqrt{\pi a}}$	$\frac{K_I(b_2)}{q\sqrt{\pi a}}$	$\frac{K_I(a_3)}{q\sqrt{\pi a}}$	$\frac{K_I(b_3)}{q\sqrt{\pi a}}$
1	-10a	-a	-	-	-	-	3	1.69324	1.92826	-	-	-	-
							1	1.68964	1.92466	-	-	-	-
							2	1.67112	1.90612	-	-	-	-
	a	10a	-	-	-	-	3	2.32390	2.64646	-	-	-	-
							1	2.31896	2.64152	-	-	-	-
							2	2.29355	2.61608	-	-	-	-
2	-10a	-a	a	10a	-	-	3	1.88048	2.49131	2.78183	2.79616	-	-
							1	1.87042	2.47955	2.77005	2.78602	-	-
							2	1.82265	2.42161	2.71172	2.73710	-	-
3	-10a	-a	a	10a	20a	30a	3	1.92422	2.57108	2.89967	2.98519	4.39051	4.99442
							1	1.90517	2.54792	2.87563	2.96332	4.36907	4.97410
							2	1.83440	2.45497	2.77469	2.86761	4.27451	4.88813
3	-100a	-80a	-60a	-40a	-20a	-a	3	0.21148	0.29481	0.67463	0.92460	1.98864	2.60378
							1	0.20194	0.28250	0.65433	0.90152	1.96318	2.57977
							2	0.18744	0.26073	0.60154	0.83307	1.85925	2.48162

Table 11. The normalized SIFs determined under fixed-grip loading $\delta h=ln10$ for a strip weakened by different numbers of cracks for $h=40a$.

N	a_1	b_1	a_2	b_2	a_3	b_3	Material i	$\frac{K_I(a_1)}{q\sqrt{\pi a}}$	$\frac{K_I(b_1)}{q\sqrt{\pi a}}$	$\frac{K_I(a_2)}{q\sqrt{\pi a}}$	$\frac{K_I(b_2)}{q\sqrt{\pi a}}$	$\frac{K_I(a_3)}{q\sqrt{\pi a}}$	$\frac{K_I(b_3)}{q\sqrt{\pi a}}$
1	-10a	-a	-	-	-	-	3	1.34426	1.74798	-	-	-	-
							1	1.33247	1.73617	-	-	-	-
							2	1.27588	1.67923	-	-	-	-
	a	10a	-	-	-	-	3	2.53213	3.29259	-	-	-	-
							1	2.50990	3.27033	-	-	-	-
							2	2.40331	3.16308	-	-	-	-
2	-10a	-a	a	10a	-	-	3	1.51653	2.32950	2.91594	3.40282	-	-
							1	1.48136	2.28605	2.87179	3.36564	-	-
							2	1.35268	2.10524	2.68141	3.21251	-	-
3	-10a	-a	a	10a	20a	30a	3	1.54026	2.39704	3.04354	3.66710	8.16037	10.8449
							1	1.47568	2.30767	2.94225	3.56538	8.04507	10.7365
							2	1.33758	2.08802	2.67410	3.26972	7.58146	10.2976
3	-100a	-80a	-60a	-40a	-20a	-a	3	0.01084	0.02210	0.12483	0.24244	1.21992	2.17301
							1	0.00992	0.01980	0.11243	0.22197	1.17011	2.12330
							2	0.00923	0.01897	0.09705	0.19859	0.99285	1.93621

4. Conclusions

In this study, SIFs at the edges of the cracks in an elastic strip weakened by *N*-collinear cracks are obtained for the special cases of $N=1$, $N=2$ and $N=3$. It is assumed that the crack sides are loaded by uniform crack surface pressure and fixed-grip loading.

The case of $N=1$ is approached by Gauss Quadrature formulas. The same problem has been solved by Iterative method and Gauss Chebyshev Quadrature in the previous studies [11,12] and the results for the normalized SIF values are compared with [13].

The case of $N=2$ is approached by Gauss Quadrature formulas and Iterative method. It is established that, when the distance between the cracks increases, i.e., $\varepsilon=a/b$ increases, normalized SIFs decrease. Also, it is obvious that the crack propagation starts at $x=\pm a$. When the crack sides are loaded by uniform crack surface pressure it has seen that $K_I(-a)/\left(q\sqrt{\pi a}\right)=K_I(a)/\left(q\sqrt{\pi a}\right)$ and $K_I(-b)/\left(q\sqrt{\pi b}\right)=K_I(b)/\left(q\sqrt{\pi b}\right)$. When the crack sides are loaded by fixed-grip loading, if loading conditions are given symmetrically as $q_1(x)=q_f e^{-\delta x}$, $q_2(x)=q_f e^{\delta x}$, then normalized SIFs at the both edges are equal, i.e., $K_I(-a)/\left(q\sqrt{\pi a}\right)=K_I(a)/\left(q\sqrt{\pi a}\right)$ and $K_I(-b)/\left(q\sqrt{\pi b}\right)=K_I(b)/\left(q\sqrt{\pi b}\right)$. Comparison of Table 4 and Table 10 shows that when the strip is weakened by symmetric two cracks, the SIFs under fixed-grip loading $\delta h=\ln 0.1$ at the left edge (a_1) are same with the SIFs under fixed-grip loading $\delta h=\ln 10$ at the right edge (b_2). The same comparison can be done for Table 6 and Table 8 for the fixed-grip loading $\delta h=\ln 0.2$ and $\delta h=\ln 5$.

The case of N is approached by Gauss Quadrature formulas. The presented results show that when crack numbers in a strip increase, cracking starts to spread faster. Also, it is obvious that increasing of crack length leads to increasing of SIFs. When the crack sides are loaded by uniform crack surface pressure, the crack starts to spread at the inner sides. On the other hand, when the crack sides are loaded by fixed-grip loading, if $\delta h<0$, cracking starts at the left crack; and if $\delta h>0$, cracking starts at the right crack.

Finally, for all cases of N from tables and figures, it can be concluded that under uniform crack surface pressure, when relative thickness of the strip increases SIFs increase, too. It causes to decrease of strip's resistance. If the crack sides are loaded by fixed-grip loading, for $\delta h<0$, while thickness of the strip decreases, the normalized SIFs on axis $-Ox$ increase; but rather, the normalized SIFs on axis Ox decrease. Another point, for $\delta h>0$, while the thickness of the strip decreases, the normalized SIFs on axis $-Ox$ decrease, conversely, the normalized SIFs on axis Ox increase. Also, the cracks in the strip composed of Material 3 starts to spread faster by comparision with the strips made from Material 1 and Material 2. So, between the strips which have same geometry and same cracks, the resistance of strip made from Material 3 is the lowest. Additionally, when material orthotrophy parameter E_1/E_2 increases, SIFs increase, too. As a result, when material anisotrophy increases, critical load decreases and so strip resistance decreases.

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