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Optimum Determination of Partial Transmission Ratios of Mechanical Driven Systems Using a V-belt and a Helical Gearbox with Second-Step Double Gear-Sets

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Abstract: This paper presents a study on the optimum determination of partial transmission ratios of a mechanical drive system using a V-belt and a helical gearbox with second-step double gear-sets in order to get the minimum size of the system. The chosen objective function was the cross section dimension of the system. In the optimization problem, the design equation for pitting resistance of a gear set was investigated and equations on moment equilibrium condition of a mechanic system including a V-belt and a helical gearbox with second-step double gear-sets and their regular resistance condition were analysed. Based on the results of the study, effective formulas for calculation of the partial ratios of the V-belt and a helical gearbox with second-step double gear-sets were proposed. By using explicit models, the partial ratios can be determined accurately and simply.

Key words: Transmission ratio, gearbox design, optimum design, V-belt drive, helical gearbox.

1. Introduction

In the problem of optimum gearbox design, optimum calculation of partial transmission ratios of the gearbox is one of the most important works. This is because the partial ratios are main factors affecting the dimension, the weight as well as the cost of the gearbox [1]. Consequently, optimum determination of the partial ratios of a gearbox has been subjected to many researches.

Till now, a lot of studies on the determination of the partial ratios of gearboxes have been done. The partial ratios were predicted for different gearbox types, such as for helical gearboxes [1-5], for bevel-helical gearboxes [1, 4, 6, 7] and for worm-gearboxes [4, 8]. In addition, many methods have been used for determining optimum partial ratios. These methods are the graph method [1, 2], the "practical method" [3] and modeling method [4-8].

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From above analysis, it is found that there have been many studies on the determination of the partial ratios for different types of gearboxes. However, until now, there was only a study for mechanical driven systems using a V-belt and a two-step helical gearbox [9]. There have not been studies for mechanical driven systems using a V-belt and a helical gearbox with second-step double gear-sets. This paper presents a study on optimum determination of partial ratios for mechanical systems using a V-belt and a helical gearbox with second-step double gear-set for getting the minimum system cross-sectional dimension.

2. Calculation of Optimum Partial Transmission Ratios

For a helical gearbox with second-step double gear-sets (Fig. 1), the cross-sectional dimension is minimum when d_{w21} and d_{w22} satisfy Eq. (1) [1].

$$d_{w21} = d_{w22} \tag{1}$$

From Eq. (1) and Fig. 1, for a mechanical drive system using a V-belt and a helical gearbox with

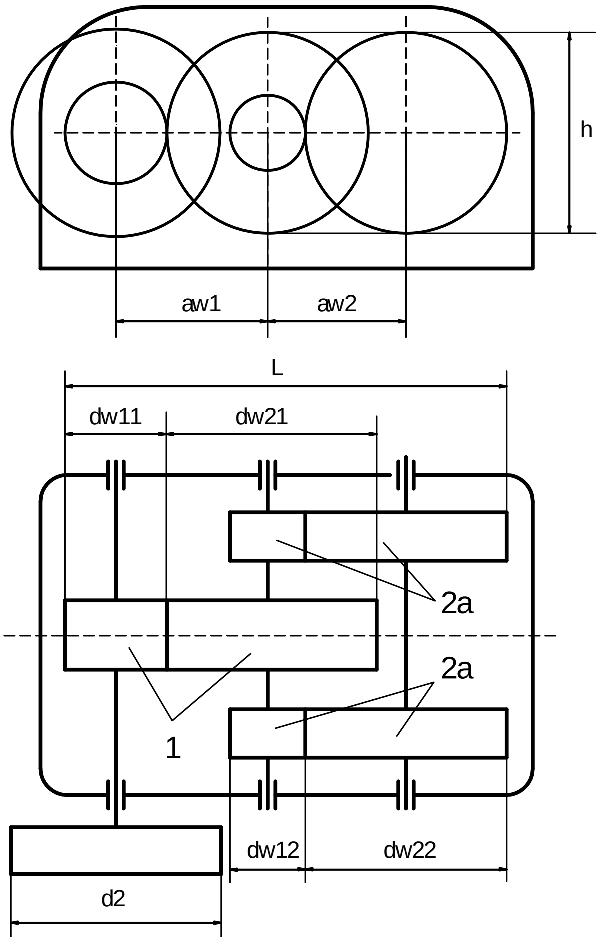

Fig. 1 Calculation chema for optimum determination of partial transmission ratios.

second-step double gear-sets, the cross-sectional dimension of the system is minimum when:

$$d_2 = d_{w21} = d_{w22} \tag{2}$$

Where d_2 is driven pulley diameter (mm); d_{w21} and d_{w22} are driven diameters of two gear units (mm).

For a helical gearbox with second-step double gear-sets, the optimum partial gear ratios u_1 and u_2 have been determined by Eq. (3) [5]:

$$u_2 = 1.2776 \cdot \sqrt[3]{\frac{K_{c2} \cdot \psi_{ba2}}{\psi_{ba1}} \cdot u_g} \tag{3}$$

In which u_g is the transmission ratio of the gearbox; ψ_{ba1} and ψ_{ba2} are coefficients of helical gear face width of steps 1 and 2; K_{c2} is coefficient; K_{c2} is 1.1 to 1.3 [5]. For this helical gearbox, it can be chosen as ψ_{ba1} = 0.3 and ψ_{ba2} = 0.35 [10]. Choosing K_{c2} = 1.2 and substituting ψ_{ba1}, ψ_{ba2} and K_{c2} into Eq. (3) gives Eq. (4):

$$u_2 = 1.3884 \cdot u_g^{1/3} \tag{4}$$

Eq. (4) is used to determine the partial ratio u_2 of the second step. The partial ratio u_1 of the first step can be calculated by Eq. (5):

$$u_1 = u_g / u_2 \tag{5}$$

From above analysis, it is found that for finding the optimum partial ratios of the systems in order to get the minimum system cross section, it is necessary to determine the diameters d_2 and d_{w22}.

2.1 Determining the Driven Pulley Diameter d_2

For a V-belt set, from tabulated data for determining allowable power [10], the regression model for calculation of driver diameter d_1 (with the determination coefficient R^2 = 0.9156) was found:

$$d_1 = 269.7721 \cdot [P_1]^{0.7042} / v^{0.5067} \tag{6}$$

Theoretically, the peripheral velocity of the belt can be determined as Eq. (7):

$$v = \pi \cdot d_1 \cdot n_1 / 60000 \tag{7}$$

From Eqs. (6) and (7), the diameter of the driver pulley can be determined by Eq. (8):

$$d_1 = 1093.8 \cdot [P_1]^{0.7923} / n_1^{0.6369} \tag{8}$$

Also, the diameter of driven pulley of a V-belt drive is calculated by Eq. (9) [10]:

$$d_2 = u_b \cdot d_1 \cdot (1 - \varepsilon) \tag{9}$$

Substituting Eq. (8) into Eq. (9) gives Eq. (10):

$$d_2 = 1093.8 \cdot u_b \cdot (1 - \varepsilon) \cdot [p_1]^{0.7923} / n_1^{0.6369} \tag{10}$$

Where ε is slippage coefficient; $\varepsilon = 0.01 \ldots 0.02$ [10]; u_b is the transmission ratio of the V-belt set; $[P_1]$ is the allowable power of the drive (kW); $[P_1]$ is calculated by Eq. (11):

$$[P_1] = n_1 \cdot [T_1] / (9.55 \cdot 10^6) \tag{11}$$

Choosing $\varepsilon = 0.015$ and substituting it and Eq. (11) into Eq. (10) gives Eq. (12):

$$d_2 = 0.0032 \cdot u_b \cdot n_1^{0.1554} \cdot [T_1]^{0.7923} \quad (12)$$

2.2. Determining the Driven Diameter d_{w22}

For the second-step with double gear-sets, the driven diameter can be determined by Eq. (13) [5]:

$$d_{w22} = \left[\frac{2.0786 \cdot \left[T_{out}\right] \cdot u_2}{\psi_{ba2} \cdot \left[K_{02}\right]} \right]^{1/3} \quad (13)$$

Where ψ_{ba2} = 0.35 (see section 2);

$$K_{02} = \left[\sigma_{H2}\right]^2 / \left(K_{H2}(Z_{M2} Z_{H2} Z_{\varepsilon 2})^2\right) \quad (14)$$

In which, $\left[\sigma_{H2}\right]$—Allowable contact stress of the second-step gear set (MPa); For a helical gear set, $\left[\sigma_{H2}\right]$ can be calculated by Eq. (15) [10]:

$$\left[\sigma_{H2}\right] = \left(\left[\sigma_H\right]_1 + \left[\sigma_H\right]_2\right) / 2 \quad (15)$$

Where $\left[\sigma_H\right]_1$ and $\left[\sigma_H\right]_2$ are the allowable contact stress of pinion and gear of the second step gear set (MPa). $\left[\sigma_H\right]_1$ is determined by Eq. (16):

$$\left[\sigma_H\right]_1 = \sigma_{H\lim1}^0 \cdot K_{HL} / S_H \quad (16)$$

In Eq. (16), $\sigma_{H\lim1}^0$ is allowable contact stress for based stress cycle life of the pinion: $\sigma_{H\lim1}^0 = 2 \cdot HB_1 + 70$; HB_1 is Brinell hardness of the pinion; K_{HL} is the stress cycle life factor; S_H is the safety factor. Since the material of the pinion is medium carbon steel 1045, it was found $HB_1 = 250$, $K_{HL} = 1$ and $S_H = 1.1$ [10].

Substituting $\sigma_{H\lim1}^0$, K_{HL} and S_H into Eq. (16) gives $\left[\sigma_H\right]_1 = 518.18$ (MPa).

Calculating the same way for the gear gives $\left[\sigma_H\right]_2 = 490.91$ (MPa).

Substituting values of $\left[\sigma_H\right]_1$ and $\left[\sigma_H\right]_2$ into Eq. (15) gives $\left[\sigma_{H2}\right] = 504.55$ (MPa).

K_{H2}—Contact load factor for pitting resistance; Since $K_{H2} = 1.1 - 1.3$ [10], authors can choose $K_{H2} = 1.2$;

Z_{M2}—Material factor; Since the pinion and gear are made from steel, Z_{M2} is 274 (MPa$^{1/3}$) [10];

Z_{H2}—Surface condition factor; With the pinion and gear are standard and the helix angles are $8^0 \div 20^0$, $Z_{H2} = 1.74 \div 1.67$ [10]. Therefore, authors can choose $Z_{H2} = 1.71$;

$Z_{\varepsilon 2}$—Loading sharing factor: $Z_{\varepsilon 2} = \left(1/\varepsilon_\alpha\right)^{1/2}$ [10] with ε_α is contact ratio. ε_α can be calculated by Eq. (17) [10]:

$$\varepsilon_\alpha = \left[1.88 - 3.2\left(1/z_1 + 1/z_2\right)\right] \cdot \cos\beta \quad (17)$$

With the helix angles are $8^0 \div 20^0$ and the number of teeth of pinion and gear are 15 to 90, the value of the transverse contact ratio is $Z_{\varepsilon 2} = 0.7628 \div 0.8344$. Therefore, the value of $Z_{\varepsilon 2}$ can be chosen as the average of these values, that is, $Z_{\varepsilon 2} = 0.7986$.

Substituting values of $\left[\sigma_{H2}\right]$, K_{H2}, Z_{M2}, Z_{H2} and $Z_{\varepsilon 2}$ into Eq. (14) gives Eq. (18):

$$K_{02} = 504.55^2 / \left(1.2 \cdot (274 \cdot 1.71 \cdot 0.7986)^2\right) = 1.5152 \quad (18)$$

Substituting $\psi_{ba2} = 0.35$ and $K_{02} = 1.5152$ into Eq. (13) gives Eq. (19):

$$d_{w22} = 1.5767 \cdot \left[T_{out}\right]^{1/3} \cdot u_2^{1/3} \quad (19)$$

2.3 Determining the Partial Ratios

Eqs. (2), (12) and (19) give Eq. (20):

$$0.0032 \cdot u_b \cdot n_1^{0.1554} \cdot [T_1]^{0.7923} = 1.5767 \cdot \left[T_{out}\right]^{1/3} \cdot u_2^{1/3} \quad (20)$$

Theoretically, the permissible torque on the drive shaft $\left[T_1\right]$ can be calculated from permissible torque on the output shaft $\left[T_r\right]$ by Eq. (21):

$$\left[T_1\right] = \left[T_r\right] / \left(u_t \cdot \eta_t\right) \quad (21)$$

Where u_t is the total transmission ratio of the system; η_t is the total efficiency of the system:

$$\eta_t = \eta_d \cdot \eta_{br}^2 \eta_o^3 \quad (22)$$

In which, η_d is V-belt efficiency (η_d is from 0.956 to 0.96 [2]); η_{br} is helical gear transmission efficiency (η_{br} is from 0.96 to 0.98 [2]); η_o is transmission efficiency of a pair of rolling bearing (η_o is from 0.99 to 0.995 [2]). Choosing $\eta_d = 0.955$, $\eta_{br} = 0.97$ and $\eta_o = 0.992$ [10] and substituting Eqs. (4), (21) and (22) into Eq. (20) with the note that

$u_g = u_t / u_b$ gives Eq. (23):

$$u_b = 266.3906 \cdot \left[T_{out}\right]^{-0.4962} \cdot n_1^{-0.1872} \cdot u_t^{0.9126} \quad (23)$$

Eq. (23) is used to calculate the speed ratio of the V-belt driver. After having u_b, the ratio of the gearbox is calculated by $u_g = u_t / u_b$ and the partial speed ratios of the gearbox u_1 and u_2 can be found by Eqs. (5) and (4), respectively.

3. Conclusions

The minimum system cross-sectional dimension of a mechanical drive system using a V-belt and a helical gearbox with second-step double gear-sets can be obtained by optimum splitting the total transmission ratio of the system.

Models for determining the partial ratios of the V-belt and the helical gearbox in order to get the minimum cross-sectional dimension of the system were proposed.

By using explicit models, the partial ratios of the V-belt driver and the helical gear units can be determined accurately and simply.

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