

Analytical Predictive Models for Lead-Core and Elastomeric Bearing

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Abstract: Constitutive models aimed at predicting the mechanical response of lead-core bearing devices for passive seismic isolation are discussed in this paper. The study is focused on single-degree-of-freedom models which provide a relation between the shear displacement (shear strain) and the shear force (shear stress) in elastomeric and lead-core rubber bearings. Classical Bouc-Wen model along with a numerical procedure for identification of the model constants is described. Alternatively, a constitutive relation introducing a damage variable aimed at assessing the material degradation is also considered.

Key words: Lead core bearing device, constitutive model, analytical models, material degradation.

1. Introduction

Among other strategies to minimize disastrous effects of seismic activity, the damping devices for seismic isolation occupy a considerable part. Damping devices have been used around the world, specifically in New Zealand, Japan, USA and Italy in the construction of bridges, large structures and residential buildings.

The use of seismic isolation results in the reduction of the peak values of the accelerations generated in a given structure during earthquake strong motion. The probability of damage to structural elements, to nonstructural components and to displacement sensitive and acceleration sensitive equipment is also reduced.

According to the generally accepted point of view, the use of passive seismic isolation aims at shifting the fundamental natural period of the isolated structure (compared to the non-isolated) to the long period range (e.g., 2 to 4 seconds). Technically, this is achieved by adding horizontally flexible damping

devices at the base of the structure in order to physically decouple it from the ground. Thus, the vibration of the structure becomes, to some extent, independent from the ground motion.

The most commonly used types of seismic isolation devices are elastomeric bearing and friction pendulum systems. This study is focused on the modeling of the mechanical response of the first type.

Elastomeric isolators consist of rubber layers separated by steel shims. The total thickness of the rubber layers provides the required low horizontal stiffness of the damping device needed to lengthen the fundamental natural period of the structure, whereas the close spacing between the intermediate steel shims provides large vertical stiffness needed to resist vertical loads.

The elastomeric isolators can be modified by adding a lead plug. The modified elastomeric bearing is often referred to as “LCR (lead-core rubber) bearing”.

The lead-plug yields relatively quickly under shear deformation and LCR bearing is then seen to have well-pronounced hysteresis response under cyclic loading paths.

The theory that the energy released by a mechanical system can be evaluated through the work done in

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hysteresis loops is widely accepted.

The dissipation capacity of LCR bearing is thus enhanced compared to elastomeric bearing since the former exhibits a more-pronounced hysteresis behavior.

Generally, the macroscopic mechanical response of a LCR bearing is modeled by using a SDOF (single-degree-of-freedom) constitutive model which defines the relation between the shear force and the shear strain. Classical Bouc-Wen [1-3] or rate independent plasticity model [4] can be used. In some cases, the time-dependent shear force could be approximated by the standard two-parameter, strain-independent Kelvin model (see for example, Ref. [5]).

LCR devices typically consist of components made of different materials (lead, rubber and steel) possessing different mechanical properties. In this context, SDOF models should provide a macro-characteristic representative for the mechanical response of this multiple-component system on the macro scale.

Constitutive models of LCR bearings have evolved from simple bi-linear models in which pre-yield stiffness is simply replaced by the post-yield stiffness after the yielding in the lead-core, to more sophisticated models involving differential equations. Within the scope of this paper, are the Bouc-Wen model and a macroscopic model in which a damage variable is introduced.

Original numerical procedures aimed at implementing these two models are developed by using the Python software. Elements of a procedure for curve fitting are also reported.

2. Methods and Materials

This study is aimed at developing numerical procedures capable of simulating the mechanical response of elastomeric bearings seen as single-degree-of-freedom systems. The procedures are based on widely accepted analytical macroscopic

models. They include appropriate algorithms for numerical integration and for identification of model constants.

The identification algorithm is inherently a sequence of comparisons between the output of the SDOF model and a target set of data points obtained experimentally or by finite element analysis.

The prospects of this study include implementation of the SDOF models in finite element models of damped structures and definition of coupling rule for the macroscopic model that should be used in case of bi-axial loading.

3. Bouc-Wen Model

The Bouc-Wen model consists of an equation that defines a “skeleton” curve (a relation between shear displacement and shear force) and another equation which determines the evolution of the hysteresis parameter.

The relation between the shear force Q in the LCR bearing device and the shear displacement u is obtained by Eq. (1). The evolution of the hysteresis parameter Z is defined by Eq. (2) (see Refs. [1-3, 6, 7]) as follows:

$$Q = \alpha \frac{Q_y}{d_y} u + (1 - \alpha) Q_y Z \quad (1)$$

$$d_y \frac{dZ}{dt} = -\gamma \left| \frac{du}{dt} \right| Z |Z|^{(\eta-1)} - \beta \frac{du}{dt} |Z|^\eta + A \frac{du}{dt} \quad (2)$$

where, in Eq. (1), u stands for the current displacement and Z for a dimensionless hysteresis component which should satisfy Eq. (2). The other model constants in Eq. (1) are defined as follows: α is the post-yielding to pre-yielding stiffness ratio, d_y is the displacement at which the yielding in the lead core takes place, and Q_y is the shear force, corresponding to the yielding. In Eq. (2), β , γ and A are dimensionless model constants that are to be identified by means of curve fitting; η controls the transition phase at yielding of the lead core.

Further, the modified mid-point method (see for

example, Ref. [8]) is used to integrate numerically the ordinary first-order differential Eq. (2):

$$\frac{dZ}{dt} = f(Z(u), \dot{u}) \quad (3)$$

$$z_0 = Z(u_i) \quad (4)$$

$$z_1 = z_0 + hf(Z(u_i), \dot{u}) \quad (5)$$

$$z_{k+1} = z_{k-1} + 2hf(Z(u_i + kh), \dot{u}) \quad (k=1, \dots, n-1) \quad (6)$$

$$Z(u_j) = \frac{1}{2}[z_n + z_{n-1} + hf(Z(u_i + H), \dot{u})] \quad (7)$$

This is an algorithm to calculate the value of the function Z at point u_j , provided the value at point u_i is known. In Eq. (7), $H = u_j - u_i$; n is the number of substeps into which the interval H is divided. Overdot denotes a derivative with respect to time. More details on this algorithm for numerical integration can be found in Ref. [8].

To solve Eqs. (1) and (2), a numerical procedure is created in Python software. The algorithm contains a subroutine for numerical integration by using the modified mid-point method (Eqs. (3)-(7)).

The output of the model in terms of shear force-shear displacement curves is presented in Fig. 1.

The model constants that yield these two curves are summarized in Tables 1 and 2.

4. Constitutive Relation Accounting for Material Degradation

An alternative approach, compared to the model discussed in the previous section can be found in Refs. [9-11].

This type of constitutive relations can be situated in line with models that are compatible with thermodynamics fundamentals [12].

An internal variable is introduced to assess mechanical degradation.

Total shear stress is split into three terms: a term which represents an elastic contribution (τ_e), and two terms describing overstresses relaxing in time

(τ_1 and τ_2):

$$\tau = \tau_e + \tau_1 + \tau_2 \quad (8)$$

The elastic contribution is evaluated as follows:

$$\tau_e = F(\gamma) \quad (9)$$

with, $F(\gamma)$ being a polynomial depending on the shear strain γ . Shear strain is usually defined as the ratio between the shear displacement and the total height of the rubber layers in the bearing device.

Overstresses are defined as functions of the shear strain, model parameters (E_1 , E_2) and internal variables (γ_1 , γ_2):

$$\tau_1 = F(E_1, \gamma, \gamma_1) \quad (10)$$

$$\tau_2 = F(E_2, \gamma, \gamma_2) \quad (11)$$

Evolutions of internal variables $\gamma_{v,1}$ and $\gamma_{v,2}$ are determined as follows (see Ref. [9]):

$$\dot{\gamma}_1 = \left(\frac{|\dot{\gamma}|}{\eta_1(\gamma, \dot{\gamma}, q_e)} + \nu_1 \right) \tau_{v,1} \quad (12)$$

$$\dot{\gamma}_2 = \left(\frac{H(-\dot{\gamma})}{\eta_2} + \nu_2 \right) \tau_{v,2} \quad (13)$$

where, ν_1 , η_2 and ν_2 are model constants. Generally, they are identified through curve fitting by using experimental data.

In Eq. (13), H stands for the Heaviside function:

$$H(q) = \begin{cases} 1 & \text{if } q > 0 \\ 0 & \text{if } q \leq 0 \end{cases} \quad (14)$$

As it can be seen in Eq. (12), it is presumed that the quantity η_1 is a function of the shear strain γ , shear strain rate $\dot{\gamma}$ and a damage parameter— q_e . Damage parameter supplies information about material degradation. Upon appropriate calibration on the basis of the current value of the damage parameter degrading phenomena in material can be rationally estimated.

In the present study, the evolution of the damage parameter is defined by using a governing equation proposed by Ref. [9]:

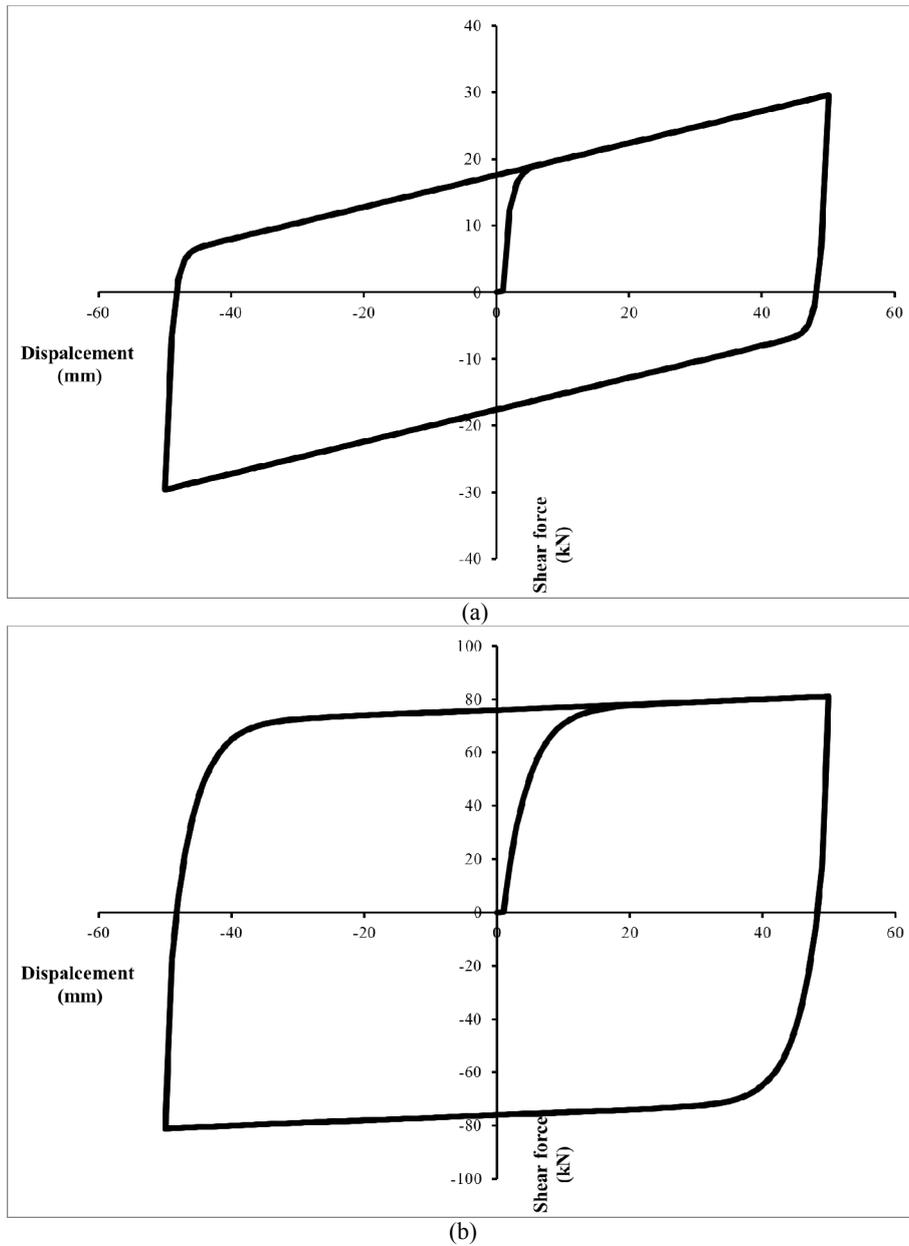


Fig. 1 Mechanical response of a lead-core bearing: shear displacement-shear force curves obtained by using a Bouc-Wen model based numerical procedure and model constants summarized in: (a) Table 1; and in (b) Table 2.

Table 1 Model constants used to obtain the shear force-shear displacement relationship plotted in Fig. 1a [6].

γ	η	β	α	A
0.3	1	0.1	0.12	2

Table 2 Model constants used to obtain the shear force – shear displacement relationship plotted in Fig. 1b [6].

γ	η	β	α	A
0.25	1	-0.15	0.05	2

$$\dot{q}_e = \begin{cases} \zeta|\dot{\gamma}|(0.5\gamma - q_e) & \text{if } q_e \leq 0.5|\gamma| \\ 0 & \text{if } 0.5|\gamma| \leq q_e \leq 1 \end{cases} \quad (15)$$

As already stated, damage parameter should enable the evaluation of the current state of mechanical damage after a given period of exploitation. The loading history which has led to the current state of mechanical degradation is taken into account through the variable γ —the shear strain and the time rate of

change of the shear strain $\dot{\gamma}$ (see Eq. (15)).

The scope of the present study is limited to numerical simulations of typical identification test, i.e., repetitive loading paths as the one shown in Fig. 2.

The evolution of the damage parameter obtained by integrating Eq. (15) is plotted in Fig. 3. Only the first quarter of one loading cycle (Fig. 2) is considered, i.e., variation of the shear strain from zero to 1.5 at a damage strain rate $\dot{\gamma} = 0.0333(3)$.

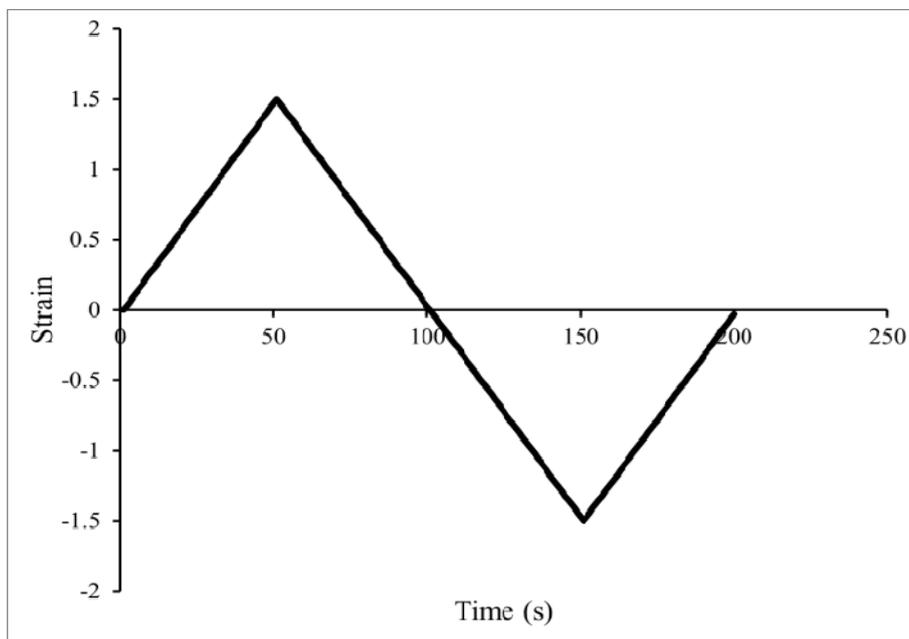


Fig. 2 Loading path.

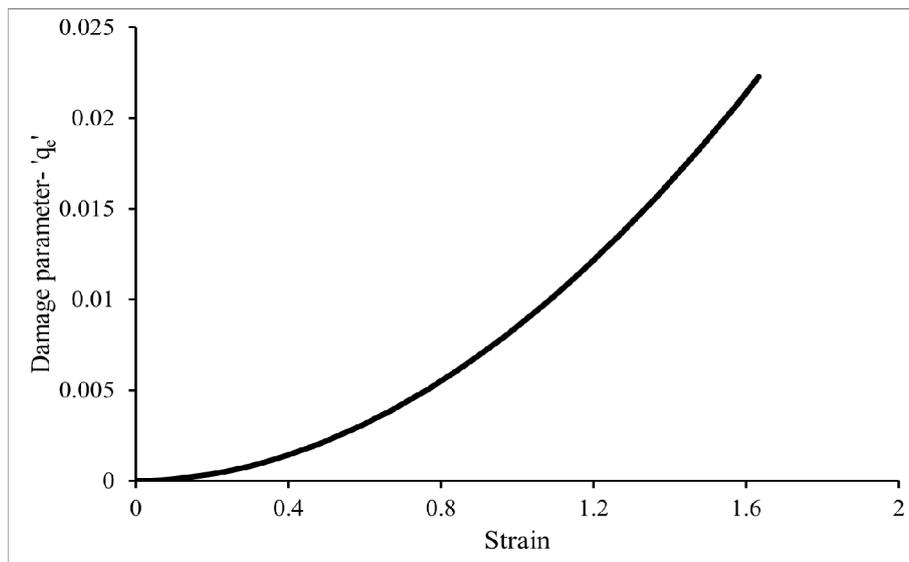


Fig. 3 Evolution of the damage variable in function of the shear strain.

5. Identification of the Model Parameters

The numerical procedure employed for the identification of model constants is outlined in this section. The identification procedure is based on genetic algorithm [13].

Curve fitting generally consists in finding a set of model parameters for which the results obtained by implementing the model fits best a target set of data points. Generally, the target set is an experimentally obtained relation that characterizes behavior of the modeled mechanical system. Alternatively, the target set of data points can be defined on the basis of results obtained by finite element analysis.

A chart-flow of the identification procedure is shown in Fig. 4.

The identification procedure starts by declaring an initial array containing a number of sets of model constants. It is important to note that expected values should be within the range of change of the initial model constants.

The initial array contains m sets of data points: $S_j^{(i)}$, $j=1\dots, m$ (the superscript “ i ” stands for initial).

In each iteration k of the identification procedure, all m sets are used consecutively as an input for the chosen constitutive model (e.g., the constitutive relation defined by Eqs. (1) and (2)). This operation is

performed in the “solution” module shown in Fig. 4.

The “adequacy” of a given output of the chosen constitutive law is evaluated in the comparison module (Fig. 4). In order to be able to conclude whether a set of model constants yields a satisfactory output, a metric should be defined in the space containing the data points. To do this, the procedure outlined in Ref. [14] can be used.

A test function $TF_k^{(j)}$ is associated to each set of model constants. The test function is normalized to give an output between 0 and 1. Thus $TF_k^{(j)}=1$ would mean a perfect match between the output of the model and the target curve whereas $TF_k^{(j)} \approx 1$ would mean that the tested set of model constant yields a set of data points that are too far from the target set of data points.

In order to improve the output of the model in the next iterations, the sets of model constants giving an output which is estimated as less convergent to the target set of data points are modified as follows:

$$TF_k^{(a)} < TF_k^{(b)} \tag{16}$$

$$S_{k+1}^{(b)} = S_k^{(b)} \tag{17}$$

$$S_{k+1}^{(a)} = \eta \cdot S_k^{(a)} + (1-\eta) \cdot S_k^{(b)} \tag{18}$$

In Eq. (18), η is a random number: $\eta \in (0, 1]$.

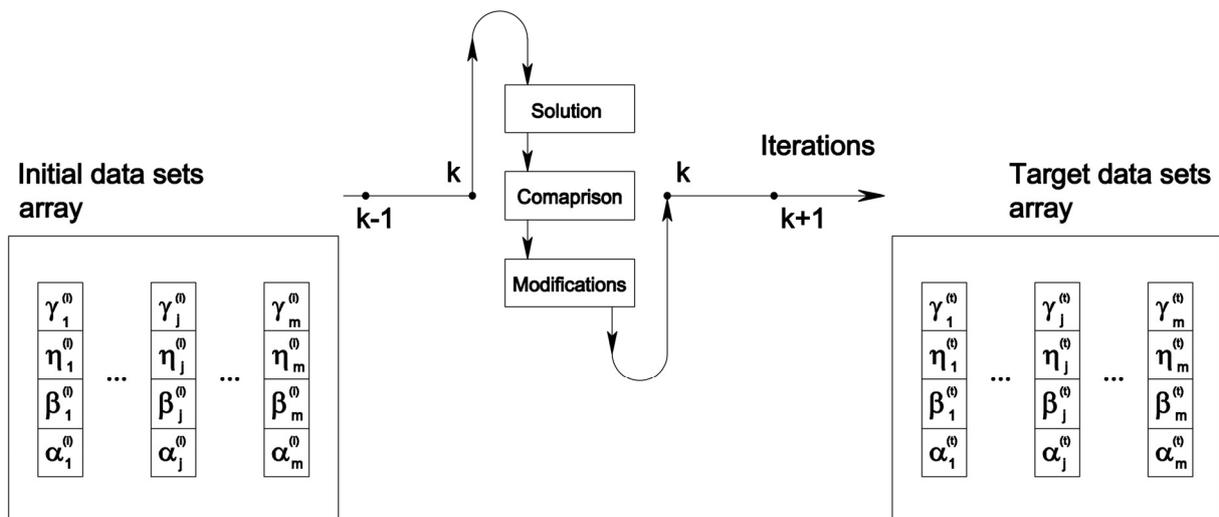


Fig. 4 Procedure for identification of the model parameters.

There are two ways to define a criterion for termination of the procedure. Either the maximum number of iterations can be limited to a predefined value or an acceptable value of the test function can

be set at the beginning of the procedure.

Finally, from the target array of sets $S_j^{(t)}$ (superscript “ t ” stands for target), the one yielding an optimal fit with the experimental data can be chosen.

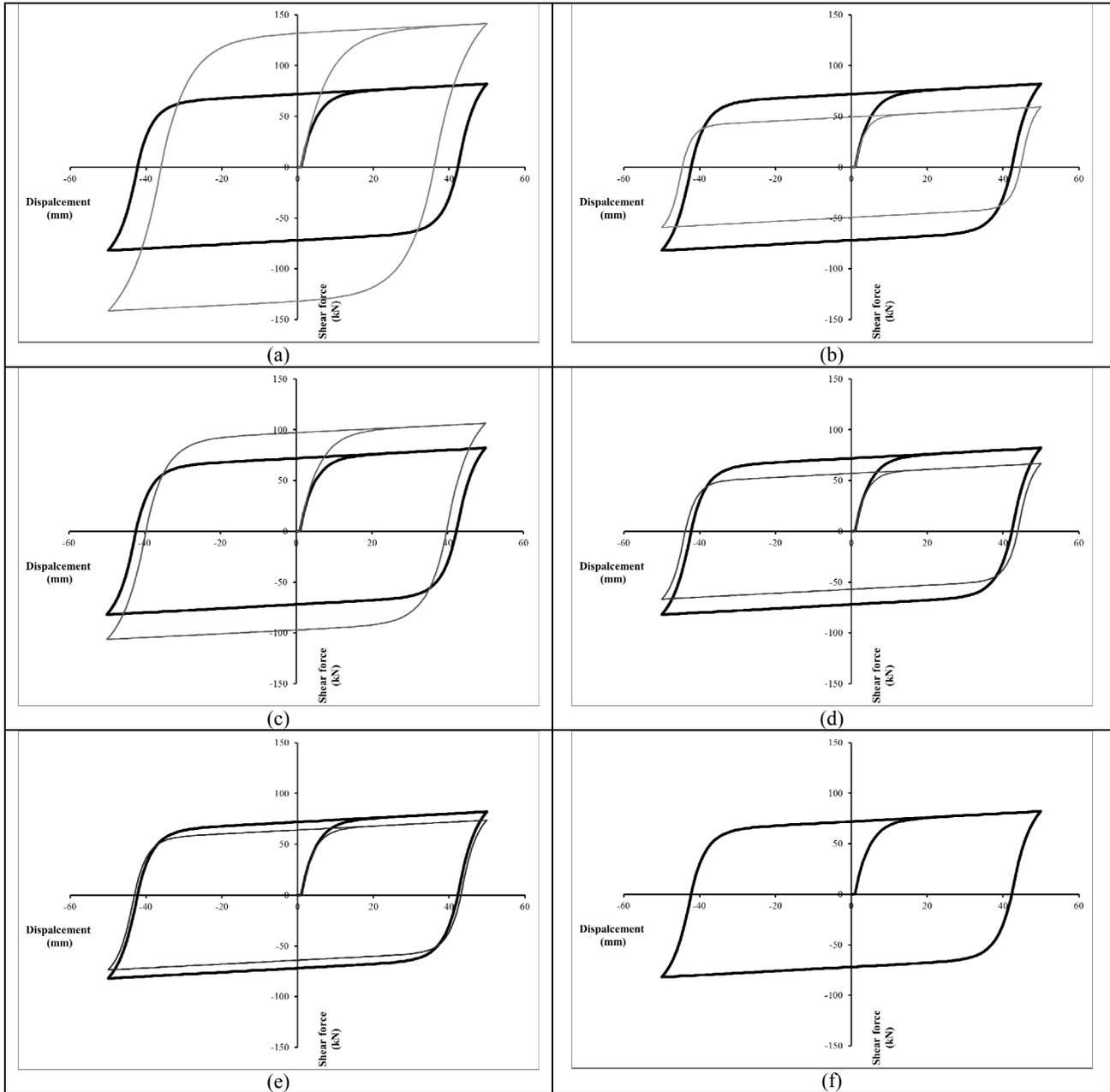


Fig. 5 Successive approximations (plotted in grey) to the trial curve (in black): (a) starting from a given set of material constant; (b) and further going through iterations k ; (c) $k + 1$; (d) $k + 2$; (e) $k + 3$; (f) the procedures eventually end by finding a set of constants that produce the best fit.

The implementation of the identification procedure is illustrated by choosing a target set of data points. Several steps of the identification procedure are shown in Figs. 5a-5f.

The target curve is plotted in black whereas the best approximation for a given step of the curve fitting procedure—in grey. It can be seen that the “best guess” for a given iteration is getting closer to the target data set with the progress of the curve fitting procedure.

6. Concluding Remarks

Analytical models aimed at reproducing the mechanical response of lead-core rubber and elastomeric bearings for passive seismic isolation have been discussed in this paper.

Numerical procedures have been developed by using Python software. These procedures have been created on the basis of models proposed in literature, namely the classical Bouc-Wen model and a constitutive relation that takes into account the mechanical degradation in an elastomeric bearing.

A curve fitting algorithm has also been presented. The procedure for identification of the model constants is based on genetic algorithms. Generally, identification is made through comparison with experimental data. The author believes that identification of single-degree-of-freedom constitutive models can be also performed by using data obtained by finite element analysis. The latter option might optimize the time-demanding and expensive experimental research program. However, it should be noted that the study outlined in this paper has to be completed by experimental tests on damping devices.

The proposed numerical procedures are to be subsequently integrated in the finite element analyses of damped structures to account for the effect of the seismic isolators. However, before implementing the discussed procedures in finite element models of

structures, a coupling rule should be defined for the case of bi-axial loading.

Acknowledgments

This paper is a result of a continuous collaboration with the Earthquake Engineering Research Centre, a part of the University of Iceland, started in 2013.

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