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Geometry Is the Common Thread in a Grand Unified Field Theory

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Abstract: The consequence of the wave-particle duality is a pointer to the fact that everything in the universe, including light and gravity, can be described in terms of particles. These particles have a property called spin. What the spin of a particle really tells us is what the particle looks like from different directions, in other words it is nothing more than a geometrical property. The motivation for this work stems from the fact that geometry has always played a fundamental role in physics, macroscopic and microscopic, relativistic and non-relativistic. Our belief is that if a GUT (Grand Unified Theory) is to be established at all, then geometry must be the common thread connecting all the different aspects of the already known theories. We propose a new way to visualize the concept of four-dimensional space-time in simple geometrical terms. It is observed that our time frame becomes curved, just as the space-frame, in the presence of a massive gravitating body. Specifically, in the event horizon of a black hole, where time seems to grind to a halt for external observers, the time frame appears to curve in on itself, forming an imaginary loop. This results in extreme time dilation, due to the strong gravitational field. Finally we adopt a descriptive view of a GUT called Quantum Necklace GUT which attempts to connect gravity together the other three fundamental forces of nature, namely the strong, weak and electromagnetic interactions.

Key words: GUT, geometry, space-time, spin, relativity, Einstein field equations.

1. Introduction

The concept of unification began in 1687 with the universal law of gravitation as established by Sir Isaac Newton. Since then unification has played a central role in physics. In the mid-19th century James Clerk Maxwell found that electricity and magnetism were two facets of electromagnetism. A century later electromagnetism was unified with the weak nuclear force governing radioactivity, in which physicists call the electroweak theory. Indeed this quest for unification is driven by practical, philosophical and aesthetic considerations. When successful, merging theories clarifies our understanding of the universe and leads us to discover things we might otherwise never have suspected. Much of the activity in experimental particle physics today, at accelerators such as the Large Hadron Collider at CERN near Geneva, involves a search for novel phenomena predicted by the unified electroweak theory. In addition to predicting new physical effects, a unified theory provides a more aesthetically satisfying picture of how our universe evolves. Numerous physicists share an intuition that, at the deepest level, all physical phenomena match the patterns of some beautiful mathematical structure.

The current best theory of non-gravitational forces—the electromagnetic, weak and strong nuclear force—was largely completed by the 1970s and has become familiar as the Standard Model of particle physics. Mathematically, the theory describes these forces and particles as the dynamics of elegant geometric objects called Lie groups and fiber bundles. It is, however, somewhat of a patchwork, this is because a separate geometric object governs each force. Over the years physicists have proposed various Grand Unified Theories, or GUTs, in which a single geometric object would explain all these forces, but no

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one yet knows which, if any, of these theories is true. One would expect that in a fully unified theory, gravity and matter should also combine naturally with the other forces, all as parts of one mathematical structure. Since the 1980s string theory, the dominant research program in theoretical particle physics has been an attempt to describe gravity and the Standard Model using elaborate constructs of strings and membranes vibrating in many spacetime dimensions.

String theory is however not the only effort. Loop quantum gravity as an alternative, uses a more minimal framework, and is closer to that of the Standard Model [1]. Building on its insights, the E8 theory [2] was proposed as a new unified theory in 2007. Here the basic idea is to extend Grand Unified Theories and include gravity as part of a consistent geometric framework. In this unified field theory, all forces and matter are described as the twisting of a single geometric object [3].

2. From Electromagnetism to Geometry

The geometric view of nature follows naturally from the way the world around us works. The simplest and most familiar examples are the forces of electricity and magnetism. Electric sparks, magnetic attraction and laser light are different manifestations of the electric and magnetic fields that pervade space. Physicists do believe that everything in the world—all the forces of nature and even all the particles of matter—arises from different kinds of fields. The behaviour of these fields suggests an underlying geometric structure.

3. Elementary Particles and Geometry

Using the wave-particle duality, it is now evident that everything in the universe, including light and gravity, can be described in terms of particles. These particles have a property called spin. One way of thinking of spin is to imagine the particles as little tops spinning about an axis. However, they do not really have any well-defined axis. What the spin of a particle really tells us is what the particle looks like from different directions. For example, a particle of spin 0 is like a dot, it appears the same from every direction. On the other hand, a particle of spin 1 is like an arrow, it appears different from different directions. It is only when it is rotated a complete revolution (360 degrees) does the particle appear the same. A particle of spin 2 can be visualized as a double-headed arrow, it appears the same if it is rotated half a revolution (180 degrees). Similarly, higher spin particles appear the same if rotated through smaller fractions of a complete revolution. Remarkably, it turns out that there are particles that do not appear the same if rotated through just one revolution: they must be rotated through two complete revolutions! Such particles are said to have spin ½. It is clear here that the concept of spin is nothing but a geometrical property of the particle under consideration.

4. Importance of Relativity Theory

A remarkable consequence of relativity [4] is the way it has revolutionized our ideas of space and time. We now know that there is no such thing as absolute time. On the other hand, one major prediction of general relativity [5] is that time should appear to be slower near a massive body like the earth. This is because there is a relation between the energy of light and its frequency, the greater the energy, the higher the frequency. As light travels upward in the earth’s gravitational field, it loses energy, and so its frequency goes down. This implies that the length of time between one wave crest and the next goes up. To an observer high up, it would appear that everything down below was taking longer to happen. Although light is made up of waves, Planck’s quantum hypothesis tells us that in some ways it behaves as if it were composed of particles: it can be emitted or absorbed only in packets, or quanta. Equally, Heisenberg’s uncertainty principle implies that particles behave in some respects like waves: they do not have a definite position but are “smeared out” with a certain probability distribution. There is thus a duality between waves and particles in quantum theory. For some purposes it is helpful to think of particles as waves and for other purposes it is better to think of waves as particles.

5. Notation

$(\mathbf{R}^4, g)$ will denote the 4-dimensional spacetime of GR, $\mathbf{R}$ is the real line and g is the metric tensor. An event in $(\mathbf{R}^4, g)$ is represented by the components of the contravariant position vector:

$$x^\mu = \left(x^0, x^1, x^2, x^3\right)$$

In the Cartesian basis the components of this vector are the time coordinate, $x^0 = t$, and the spatial coordinates, $x^i = \left(x^1, x^2, x^3\right) = \left(x, y, z\right)$. Differentiation of a function F with respect to the space-time coordinate x^μ is written as $\partial_\mu F \equiv \dfrac{\partial F}{\partial x^\mu}$. Performing a coordinate transformation simply computes the values of the coordinates at that event, from a different reference frame; this different frame could correspond to an observer that is accelerating relative to the first observer, or where the second observer is at a different point in a gravitational field. The coordinate transformation is:

$$x^\mu \to x'^\mu\left(x^\nu\right)$$

The Jacobian of the transformation $J^\mu{}_\nu$, and its inverse $J_\nu{}^\mu$, are given by:

$$J^\mu{}_\nu = \frac{\partial x'^\mu}{\partial x^\nu}, \quad J_\nu{}^\mu = \frac{\partial x^\mu}{\partial x'^\nu}, \quad J^\mu{}_\alpha J_\nu{}^\alpha \equiv \delta^\mu{}_\nu$$

6. Remark

- The components of a tensor may not be independent. If for example $A_{\mu\nu} = A_{\nu\mu}$ then we say that $A_{\mu\nu}$ is a symmetric tensor, and if $B_{\mu\nu} = -B_{\nu\mu}$, then $A_{\mu\nu}$ is antisymmetric.
- Einstein summation convention applies throughout this work.
- The quantity $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$ is referred to as a line element and $g_{\mu\nu}$ is the metric tensor.

7. The Flow of Spacetime

An elegant way of visualizing the curvature of spacetime $\left(x^0, x^1, x^2, x^3\right)$, is illustrated in Fig. 1. Here spacetime is composed of time axis pointing upwards in time and carrying with it at each point a triad $\left(x^1, x^2, x^3\right)$ subsequently referred to as the spaceframe. In the vicinity of a massive object both the time frame and the space frame become curved as shown. Hans von Baeyer [6], in a prize-winning essay described spacetime as an invisible stream flowing ever onward,

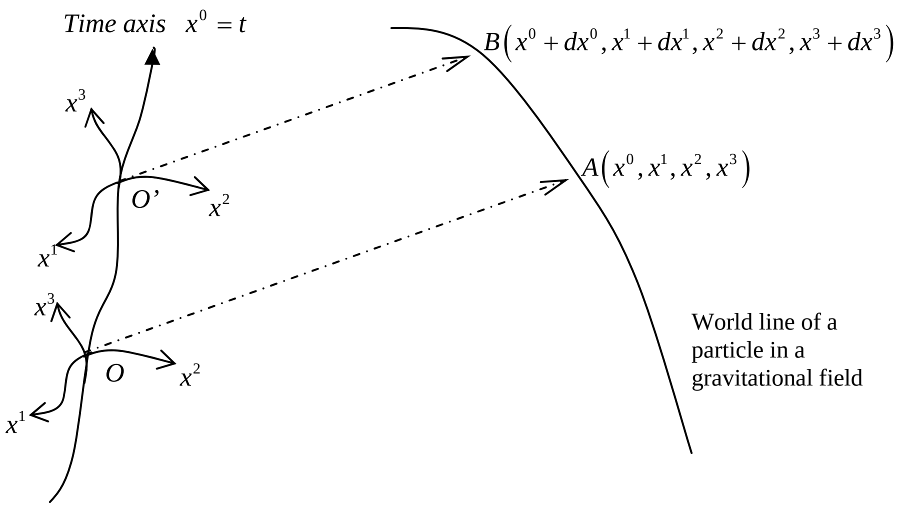

Fig. 1 The flow of curved spacetime.

bending in response to objects in its path, carrying everything in the universe along its twists and turns. The elemental distance between the two infinitesimally separated points A and B is given by the line element:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \tag{1}$$

The curvature of the region of space is specified by the metric tensor $g_{\mu\nu}$.

The case of flat spacetime Fig 2, is similar except for the fact that there is no curvature, and this situation corresponds very well with the standard Minkowski's four-dimensional space. In flat spacetime a particle moves from point A to a point B with coordinates as shown. The observer in his frame of reference initially at O, monitors the particle until it reaches point B by which time his frame has traveled upwards to reach O' in time by $dx^0 = dt$. We define the line element:

$$ds^2 = \left(dx^0\right)^2 + \left(dx^1\right)^2 + \left(dx^2\right)^2 + \left(dx^3\right)^2 \tag{2}$$

as the elemental distance (squared) between the two infinitesimally separated points A and B. By introducing the Kronecker-delta $\delta_{ij} = 0,\ (i \neq j),\ 1\ (i = j)$, the above can be written more compactly as:

$$ds^2 = \delta_{ij} dx^i dx^j \tag{3}$$

An important feature of the Kronecker-delta is that its determinant is positive definite. A manifold endowed with a metric whose determinant is positive-definite is called a Riemannian manifold. It is important to know that the metric actually defines the geometry of a space.

Our definition here coincides with the Minkowski's four-dimensional spacetime if we take;

$$x^0 = it \ \Rightarrow\ dx^0 = idt,\ i = \sqrt{-1}$$

Hence $\left(dx^0\right)^2 = -dt^2$ and

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu = -dt^2 + dx^2 + dy^2 + dz^2$$

with $\eta_{\mu\nu} = \mathrm{diag}\left(-1,1,1,1\right)$.

8. Properties of the Einstein Field Equations

We write the Einstein Field equations of general relativity as:

$$G_{\mu\nu} = \gamma T_{\mu\nu} \tag{4}$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$ is the Einstein tensor, $R = g^{\mu\nu} R_{\mu\nu}$ is the Ricci tensor. $T_{\mu\nu}$ is the energy-momentum tensor $\gamma = 8\pi G/c^4$, G being the Newtonian gravitational constant. At this point we consider the following three basic properties of Eq. (4):

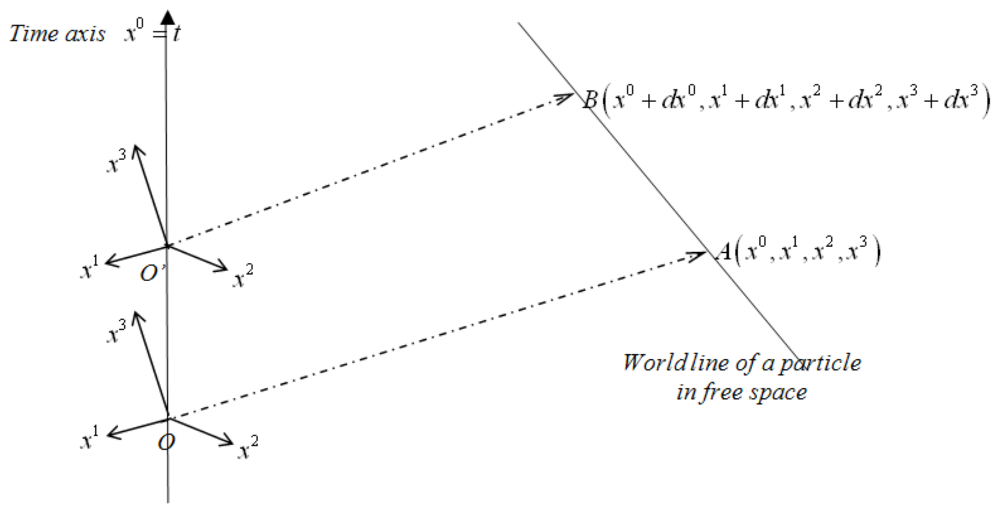

Fig. 2 The flow of flat spacetime.

(a) Eq. (4) is a tensor equation. This is necessary so, since the principle of invariance under coordinate transformations must hold, in other words the equations of physics must look the same in any frame of reference.

(b) We can interpret Eq. (4) more simply as:

Tensor representing geometry of space = Tensor representing energy content of space

i.e., it is the presence of matter (or energy) in space that distorts the neighbouring geometry. Most equations of mathematical physics can be interpreted similarly.

(c) The solution to Eq. (4) is a geometrical object, namely a line element given by:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

where $g_{\mu\nu}$ is the metric tensor to be solved for in Eq. (4).

GR (General Relativity) is one of the greatest advances in theoretical physics from the past century. Remarkably, Albert Einstein was able to understand that the geometry of the universe is determined by, and responds to, the gravitating content of the universe. Einstein proceeded to construct the dynamical rules for spacetime with arbitrary geometry; these rules are precisely the Einstein's field equations, Eq. (4) of GR. It became evident that gravity is just a manifestation of the location-dependence of the metric $g_{\mu\nu}$. The field equations determine the values of the components of the metric of a spacetime for some known content. Once the metric is known one can begin to compute geodesics in the spacetime; these are the paths that bundles of light rays travel along or the orbits that planets trace out. This enables GR, as a gravitational theory, to predict directly observable quantities.

9. Exact Analytic Solutions of Einstein's Field Equations

No general solution to Einstein's field equations, Eq. (4), is known to exist. However, solutions are known for rather specific configurations. The simplest solution is for a spacetime which is completely empty (of gravitating matter) is called a vacuum, and whose metric is given by the Minkowski metric, viz:

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

The next simplest metric is that for a spacetime containing a homogeneous and isotropic fluid, whose energy-momentum tensor is of the form:

$$T_{\mu\nu} = \rho u_\mu u_\nu + P\gamma_{\mu\nu} \tag{5}$$

where $\rho = \rho(t), P = P(t)$, the time dependent density and pressure respectively, and is given by the Freidmann-Robertson Walker's solution,

$$ds^2 = -dt^2 + a^2(t)\left(dx^2 + dy^2 + dz^2\right) \tag{6}$$

The metric of a spacetime [8] which in the limit tends to a vacuum solution at infinity and containing a single stationary black hole of mass *M* at the origin, is given by Schwarzschild's solution [9],

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 \tag{7}$$

$d\Omega^2 \equiv d\theta^2 + \sin^2 d\phi^2$ is the solid angle element, and *M* is the mass of the blackhole.

The metric for a spacetime containing a stationary black hole of mass *M* in a universe containing a cosmological constant Λ is given by Kottler's solution [10]:

$$ds^2 = -\left(1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2\right)dt^2 + \left(1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2\right)^{-1} dr^2 + r^2 d\Omega^2 \tag{8}$$

This is the well known Schwarzschild de-Sitter metric. The cosmological constant Λ [7], was introduced by Einstein deliberately to cancel out the expansion of the universe, predicted by the field equations, at the time this did not make sense. However, Edwin Hubble [11] in 1929, proved that the universe was indeed expanding.

A single black hole of mass M immersed in a homogeneous isotropic fluid has a metric given by McVittie's solution [12, 13]:

$$ds^2 = -\left(\frac{2a(t)r - M}{2a(t)r + M}\right)dt^2 + a^2(t)\left(1 + \frac{M}{2a(t)r}\right)^4\left(dr^2 + r^2 d\Omega^2\right) \quad (9)$$

The important thing to note in each of the above solutions is the fact that the coefficient of the time elemental squared is non constant and this leads us to conclude that the time frame is not flat, but naturally curved just as the space frame is.

In the case of the Schwarzschild's solution we have a singularity at $r = 2M$ in the space frame, but in the time frame, time disappears for this same value. Now $r = 2M$ is the event horizon in Fig. 3: a region in the vicinity of the stationary black hole where time seems to grind to a halt.

Our proposition here is that the time frame has completely curved in on itself, forming a kind of imaginary loop, resulting in extreme time dilation, due to the strong gravitational field of the black hole. This is also indirectly as a result of the fact that light cannot escape from the black hole. In the case of Kottler's solution incorporating a cosmological constant Λ, the event horizon will be given by the real roots of the cubic:

$$\Lambda r^3 - 3r + 6M = 0 \quad (10)$$

In this case there is a possibility of three singularities. For McVittie's solution, time disappears for $r = M/2a(t)$, however this does not give rise to a singularity in the space frame, though it is curved to a certain degree, and this can be measured.

10. Quantum Gravity and Unified Theories

The unification of general relativity and quantum mechanics into a theory of quantum gravity is of great interest to theoretical physicists and mathematicians. This project is vigorously being pursued in numerous scientific institutes and laboratories all around the world. If successful, it should also provide a consistent framework for incorporating all other fundamental forces of nature. Unfortunately despite intense efforts over the last years it is far from clear at this time what a consistent theory of quantum gravity will look like, and what its main features ought to be. In the light of these uncertainties, the best strategy appears to be one which is both diversified and interdisciplinary. To that extent, it would make sense

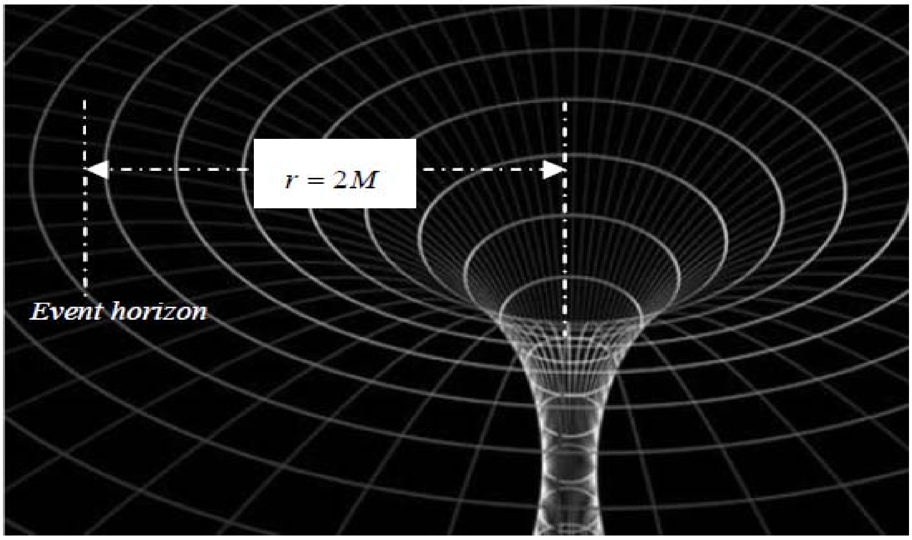

Fig. 3 The event horizon.

to incorporate all the major current approaches to quantum gravity, in particular supergravity [14] and string theory [15] and their modern developments, as well as canonical quantization (e.g. loop quantum gravity [16]) and discrete models of quantum gravity. We do not intend to go into the theory of quantum gravity here, however it is important to note that the current canonical approaches to this theory, emphasize the geometrical aspects which appear well suited to deal with unsolved conceptual issues of quantum gravity, such as for example, the problem of time or the interpretation of the wave function of the universe [17].

11. Quantum Geometry

Every geometry is associated with some kind of space. Quantum (or noncommutative) geometry [18-20] deals with quantum spaces, including the classical concept of space as a very special case. In classical geometry spaces are always regarded as collections of points equipped with the appropriate additional structure (as for example a topological structure given by the collection of open sets, or a smooth structure given by the atlas). In contrast to classical geometry, quantum spaces are not interpretable in this way. In general, quantum spaces have no points at all! They exhibit non-trivial quantum fluctuations' of geometry at all scales. Quantum geometry as constructed by Alain Connes [21] is a well developed field of active research with interesting applications. What follows is a simple example of what quantum geometry is.

12. The Quantum Plane

A simple example of quantum geometry is the quantum plane (Fig. 4). Usually, a plane is described by two coordinate functions x, y. Naturally, the functions xy and yx are the same since it does not matter whether you measure *x* first and then *y* or *y* first and then *x*. This is precisely what is lost in the quantum world.

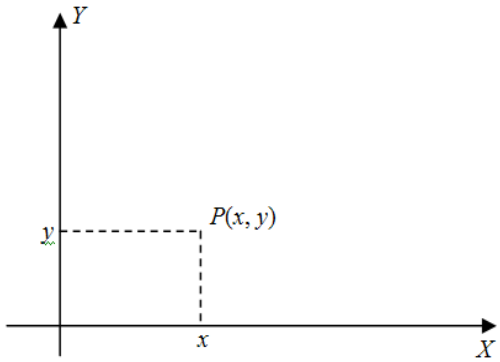

Fig. 4 The quantum plane $(xy \neq yx)$.

In the quantum plane we replace the property $xy = yx$ by $xy = qyx$ where *q* is some parameter. We no longer have points, however we can continue to work algebraically with *x* and *y*.

Define $[x, y] = xy - yx$, the commutator bracket. Hence $xy = qyx$ can be rewritten as:

$$[x, y] = (q-1)\, yx$$

The commutative case is obtained when $q = 1$.

13. Summary and Conclusion

From the foregoing, it is clear that our objective was not to postulate a GUT, but rather to examine existing efforts towards this goal and the inherent difficulties. More importantly we emphasize that geometry is intrinsic to all aspects of physics. Our view is that a successful construction of a GUT must incorporate geometry as the common thread connecting all the different aspects of the already established theories.

It is now a general belief that all four of the fundamental forces of nature are, in fact, the manifestations of a single unifying force which has yet to be discovered. Just as electricity, magnetism, and the weak force were unified into the electroweak interaction, they work to unify all of the fundamental forces. From our current understanding of physics, forces are not transmitted directly between objects, but instead are described by intermediary entities called fields. All four of the known fundamental forces are mediated by fields, which in the Standard Model [18]

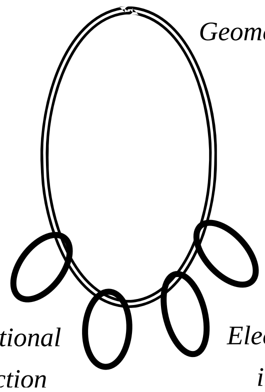

Fig. 5 Quantum Necklace GUT.

of particle physics result from exchange of gauge bosons. Specifically the four interactions to be unified are:

- **Strong interaction**: the interaction responsible for holding quarks together to form hadrons, and holding neutrons and also protons together to form nuclei. The exchange particle that mediates this force is the gluon.
- **Electromagnetic interaction**: the familiar interaction that acts on electrically charged particles. The photon is the exchange particle for this force.
- **Weak interaction**: a short-range interaction responsible for some forms of radioactivity, which acts on electrons, neutrinos, and quarks. It is governed by the W and Z bosons.
- **Gravitational interaction**: a long-range attractive interaction that acts on all particles. The postulated exchange particle has been named the graviton.

We also recall that the quest to unify gravity with the other three fundamental forces is known as quantum gravity. While research efforts are still ongoing in this field, we adopt the following descriptive view of a GUT which we call Quantum Necklace GUT, as shown in Fig. 5.

In the above figure we propose *Quantum Geometry* as described in Section 11, as a good candidate for the geometry connecting all the fundamental forces of nature. Our research focus for the future is to show in a mathematically rigorous way that quantum geometry does indeed unify all the fundamental forces of nature.

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