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Gas Turbines Dimensions Optimization by Scale Factor Design Principle Analysis

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Abstract: In the present work and on the basis of a previous study, we expect to determine optimum methods of gas turbine design. We will look for a design of more efficient machines using the principle of scale factor. It is a question of modifying of machines dimensions that we are going to look for.

Key words: Gas turbine, scale factor, rate flow, performances.

1. Basic Equation, Equation Resolution: [1]

From the meridien flow equation of the axial turbo machinery with single stage and if it is supposed that the forces of blading are negligible, the general equation in co-ordinates(z, r, θ) is written for multistage machines as Eq. (1) [2]:

$$\frac{\partial 2\psi}{\partial r2}-\frac{1}{r}\frac{\partial\psi}{\partial r}+\frac{\partial 2\psi}{\partial z2}=0 \qquad (1)$$

ψ is stream function; this is by holding account that the radial component of the force of blading is negligible, in particular in the following cases:

(1) The blades are radial,

(2) The flow is in free vortex,

(3) We are outside the blade zone.

It is the Laplace equation in cylindrical co-ordinates which governs flows in free vortex, in which many works treat its resolution. We try on this study to proceed to a new resolution.

By considering the boundary conditions of Dirichlet and Neumann, the expression of the steam function is written for nominal rate flow *Q*, which crosses the machine, in Eq. (2) [1]:

$$\psi=-\frac{QR_1^2R_2^2}{2\pi(R_1^2-R_2^2)}\cdot\frac{1}{r^2}+\frac{QR_1^2}{2\pi(R_1^2-R_2^2)} \qquad (2)$$

For two D:

$$r=R_1R_2\sqrt{\frac{Q}{2\pi(R_2^2-R_1^2)}}z_1^2 \qquad (3)$$

$z_1^4 = z^2 + 4H$

In Eq. (3), R_1 and R_2 are the hub and the casing radius of last stage of the machine.

If we note Eq. (4):

$$\lambda=\frac{R_2^2}{R_1^2}=2 \qquad (4)$$

Then the stream function does not depend on R_1 and depend on flow rate *Q* (of the flow velocity), then Eq. (5):

$$\psi=\frac{Q}{2\pi}(\frac{R_2^2}{r^2}-1) \qquad (5)$$

And Eq. (6):

$$r=R_2\sqrt{\frac{Q}{2\pi}}z_1^2 \qquad (6)$$

2. Stream Function

Stream function: *(r=f(z))* is Eq. (7):

$$r\big|_{z=H}=k\times r\big|_{z=0} \qquad (7)$$

$$k=\sqrt{1+H/4}$$

For gas turbine with two stages and if $R_{2.2}$ is the casing radius of second stage and $R_{1.1}$ of the first stage, we pose:

$$\frac{R_{2.2}}{R_{1.1}}=\delta \qquad (8)$$

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Let us suppose that the line which passes through the two points of the axis has the length H'.

Flow is on free Vortex:

$$\frac{\Delta r}{\Delta z} = cte$$

If we take the axis that passes through the center of gravity (Fig. 1), the slope of this line is:

$$a = \frac{R_{2.2} - R_{1.1}}{H} = \frac{R_{1.1}(\delta - 1)}{H}$$

For all machines designed by scale factor, and by multiplying dimensions with the coefficient ξ we have:

$$a = Cte$$

Then:

$$r = a.z = \frac{R_{1.1}(\delta - 1)}{H}.z + b$$

For:

$$z = 0$$

We have:

$$r = R_{1.1}$$

And:

$$r = a.z = \frac{R_{1.1}(\delta - 1)}{H}.z + R_{1.1}$$

If:

$$z = H$$

Then:

$$R_{2.2} = R_{1.1}\delta$$

Then:

$$r = a.z = (\frac{\delta - 1}{H}.z + 1)R_{1.1} \quad (9)$$

If:

$$\delta = 2$$

We have Eq. (10):

$$r = a.z = (\frac{z}{H} + 1)R_{1.1} \quad (10)$$

Casing radius do not depend on δ, it depends on $R_{1.1}$.

3. Dimensions Optimization

3.1 Rate Flow Optimization

Rate flow expression is:

$$Q = V_z.S = V_z.\pi(r^2 - r_1^2) = V_z.\pi[(\frac{z}{H}+1)^2 R_{1.1}{}^2 - r_1^2]$$

With r_1 the inferior radius(hub radius).

Maximal rate flow is for:

$$z = H$$

And:

$$r_1 = R_1$$

Then Eq. (11):

$$Q_{\max} = V_z.[4\pi R_{1.1}{}^2 - \pi R_1{}^2] = V_z[4\pi.R_{1.1}{}^2 - 4\pi.R_{1.2}{}^2] = 4.V_z.S_1 = 4.Q \quad (11)$$

With $R_{1.2}$ the radius of hub for the first stage of rotor and:

$$S_1 = \pi(R_{1.1}{}^2 - R_{1.2}{}^2)$$

Then Eq. (12):

$$Q_{\max} = 4Q \quad (12)$$

If we design a machine from an initial turbine and if we use the scale factor principle and with:

$$\delta = 2$$

This factor is constant for all machines designed.

3.2 Scale Factor

The value of axial velocity V_z in the meridian stream plane is: [1]

$$V_z = \frac{QR_1{}^2 R_2{}^2}{\pi(R_1{}^2 - R_2{}^2)}.\frac{1}{r^4} \quad (13)$$

Eq. (13) is negative because the large radius to the small direct sense of the velocity; the maximal value of the axial velocity is for $r = R_1$.

For expression of the maximum of volume flow rate, we take the absolute value of axial velocity as Eq. (14):

$$Q_{\max} = \lambda.Q \quad (14)$$

With: $\lambda = \frac{R_2{}^2}{R_1{}^2}$

And: $V_{z\max}$: Maximal axial velocity.

S: Section of exhaust turbine.

Q: Total volume flow rate (nominal).

When we multiply the hub and the casing radius by the coefficient ξ such as $\xi^2 = \lambda = \frac{R_2{}^2}{R_1{}^2}$, the maximal value of the flow rate is proportional to the square of ξ, and the maximum variation of the flow rate according to the nominal flow rate is thus linear. The slope is the ratio of the square of casing to hub radius. The maximum of flow rate corresponds to a nominal of the new machine [1].

We have machines dimensions with:

$Q_{\max} = 4Q$

A geometric progression of expression $\lambda = 4^n$ of reason, the relative number $\lambda = 4$ will be the subject of a multitude of machines, among which we can choose which is appropriate for our need.

We choose two factors:

– For $n = 1$, we have $\xi = 2$
– For $n = -1$, we have $\xi = 0.5$

We have the same manner [01] by multiplying the dimensions with the factor $\xi = 2$, the rate flow will be:

$$Q_\lambda = 4Q$$

We can design machines with less dimensions, but better performances when we choose the conditions:

$$\lambda = \frac{R_2}{R_1} = 2$$

And:

$$\delta = \frac{R_{2.2}}{R_{1.1}} = 2$$

And length H:

$$H = Cons\tan te$$

The stream function is then preserved:

$$\psi_{r.\lambda} = \lambda.\psi\,; \quad r_\lambda = \xi.r$$

When we use scale factor principle [1], we can say that by varying all dimensions by ξ time, we obtain stream function proportional to the square of the factor ξ. The stream function is then preserved [1]:

$$\psi_{r.\lambda} = \lambda.\psi\,; \quad r_\lambda = \xi.r$$

With:

$$z_1^4 = z^2 + 4H$$

And:

$$\xi.z_1^2 = z^2_{1.\lambda}$$

The radius r_λ is proportional to $z^2_{1.\lambda}$ and the factor of proportionality is the same that of the initial machine.

With identification, we obtain Eq. (15):

$$Q_{\lambda.ait} = \lambda.Q_{air} \qquad (15)$$

And Eq. (16):

$$Q_{\lambda.g} = \lambda.Q_g \qquad (16)$$

With $Q_{\lambda.air}$ and $Q_{\lambda.g}$ respectively the air flow rate and the flow rate of natural gas of the new machine, Q_{air} and Q_g are for the initial machine.

We will proceed of the same manner to design the axial compressor of the new machine by multiplying its radius by the same factor ξ.

We can determine machine designs by combining the data in Table 1.

4. Design Cases

(1) We can do calculation for a new machine using our principle such as: $Q_{\max} = 4.Q$ and multiplying the dimensions of the machine designed by the coefficient $\xi = 0.5$ ($\lambda = 0.25$). We have designed another machine of the same rate $Q_\lambda = Q$ as that of the initial machine but with smaller dimensions.

(2) With the same manner instead of using the scale factor $\xi = 2$ ($\lambda = 4$) [02], which satisfies: $Q_{\lambda.1} = 4.Q$, we use our concept, and some will have smaller dimensions.

Table 1 Coefficient λ and scale factors.

Scale factor ξ	Coefficient λ	Our machine rate flow	Rate flow [2] $Q_{\lambda.1} = \lambda.Q$
0.5	$\lambda = 0.25$	$Q_\lambda = Q$	$Q_{\lambda.1} = 0.25Q$
1	$\lambda = 1$	$Q_{\max} = 4.Q$	$Q_\lambda = Q$
2	$\lambda = 4$	$Q_\lambda = 16.Q$	$Q_{\lambda.1} = 4.Q$

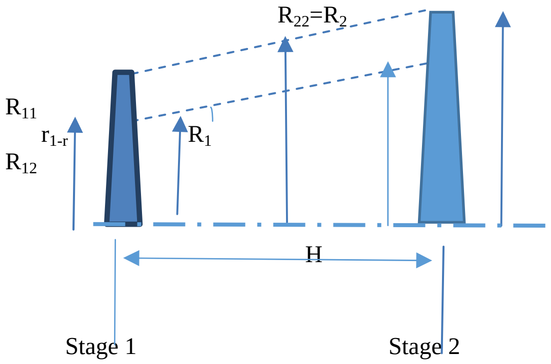

Fig. 1 Gas turbine rotor longitudinal section (two stages machine).

5. Performances: [1]

The specific consumption η of initial machine is Eq. (17):

$$\eta=\frac{Q_g.\rho}{P}.PCI \tag{17}$$

PCI is the lower calorific value of natural gas, ρ is the density of the natural gas.

The power delivered by an alternator driven by the machine according to the scale factor concept is the product of the power corresponding to the initial machine and square of the scale factor. [3]

The variation in power is Eq. (18):

$$\varepsilon_{P.1}=\frac{\lambda-1}{\lambda} \tag{18}$$

That in η is Eq. (19):

$$\varepsilon_1=\frac{\lambda^2-1}{\lambda^2} \tag{19}$$

6. Numerical Calculation

6.1 Numerical Calculation of Axial Velocity: [1]

In technique of the finished differences, we divided the flow field in a system of the point's grid; we take the differences between the values of the grid to adjacent grid of points. The punctual coordinates of the flow are:

$r=i.\Delta r$ and $z=j.\Delta z$; ($\Delta r=R_i-R_{i-1}$ and $\Delta z=z_j-z_{j-1}$).

Δr And Δz are the dimensions of the grid element, *I* and *j* are the associated index to the points gird. The value of the variable $\psi(r,z)$ can be represented by $\psi_{i,j}$, the value of ψ in the grid (i,j).

By Eq. (1) and after the estimate of the first and second derivate of ψ, we obtain Eq. (20) by taking $\Delta z=\Delta r$:

$$\psi_{i,j}=\frac{1}{4}\left(\psi_{i-1,j}\left(1+\frac{\Delta r}{2r}\right)+\psi_{i+1,j}\left(1+\frac{\Delta r}{2r}\right)+\psi_{i,j+1}+\psi_{i,j-1}\right) \tag{20}$$

For $i=2,k,m-1$ and $j=2,k,n-1$

We pose: $c=\frac{\Delta r}{2}$

The error on the stream function $\Delta\psi_{i,j}$ corresponds to the residues given by Eq. (21):

$$res_{,j}=\psi_{i,j}-\frac{1}{4}\left(\psi_{i-1,j}\left(1+\frac{c}{n}\right)-\psi_{i+1,j}\left(1+\frac{c}{n}\right)+\psi_{i,j+1}+\psi_{i,j-1}\right) \tag{21}$$

For: $i=2,k,m-1$ And $j=2,k,n-1$

The approximate value of maximum axial velocity according to the stream function is Eq. (22):

$$V_{z.\max}=\frac{1}{R_1}.\frac{\psi_{2,j}+\psi_{1,j}}{2\Delta r} \tag{22}$$

The errors on axial velocity can be obtained by Eq. (23):

$$\Delta V_z=\frac{1}{r\Delta r}(\Delta\psi_{i+1,j}+\Delta\psi_{i-1,j}) \tag{23}$$

where $\Delta\psi_{i,j}$ is given by the expression Eq. (20).

To estimate the errors on axial velocity, we use Eq. (24):

$$\Delta V_z=\frac{Max(res_{i,j})}{2r\Delta r} \tag{24}$$

6.2 Limits Conditions: [1]

Dirichlet conditions:

On (1) Hub: $r=R_1$, $i=1\Rightarrow\psi(z,r)=\psi_1=\frac{Q}{2\pi}$,

On (2) Casing: $r=R_2$, $i=m\Rightarrow\psi(z,r)=\psi_2=0$,

Neumann conditions:

On (3) In the upstream: $z=0$, $j=1\Rightarrow\frac{\partial\psi}{\partial z}=0$

On (4) In the downstream: $z=H$, $j=n\Rightarrow\frac{\partial\psi}{\partial z}=0$

6.3 Equation Resolution: [4]

To resolve the equation of finished difference (20), the iterative method is used. Let us suppose the rectangular sector on the plan represented by Fig. 2, on which the flow is divided into grid of 16 points. The values of ψ are known on 12 points to the borders of grid, and the values inside the grid indicated in black are unknown. For these internal points we use Eq. (25):

$$\begin{aligned}
\psi_{2,2} &= \frac{1}{4}\left(\psi_{1,2}{}^{B}\left(1+\frac{c}{r_2}\right)+\psi_{3,2}\left(1+\frac{c}{r_2}\right)+\psi_{2,3}+\psi_{2,1}{}^{B}\right)\\
\psi_{3,2} &= \frac{1}{4}\left(\psi_{2,2}\left(1+\frac{c}{r_3}\right)+\psi_{4,2}{}^{B}\left(1+\frac{c}{r_3}\right)+\psi_{3,3}+\psi_{3,1}{}^{B}\right)\\
\psi_{2,3} &= \frac{1}{4}\left(\psi_{1,3}{}^{B}\left(1+\frac{c}{r_2}\right)+\psi_{3,3}\left(1+\frac{c}{r_2}\right)+\psi_{2,4}{}^{B}+\psi_{2,2}\right)\\
\psi_{3,3} &= \frac{1}{4}\left(\psi_{2,3}\left(1+\frac{c}{r_3}\right)+\psi_{4,3}{}^{B}\left(1+\frac{c}{r_3}\right)+\psi_{3,4}{}^{B}+\psi_{3,2}\right)
\end{aligned} \tag{25}$$

On the previous equations, the known values are indicated by the indication B. The Eq. (25) represents a system of equation to 4 unknowns and it is soluble. They represent four equations with four unknowns. One of resolution techniques is the iteration method.

6.4 Scale Factor Calculation, Performances Gains

Coefficient value λ cf. (3.2) can be calculated by Eq. (26):

$$Q_a = Q_{\max} = V_{z\max}.S = \lambda_1 Q \tag{26}$$

With the new section which verifies Eq. (27):

$$\frac{R_2}{R_1} = 2 \tag{27}$$

With:

$S = \pi(R_2{}^2 - R_1{}^2/4)$ And $\lambda_1 = \xi_1{}^2$

The first coefficient $\lambda_1 = \xi_1{}^2$

Q_a corresponds to flow rate of a new machine and Q of the initial machine.

$V_{z.\max}$ is given by Eq. (22).

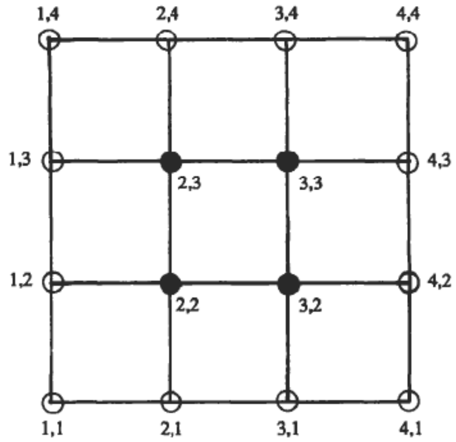

Fig. 2 Point's grid on rectangular region.

We can easily calculate the value of ξ (of λ) using the value of rate flow Q_1, the specific consumption gain is like Eq. (28):

$$\varepsilon = 1 - \frac{1}{\lambda^2} \tag{28}$$

For Power Eq. (29):

$$\varepsilon_P = 1 - \frac{1}{\lambda} \tag{29}$$

7. Conclusions

The use of the scale factor notion and a result of the calculations carried out on our present study allow us to determine optimum design methods for gas turbines. From this study, we will have gain on dimensions and performances. We hope that our modest work will be the subject of scientific research in the future.

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Appendix

Table N02

Performances Gains

Data:

- Scale factor $\xi = 2$
- Coefficient $\lambda = 4$
- Gain in specific consumption

$$\varepsilon = 1 - \frac{1}{\lambda^2} = 93.75\%$$

- Gain in power $\varepsilon_P = 1 - \frac{1}{\lambda} = 0.75 = 75\%$

Essays Data:

- Initial machine specific consumption:

$\eta = 3362.9 kcal / kwh$

- Initial machine Electrical power:

$P = 19751.1 Kw$

Gains:

- New machine power: $P_1 = 24668.8 Kw$
- New machine specific consumption:

$$\eta_1 = (\eta - \varepsilon.\eta) = 315272\ Kcal\ / Kwh$$

- Gain in specific consumption:

$\Delta\eta = 210,18 Kcal\ / Kwh$

- Difference in natural gas rate flow:

$\Delta Q_g = \Delta\eta x(P_1 - P)) = 1037.805.th$

- Gain in power: $\Delta P = 4937.7 Kw$
-