

Serpentine Design on Forest Roads by the Internal Circular Curve Method: A Case Study in Serbia

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Abstract: The paper provides an overview of geometric solutions of marking all types of serpentine by the method of internal circular curve in designing forest roads directly in the field. The main objective of presenting this original method for marking all serpentine types in one place is to show similarities and differences in marking different types of serpentine, and identify opportunities for further research of this type. The method is based on the establishment of the minimum number of elements necessary to mark the serpentine on the forest roads and other budget elements and their design in the field. By using this method, construction errors or the number of attempts of serpentine marking are reduced, which increases the effects of design compared to the ones reached by the previous method of marking the serpentine on forest roads.

Key words: Forest roads, symmetric serpentine, asymmetric serpentine, full serpentine, half serpentine, internal circular curve method, Serbia.

1. Introduction

The basic geometric elements of serpentines (complex or combined curves) on forest roads are: base curve, input and/or output curves and directions. Directions connect input and output curves with the base curve. According to the relative disposition of the geometric elements, serpentines can be divided into 1st order serpentines—symmetric and asymmetric, and 2nd order serpentines—full and half serpentines [1].

In *symmetric serpentines* (Fig. 1), the center of the base curve (O) is located on the centerline of refraction angle β_n , while input and output curves have the same direction (left or right) and equal radii $R_u = R_i$.

In *asymmetric serpentines* (Fig. 2), the center of the base curve (O) is not on the centerline of refraction angle β_n , but input and output curves have the same direction (left or right) and equal $R_u = R_i$ or different radii $R_u \neq R_i$.

In *full serpentines* (Fig. 3), the center of the base curve (O) is not on the centerline of refraction angle β_n ,

and input and output curves have different directions (left and right) and different radii $R_u \neq R_i$.

In *half serpentines* (Fig. 4), the center of the base curve (O) is not on the centerline of refraction angle β_n , but on the line perpendicular to one of the tangents, so there are no input or output curves. The half serpentine on forest roads is the most common mark on the field [2].

The selection of the serpentine type in forest road construction depends on the characteristics of the route section suitable for its location, i.e. on its position relative to the vertex and the center of the serpentine base curve. The criteria for the selection of sites suitable for the location of a serpentine and determining the position of its center depend on local terrain conditions. The construction design of the serpentine should enable the use of the largest possible radius of the base curve with as little earthworks and artificial terrain protection as possible. These road sites include terrains with a moderate slope, natural plateaus, terrains that require smaller volume of rock works, sites with better drainage, stable terrains, etc [3].

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Fig. 4 Half serpentine.

1.1 The Problem of Serpentine Design

The design of forest road serpentines in Serbia is done while marking the route, directly in the field by the so-called *simplified method of forest road design* [4]. The theory and practice of forest road engineering in Serbia have given a positive opinion on this method of forest road design [5], because it is simpler in comparison to the public transport road design, and its use ensures sufficient accuracy needed for the construction of forest roads [6]. The current method of forest road design should be improved [7] by simplifying certain operations of data surveying in the field. It would further facilitate the designing process and increase its efficiency.

The principles and the geometry of the existing method are thoroughly explained and presented in figures for symmetric [8], asymmetric [9], half serpentine [10] and full serpentine [11] in order to analyze them and identify their methodological shortcomings with an aim to investigate, find and offer a new method of marking forest road serpentines. Namely, the author's experience in the design of serpentine [12-14] shows that they are, in terms of design, very complicated elements of road routes, which require not only solid technical knowledge but extensive experience in route marking and selecting the geometry of serpentine, as well as good spatial orientation in the forest [15]. In addition, it has been observed that the serpentine marking directly in the field by the so-called simplified method of forest road design has organizational shortcomings which often result in the failure to mark the serpentine at the first attempt so that the entire process has to be repeated several times [16]. This is especially true with inexperienced designers because the design of forest roads is a highly subjective and intuitive job.

In the design of serpentines directly in the field by the simplified method, we first select the center of the base curve, then mark the entire circle of the base curve and finally position the beginning point (PK_o) and the end point (KK_o) of the base curve on the circle. These points are then used to design the direction of auxiliary curves and determine the position of their vertices. It is only then that we can calculate the values of the geometric elements necessary for the marking of serpentines. The biggest problem in the process of serpentine design is to meet the requirement of the minimum radius of the auxiliary curves that has been set in advance as part of the project task. If the designed serpentine elements do not meet the requirement of the minimum radius, another beginning point (PK_o) and end point (KK_o) of the base curve have to be selected. If several attempts of marking fail, the whole procedure must be repeated from the point of selecting the position of the center or the radius of the base curve, or even the whole serpentine has to be repositioned.

1.2 The Idea of New Serpentine Design

This old method of serpentine marking is complex and it is not effective. Therefore, we have come to an idea to explore a new method of serpentine design which would overcome the observed problems. The resulting methodology solution is presented as a new method of marking symmetric [8], asymmetric [9], full [11] and half serpentines [10] on forest roads, called the internal circular curve method.

The main idea of the proposed method is to make the field work more effective and to optimize the design of serpentine. The effectiveness is achieved by changing the sequence of the marking operations and by surveying the minimum amount of data necessary to calculate the serpentine elements. The possibility of errors and the time needed for the design are thus reduced. As its effectiveness increases, the serpentine design becomes more optimized.

The paper provides an overview of the methodology of marking all types of serpentine on forest roads by the method of internal circular curve. The main goal is to make a comparative analysis of the required length and angle measurements as well as the calculations of the elements necessary for the marking and to present the process of marking all types of serpentine in the field by showing the similarities and differences in marking certain types of

serpentine. In addition, previous studies related to this issue have been thus systematized and presented to the relevant scientific and professional community [17].

2. Material and Method

The design and marking of all types of serpentine by the method of internal circular curve was primarily performed by using the method of comparative analysis. Apart from it, we used the modeling method, by applying mathematical, geometric and trigonometric relations and visualization.

The marking out of the serpentines by the internal circular curve method is based on the necessary amount of measured data and the calculation of the necessary elements for the serpentine marking. The data for serpentine marking by this method are obtained on the basis of the calculations related to the "interior part of the serpentine base curve". Namely, beginning point (PKo) and end point (KKo) of the serpentine base curve divide the base circle into two circular arcs. One of them is an integrate part of the serpentine, being the circumference of the base curve. The other curve is on the interior side of the circle in relation to the whole serpentine (this part of the circle is in all figures given in this paper marked with a dotted line). If we set the tangents to the circular curve that is on the interior side of the circle at beginning (PK_o) and end (KK_o) points of the base curve and intersect them we obtain vertex (T_0) of the interior curve with refraction angle β_0 .

3. Results and Discussion

For the successful application of the internal circular curve method, there is one basic condition that has to be satisfied in the field: refraction angle β_n of the route directions should be measurable, i.e. the position of vertex (T_n) must be determined in the field.

3.1 Results

As with the simplified method of laying out forest roads, the first step is to determine the directions of the route intersecting in vertex (T_n) using at least three poles. Pole \bigcirc is set on the serpentine vertex (T_n) , and in the direction of the previous (T_{n-1}) and the subsequent vertex (T_{n+1}) poles \bigcirc and \bigcirc are aligned.

Marking all types of serpentine in the field by the method of internal circular curve includes the following stages:

1. From the serpentine vertex (T_n) in the direction of the previous (T_{n-1}) and the following vertex (T_{n+1}) measure pre-selected lengths b_p , b_u and b_i and with poles ④ and ⑤ mark the vertices of auxiliary curves (T_p) . In symmetric serpentine (Fig. 5) lengths b_p are equal, while asymmetric (Fig. 6) and full (Fig. 7) serpentine have different lengths b_u and b_i . In half serpentine (Fig. 8), one vertex of auxiliary curve (T_p) is marked with pole ④ and length b_p is measured.

2. From the vertex of auxiliary (T_p) , input (T_u) and output (T_i) curves, directions of the tangents to the future base circle are marked and the turning angles of



Fig. 5 Marking of symmetric serpentine by internal circular curve method.



Fig. 6 Marking of asymmetric serpentine by internal circular curve method.



Fig. 7 Marking of the full serpentine by internal circular curve method.



Fig. 8 Marking of half serpentine by internal circular curve method.

input α_u and output α_i curves are measured for asymmetric (Fig. 6) and full (Fig. 7) serpentine. In symmetric serpentine (Fig. 5), turning angles α_p of auxiliary curves are equal, so only one of them is measured. The same applies to half serpentine which has only one turning angle α_p of the auxiliary curve (Fig. 8).

3. Calculation of geometric elements for serpentine marking

The measured lengths b_p , b_u , and b_i and angles β_n , α_p , α_u and α_i are used to calculate the elements of serpentine marking:

• Calculating the length of inner curve tangent t_0 (m)

$$t_{\rm o} = R_{\rm o} \cdot ctg \,\frac{\beta_{\rm o}}{2} \tag{1}$$

The length of radius R_0 of the base curve is determined by the designer based on the characteristics of the serpentine site. The minimum length of radius R_0 of the serpentine base curve is 12 m [18].

Calculating refraction angle β_0 (°)

symmetric serpentine

$$\beta_{\rm o} = 2 \cdot \alpha_{\rm p} - \beta_{\rm n} \tag{2}$$

asymmetric serpentine

$$\beta_{\rm o} = (\alpha_{\rm i} + \varphi + \alpha_{\rm u} + \varepsilon) - 180^{\circ} \tag{3}$$

full serpentine

$$\beta_{\rm o} = \left(\varphi + \varepsilon \pm \alpha_{\rm i} \mp \alpha_{\rm u}\right) - 180^{\circ} \tag{4}$$

half serpentine

$$\beta_{\rm o} = \alpha_{\rm p} - \beta_{\rm n} \tag{5}$$

Angles φ (°) and ε (°) are calculated based on the law of sines in $\Delta T_u T_n T_i$ by the following formulas:

$$\sin \varphi = \frac{b_{\rm u}}{L} \sin \beta_{\rm n} \Rightarrow \varphi = \arcsin \varphi \tag{6}$$

$$\sin \varepsilon = \frac{b_{\rm i}}{L} \sin \beta_{\rm n} \Longrightarrow \varepsilon = \arcsin \varepsilon \tag{7}$$

and the length L (m) between the vertices of the input and output curves is calculated based on the law of cosines in $\Delta T_u T_n T_i$ by the following formula:

$$L = \sqrt{b_{\mathrm{u}}^{2} + b_{\mathrm{i}}^{2} - 2 \cdot b_{\mathrm{u}} \cdot b_{\mathrm{i}} \cdot \cos \beta_{\mathrm{n}}}$$
(8)

Calculating lengths $l_p(m)$, $l_u(m)$ and $l_i(m)$ needed to mark points PK_o and KK_o

symmetric and half serpentine

$$l_{\rm p} = t_{\rm o} - d_{\rm p} \tag{9}$$

asymmetric and full serpentine

$$l_{\rm u} = t_{\rm o} - d_{\rm u} \tag{10}$$

$$l_{\rm i} = t_{\rm o} - d_{\rm i} \tag{11}$$

where d_p is the length between the vertices of auxiliary curves and vertex T_o , d_u is the length between the vertex of input curve and vertex T_o and d_i is the length between the vertex of output curve and vertex T_o .

Lengths d_p (m), d_u (m) and d_i (m) are calculated based on the law of sines in $\Delta T_u T_o T_i$ by the following formulas:

symmetric serpentine

$$d_{\rm p} = L \frac{\sin \gamma}{\sin \beta_{\rm o}} \tag{12}$$

half serpentine

$$d_{\rm p} = b_{\rm p} \, \frac{\sin \beta_{\rm n}}{\sin \beta_{\rm o}} \tag{13}$$

asymmetric and full serpentine

$$d_{\rm u} = L \frac{\sin \gamma}{\sin \beta_{\rm o}} \tag{14}$$

$$d_{\rm i} = L \frac{\sin \delta}{\sin \beta_{\rm o}} \tag{15}$$

Angle γ (°), depending on the type of serpentine, is calculated by the following formulas: symmetric serpentine

$$\gamma = 180^{\circ} - \left(\varphi + \alpha_{\rm p}\right) \tag{16}$$

asymmetric and full serpentine

$$\gamma = 180^{\circ} - \left(\varphi + \alpha_{i}\right) \tag{17}$$

While angle δ (°), in the asymmetric and full serpentine is calculated by the formulas: asymmetric serpentine

$$\delta = 180^{\circ} - (\varepsilon + \alpha_{\rm u}) \tag{18}$$

full serpentine

$$\delta = 180^{\circ} - (\varepsilon - \alpha_{\rm u}) \tag{19}$$

Calculating the length of tangent of auxiliary t_p (m) input t_u (m) and output t_i (m) curves symmetric and half serpentine

$$t_{\rm p} = l_{\rm p} - a_{\rm p} \tag{20}$$

asymmetric and full serpentine

$$t_{\rm u} = l_{\rm u} - a_{\rm u} \tag{21}$$

$$t_{\rm i} = l_{\rm i} - a_{\rm i} \tag{22}$$

where a_p is the length between the base and auxiliary curves, a_u is the length between the base and the input curve and a_i is the length between the base and output curve.

The minimum length of the straight section a_{\min} (m) of forest roads is equal to the double length of the transition ramps l_r (m). The length of the transition ramps on the forest roads is $l_r = 10$ m [4], so that the *min* length of the straight sections between the base and the auxiliary curve is $a_{\min} = 20$ m.

Calculating the required length of the radius for auxiliary R_p , input R_u and output R_i curves

The accuracy of the design of all types of serpentine by the method of internal circular curve is checked by calculating the length of auxiliary R_p (m), input R_u (m) and output R_i (m) curve radii, by the following formulas:

symmetric and half serpentine

$$R_{\rm p} = \frac{t_{\rm p}}{tg \frac{\alpha_{\rm p}}{2}} \tag{23}$$

asymmetric and full serpentine

$$R_{\rm u} = \frac{t_{\rm u}}{tg \,\frac{\alpha_{\rm u}}{2}} \tag{24}$$

$$R_{\rm i} = \frac{t_{\rm i}}{tg \frac{\alpha_{\rm i}}{2}} \tag{25}$$

If the calculated radii of auxiliary curves are at least two or three times greater than the radii of the base curve [4] the serpentine can be marked. Otherwise, survey of the data necessary for a repeated calculation of geometric elements should be performed.

The elements necessary for serpentine marking are calculated in the following order:

- calculating length *L* by formula 8,

- calculating angles φ and ε by formulas 6 and 7,

- calculating refraction angle β_0 by formulas 2, 3, 4 and 5,

- calculating the length of tangent t_0 by formula 1,

- calculating angles γ and δ by formulas 16, 17, 18 and 19,

calculating lengths d_p, d_u and d_i by formulas 12, 13, 14 and 15,

- calculating length $l_{\rm p}$, $l_{\rm u}$ and $l_{\rm i}$ by formulas 9, 10 and 11,

- calculating the length of tangents t_p , t_u and t_i by formulas 20, 21 and 22,

- calculating the length of radii R_p , R_u and R_i by formulas 23, 24 and 25.

4. Beginning point (PK_o) and end point (KK_o) of the base curve are marked based on the calculated lengths l_p , l_u and l_i . We measure the lengths l_p , l_u and l_i from the vertex of auxiliary (T_p), input (T_u) or output (T_i) curves in the direction of turning angles α_p of auxiliary input α_u and output α_i curves, and mark beginning (PK_o) and/or end points (KK_o) of the base curve. In the field, these points are on the symmetric (Fig. 5), asymmetric (Fig. 6) and full (Fig. 7) serpentines marked with poles (Fig. 8).

5. Perpendicular lines are constructed at beginning point (PK_o) and end point (KK_o) of the base curve to tangent PK_o-T_u and KK_o-T_i. The intersection of perpendicular lines in symmetric (Fig. 5), asymmetric (Fig. 6) and full (Fig. 7) serpentine produces central point (O) of the base curve, which is in the field marked with a peg and pole \circledast . The distance between PK_o or KK_o and center (O) of the base curve is R_o , which should equal the selected value of the radius used in formula 1 calculation.

In half serpentine the length of radius R_0 of the base curve is added to the normal line in the direction of tangent line PK_0-T_p or KK_0-T_p and center (O) of the base curve is determined. The center of the base curve is marked with a peg and pole (6) (Fig. 8). From the center point (O) of the base curve a normal line is constructed in the opposite direction of the route (whose length b_p has not been measured) and beginning point (PK_o) or end point (KK_o) of the base curve are marked. This point is in the field marked with pole \bigcirc . The accuracy of the calculation and marking can be tested by checking whether the length of the normal from center point (O) to start point (PK_o) or end point (KK_o) of the base curve equals the pre-selected length of radius R_o of the base curve.

6. Directions of radius R_0 of base curve O-PK₀ and O-KK₀ make central angle α_0 . When the centerline of this angle is marked in the field and the length of radius R_0 of the base curve added along it from center (O) of the base curve, we obtain mid-point (SK₀) of the base curve. This point is on the symmetric (Fig. 5), asymmetric (Fig. 6) and full (Fig. 7) serpentine marked with pole ⁽¹⁾, and this point is on the half serpentine marked with pole ⁽²⁾ (Fig. 8). By observing its position in the field and the position of other points marked on the serpentine: center (O), beginning point (PK₀) and end point (KK₀) of the base curve, we can observe the position of the whole serpentine in the space.

3.2 Discussion

The internal circular curve method uses the minimum amount of necessary pre-required data to calculate the values of other elements necessary to mark all types of serpentine. The marking out process begins by determining the position of the vertices of input (T_u) and output (T_i) curves, as well as input and output directions, taking into account the lengths of auxiliary curve tangents and straight sections. In this way, it is possible to mark a curve of specific base R_o , input R_u and output R_i radius. When we mark beginning point (PK_o), end point (KK_o) and mid-point (SK_o) of the base curve, we can see the possibility of marking the entire serpentine. This practice can reduce errors in determining the beginning and end points of the base curve and it can also decrease the amount of

work, because we do not mark the entire base curve, but only its main points which are then used to perceive the whole serpentine.

The internal circular curve method is an innovation based on an original idea that originated from the attempts to overcome the perceived problems with serpentine marking on forest roads. The method is named internal circular curve because it is used for the calculation of the basic elements of the inner circle of the serpentine curve and it is similar for all types of serpentine.

The proposed method of serpentine marking, presented in this paper, needs to pass various tests and checks before it is applied and verified in practice. Its application and improvement open a range of issues related to technical constraints and organizational solutions such as: research into the relations between individual elements of the serpentine marking, testing the limit values of serpentine geometric elements, comparing the time needed for serpentine marking by the old and the new method and assessing the performance. It is assumed that the application of this method can shorten, facilitate and rationalize marking forest road serpentines. However, these questions should be answered by further research.

4. Conclusions

Based on the presented method of marking all types of serpentine on forest roads we can draw the following conclusions:

The previous method of marking different types of serpentines in the design of forest roads in the field had both organizational and methodological shortcomings. Due to its shortcomings, it was impossible or very difficult to achieve a satisfactory solution at the first attempt. Therefore it was necessary to repeat the process of marking until the best solution had been reached. It increased the time necessary to mark the serpentine, reduced the level of design performance, and consequently increased the cost of the design process in the field. Taking into account the perceived problems in marking all types of serpentine on forest roads, a new method of serpentine marking called the internal circular curve method has been innovated. This method is based on the minimum amount of pre-required field data that are used to calculate the values of geometric elements necessary for marking the serpentine and then to mark them in the field. This practice can reduce design errors and decrease the amount of work because the serpentine geometric elements are calculated before they are marked in the field, without marking the entire base curve, but only its main points, based on which the entire serpentine can be marked.

The internal circular curve method of serpentine marking uses the elements of the inner circle of the serpentine base curve which makes it similar for all types of serpentine. However, the geometric construction of each type of serpentine is different, so the amount of data to be surveyed is different, as well as the method of calculation. However, the procedure and the sequence of marking do not differ significantly. When surveying data for symmetric serpentine, 3 data items are recorded, and 5 elements are calculated to obtain the marking elements. In asymmetric and full serpentine 5 data items are recorded and 13 elements calculated, while half serpentine requires 3 data items and 4 elements.

4. The use of the internal circular curve method for marking all types of serpentine on forest roads requires verification and application in forest engineering practice, but it opens a series of questions related to further research and improvement of this method of designing forest road serpentines.

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